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Unsteady laminar hydromagnetic flow and heat transfer in porous channels with temperature-dependent properties

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Abstract *The problem of unsteady laminar fully-developed flow and heat transfer of an electrically-conducting and heat-generating or absorbing fluid with variable properties through porous channels in the presence of uniform magnetic and electric fields is formulated. The general governing equations which include such effects as magnetic field, electric field, porous medium inertia and heat generation or absorption effects are non-dimensionalized and solved numerically by the implicit finite-difference methodology. A representative set of numerical results for the transient and steady-state velocity and temperature profiles, the skin-friction coefficients at both the upper and lower walls of the channel as well as the heat transfer coefficient at the lower wall are presented graphically and discussed. Favorable comparisons with previously published work are performed and the results are in excellent agreement. A comprehensive parametric study is performed to show the effects of the Hartmann number, electric field parameter, inverse Darcy number, inertia parameter, heat generation or absorption coefficient, exponents of variable properties and the Eckert number on the solutions.*

Nomenclature

a = exponent of dimensionless viscosity coefficient
b = exponent of dimensionless electrical conductivity coefficient
B_0 = magnetic induction
c = exponent of dimensionless porous medium thermal conductivity coefficient
c_p = specific heat at constant pressure
C_l = skin-friction coefficient at the lower wall of the channel
C_u = skin-friction coefficient at the upper wall of the channel
D_i = inverse Darcy number, $h^2\varepsilon/K$
E = electric field
Ec = Eckert number, $\nu_0/(hc_p(T_2 - T_1))$
F = dimensionless fluid velocity, Vh/ν_0
G = dimensionless pressure gradient, $-h^3/(\rho\nu_0^2)\partial P/\partial z$
h = channel half width
Ha = Hartmann number, $\sqrt{\sigma_0/\mu_0}B_0h$
I = porous medium inertia coefficient
k_e = effective thermal conductivity of the porous medium
K = permeability of the porous medium
P = pressure
Pr = Prandtl number, $\mu_0 c_p/k_{e0}$
Q_0 = heat generation or absorption coefficient
R_e = electric field parameter, $Eh / (\nu_0 B_0)$
Nu_l = heat transfer coefficient at the lower wall of the channel
Nu_u = heat transfer coefficient at the upper wall of the channel
y = distance in the normal direction
t = time
T = temperature
T_0 = mean temperature, $(T_1 + T_2)/2$
T_1 = temperature at the lower wall of the channel

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T_2 = temperature at the upper wall of the channel
V = velocity
z = distance in the axial direction

Greek symbols
ε = porosity
η = dimensionless radial distance, y/h
ϕ = dimensionless heat generation or absorption coefficient
Γ = dimensionless porous medium inertia parameter, $I\varepsilon^2 h$
μ = dynamic viscosity of the fluid
ν_0 = kinematic viscosity of the fluid, μ_0/ρ
θ = dimensionless fluid temperature, $(T-T_1)/ T_2 -T_1)$
ρ = fluid density
σ = fluid electrical conductivity
τ = dimensionless time, $\nu_0 t/h^2$

Introduction

The flow and heat transfer of fluids in porous media in the presence or absence of magnetic field and heat generation or absorption effects are of special technical significance due to their frequent occurrence in many industrial applications such as geothermal systems, cooling of nuclear reactors, thermal insulation, petroleum reservoirs, coal and grain storage, solid matrix heat exchangers, nuclear waste disposal and others. Most early studies on flow in porous media have employed the Darcy law which is an empirical relation between the pressure drop across the porous medium and the viscous and gravitational forces. More recently, Vafai and Tien (1981, 1982) and Plumb and Huenefeld (1981) have studied the effects of the presence of a boundary adjacent to a porous medium (Brinkman-extension of Darcy law) and the inertial effects (Forchheimer-extension of Darcy law) which would occur when the velocity in the porous medium becomes high (that is, the Reynolds number based on the mean pore size is greater than unity). In this case, the pressure gradient across the porous medium is a quadratic relation with the volume-averaged velocity.

There has been considerable published work dealing with steady and transient flow and heat transfer problems from different geometries embedded in porous or non-porous media in the presence or absence of a magnetic field. Raptis and Kafoussias (1982) have considered heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Kaviany (1985) has presented an analytical solution for laminar flow through a porous channel bounded by isothermal parallel plates based on the Brinkman-extended Darcy law. Vafai and Kim (1989) have reported a closed-form solution for forced convection flow in a porous channel with isoflux boundaries using the Brinkman-Forchheimer-extended Darcy law. Poulikakos and Renken (1987) and Renken and Poulikakos (1988) have studied, respectively, numerically and experimentally forced convection in a channel filled with a porous medium using the Brinkman-Forchheimer-extended Darcy model with variable porosity and allowing for viscosity variations. Kafoussias (1992) has reported on hydromagnetic free convection flow through a non-homogeneous porous medium over an isothermal cone surface. Kou and Lu (1993) have analyzed combined boundary and inertia effects for fully-developed mixed convection in a vertical channel embedded in porous media. Takhar and

Ram (1994) have studied magnetohydrodynamics free convection flow of water at 4°C through a porous medium. Cheng *et al*. (1990) have discussed flow reversal and heat transfer of fully-developed mixed convection in vertical channel. Garandet *et al*. (1992) have investigated buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Neild *et al*. (1996) have studied forced convection in a fluid-saturated porous channel with isothermal or isoflux boundaries. Gulab and Mishra (1977) have examined unsteady flow through magnetohydrodynamic porous media. Sacheti *et al*. (1994) have reported an exact solution for unsteady magnetohydrodynamics free convection flow with constant heat flux. Chamkha (1996) has investigated unsteady hydromagnetic natural convection in a fluid-saturated porous medium channel.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids (see, for instance, Vajravelu and Nayfeh, 1992 and Vajravelu and Hadjinicolaou, 1993). Also, the design of placing many electronic circuits into one small chip and more chips into a package, which can be modelled as a porous medium, results in high volumetric heat generation in the electronic equipment (see, Kou and Lu, 1993). Recently, Chamkha (1997) has analyzed the problem of hydromagnetic non-Darcy mixed convection flow through a porous medium channel in the presence of heat generation effects.

Almost all of the above work has assumed that the fluid properties are constant. In many high temperature applications such as nuclear reactors and Magnetohydrodynamic (MHD) generators, the fluid properties are highly dependent on temperature. In these situations, the variations of properties with temperature must be taken into account in their analysis. Thomson and Bopp (1970) have shown that the consideration of variable fluid properties in MHD channels results in flow rates and friction factors which differ markedly from constant property solutions. The studies by Fillippov (1970), Lohrasbi (1980) and Setayeshpour (1979) have shown that departures from constant transport properties have significant effect on the heat transfer characteristics of MHD channel flows (Setayeshpour, 1984). In addition to that, the presence of a magnetic field such as in MHD power generators, plasma MHD accelerators, and nuclear reactors can have a great effect on the performance of such systems. For example, in an MHD generator, coal is fed into a combustor which is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas flowing through the MHD channel consists of a condensible vapor (slag) and a non-condensible gas. Both the slag and the non-condensible gas are electrically conducting (see, Lohrasbi, 1980). The presence of a magnetic field in this high temperature application significantly influences the flow and heat transfer characteristics in the MHD channel. By considering the MHD generator start-up condition and the dependence of properties on temperature, the problem reduces to unsteady variable properties flow and heat transfer in an MHD channel.

Klemp *et al.* (1990) have investigated the effects of temperature-dependent viscosity on the flow in the entrance region of a channel. Attia and Kotb (1996) have considered steady MHD flow and heat transfer between two parallel plates with temperature-dependent viscosity. Recently, Attia (1999) has extended the work of Attia and Kotb (1996) to the unsteady case. The aim of this paper is to consider transient, fully-developed, electromagnetic flow and heat transfer with temperature-dependent properties in a channel embedded in a non-Darcian porous medium (called herein a porous medium channel) in the presence of heat generation or absorption effects. The walls of the channel are assumed to be isothermal and are kept at different temperatures.

Governing equations

Consider unsteady, laminar, hydromagnetic, fully developed, plane flow of an electrically-conducting and heat-generating or absorbing fluid in a horizontal channel filled with a uniform porous medium due to the action of a constant pressure gradient. A uniform transverse magnetic field is applied normal to the flow direction (see Figure 1). While the fluid is assumed to be electrically conducting, the channel walls are assumed to be electrically non-conducting or insulating. In addition, a uniform electric field is assumed to exist and the magnetic Reynolds number is assumed to be small so that the induced magnetic field and the Hall effect of magnetohydrodynamics are negligible. Both the fluid and the porous medium are assumed to be in thermal equilibrium. The governing equations for this study are based on the balance laws of mass, linear momentum and energy. Under these assumptions, the governing equations for this problem can be written in dimensionless form as

$$\frac{\partial \mathrm{F}}{\partial \tau} = \mu(\theta)\frac{\partial^2 \mathrm{F}}{\partial \eta^2} + \frac{\partial \mu}{\partial \eta}\frac{\partial \mathrm{F}}{\partial \eta} - \sigma(\theta)\mathrm{Ha}^2(\mathrm{F}+\mathrm{R_e}) - \mu(\theta)\mathrm{DiF} - \Gamma \mathrm{F}^2 + \mathrm{G} \quad (1)$$

$$\begin{aligned}\frac{\partial \theta}{\partial \tau} = \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial \eta}(\mathrm{k_e}(\theta)\ \frac{\partial \theta}{\partial \eta}) + \mathrm{Ec}\mu(\theta)(\frac{\partial \mathrm{F}}{\partial \eta})^2 \\ + \mathrm{Ec\,Ha}^2\sigma(\theta)(\mathrm{F}+\mathrm{R_e})^2 + \phi(\theta - 0.5)\end{aligned} \quad (2)$$

where $\mathrm{Ha} = \sqrt{\sigma_0/\mu_0}\,\mathrm{B_0 h}$, $\mathrm{R_e} = \mathrm{Eh}/(\nu_0 \mathrm{B_0})$, $\mathrm{D_i} = \mathrm{h}^2\varepsilon/\mathrm{K}$, $\Gamma = \mathrm{I}\varepsilon^2\mathrm{h}$, $\mathrm{Pr} = \mu_0 \mathrm{c_p}/\mathrm{k_{e0}}$, $\mathrm{Ec} = \nu_0/(\mathrm{hc_p}(\mathrm{T_2} - \mathrm{T_1}))$, and $\phi = \mathrm{Q_0 h^2}/(\mu_0 \mathrm{c_p})$ are the Hartmann number, electric field parameter, inverse Darcy number, dimensionless porous

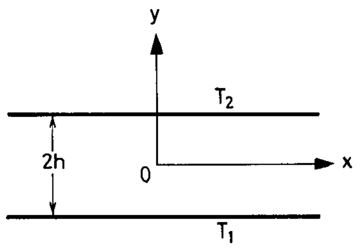

Figure 1. Problem schematics and coordinate system

medium inertia parameter, Prandtl number, Eckert number, and the heat generation or absorption coefficient, respectively. The parameters $\mu(\theta)$, $\sigma(\theta)$ and $k_e(\theta)$ are the dimensionless viscosity, electrical conductivity, and effective thermal conductivity coefficients, respectively. For the special case of constant properties, the coefficients μ, σ and k_e are all equal to unity. It should be mentioned that the temperature-dependent heat generation/absorption term (last term of equation (2)) was based on the difference between the flow temperature and the mean temperature $T_0 = (T_1 + T_2)/2$. Also, the effective viscosity was assumed equal to that of the fluid.

In high temperature applications such as MHD generators, the viscosity and electrical and thermal conductivities are highly dependent on temperature. For MHD channel flow and heat transfer, Rosa (1971) and Doss *et al.* (1981) have suggested general exponential functions for the viscosity and electrical conductivity. Attia (1999) has used a slightly different exponential form for the viscosity coefficient. In the present work, the following general coefficients are employed.

$$\mu(\theta) = \exp(-a\theta), \sigma(\theta) = \exp(b\theta), k_e(\theta) = \exp(c\theta) \tag{3}$$

where the exponents "a", "b" and "c" are constants. The viscosity coefficient is the same as employed by Attia (1999).

In obtaining equations (1) and (2), the following parameters are employed to make the governing equations dimensionless:

$$y = h\eta, \quad t = h^2\tau/\nu_0, \quad \frac{\partial P}{\partial z} = -\rho\nu_0^2/h^3 G, \mu(T) = \mu_0\mu(\theta), \sigma(T) = \sigma_0\sigma(\theta)$$
$$k_e(T) = k_{e0}k_e(\theta), V(r,t) = \nu_0/hF(\eta,\tau), \quad T(r,t) = T_1 + (T_2 - T_1)\theta(\eta,\tau) \tag{4}$$

where the meaning of these parameters is given in the nomenclature section.

Equations (1) and (2) are supplemented by the following dimensionless initial and boundary conditions:

$$\begin{aligned} &F(\eta,0) = 0, \quad F(-1,\tau) = 0, \quad F(1,\tau) = 0 \\ &\theta(\eta,0) = 0, \quad \theta(-1,\tau) = 0, \theta(1,\tau) = 1 \end{aligned} \tag{5}$$

Of special interest in the present work are the skin-friction coefficients at the upper and lower walls of the channel (C_u and C_l) and the wall heat transfer coefficients at the upper and lower walls of the channel (Nu_u and Nu_l). These physical parameters can, respectively, be defined as follows:

$$\begin{aligned} &C_u = \frac{\partial F}{\partial \eta}(1,\tau), \quad C_l = \frac{\partial F}{\partial \eta}(-1,\tau) \\ &Nu_u = -\frac{\partial \theta}{\partial \eta}(1,\tau), \quad Nu_l = -\frac{\partial \theta}{\partial \eta}(-1,\tau) \end{aligned} \tag{6}$$

Numerical results

The general physical effects of the various parameters on the solutions are illustrated through a representative set of hydrodynamic and thermal results presented below. All subsequent results are obtained numerically using the implicit and iterative finite-difference methodology discussed by Blottner (1970). Constant step sizes of 0.01 are used in the normal direction and variable step sizes in time with an initial time step of 0.001 and a growth factor of 1.0375. These values are arrived at after many numerical experimentations which were performed to assess grid independence. For example, when $\triangle_\eta$ was set to 0.001, no significant changes in the results were observed in which the error was less than 1 per cent. In addition, when constant step sizes in τ were used, an average error of less than 1.5 per cent was predicted. The solution convergence criterion was based on the difference between the current and the previous iterations and convergence was assumed achieved when this difference reached 10^{-5}. The numerical solutions were checked against those reported earlier by Attia (1999) and were found to be in excellent agreement. For more details of the numerical method, the reader is advised to read the paper by Blottner (1970).

Figures 2 and 3 present the evolution of the velocity and temperature profiles in a non-porous channel ($D_i = 0$ and $\Gamma = 0$) for both short ($R_e = 0$) and open ($R_e = -1$) circuit electric conditions, respectively. It is observed that both the fluid velocity and temperature in the channel increase as time progresses until they reach their steady-state conditions. Also, while the electric field is seen to have no effect on the temperature profiles since the Eckert number Ec is taken to be zero, it is predicted to cause the fluid velocity to increase. This is due to the fact that for an open circuit condition, the electric field produces a motive

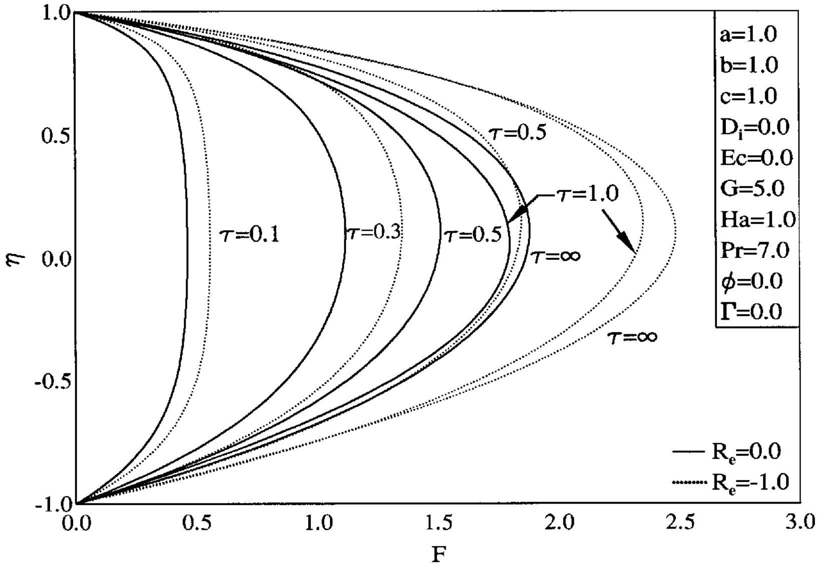

Figure 2. Temporal development of velocity profiles

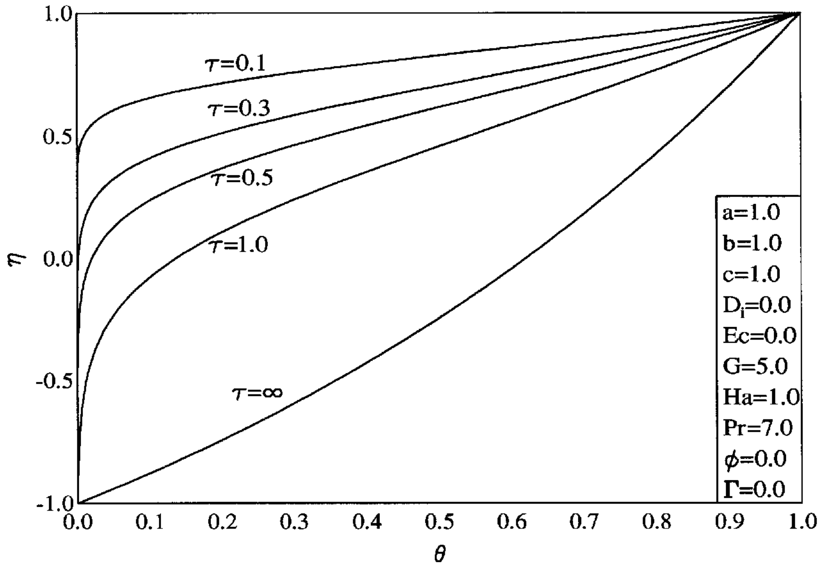

Figure 3.
Temporal development of temperature profiles

force which aids the flow. These behaviors are clearly illustrated in Figures 2 and 3.

Figures 4 and 5 illustrate the influence of the Hartmann number Ha on the steady-state velocity profiles and the time histories of the skin-friction coefficients at both the upper and lower walls of the channel (C_u and C_l), respectively. Imposition of a magnetic field normal to the flow direction gives rise to a drag-like or resistive force in the direction of flow. This force is called

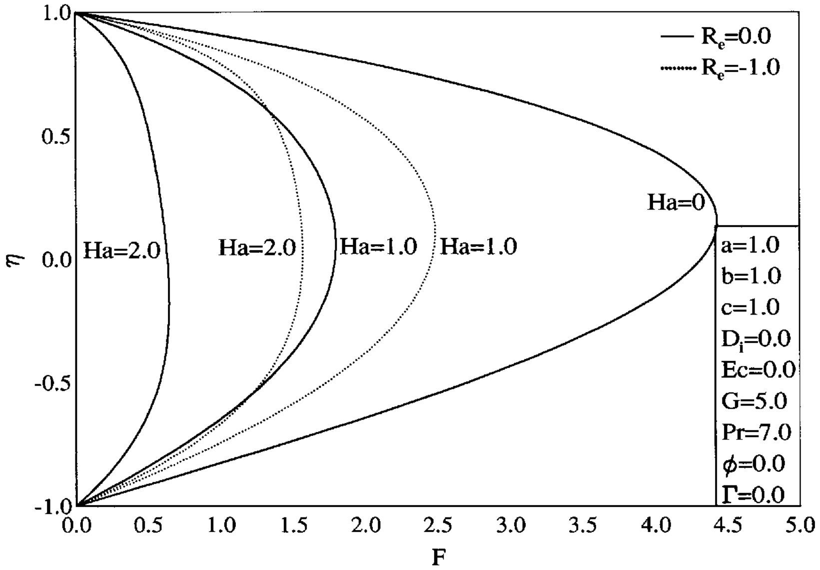

Figure 4.
Effects of Ha on steady-state velocity profiles

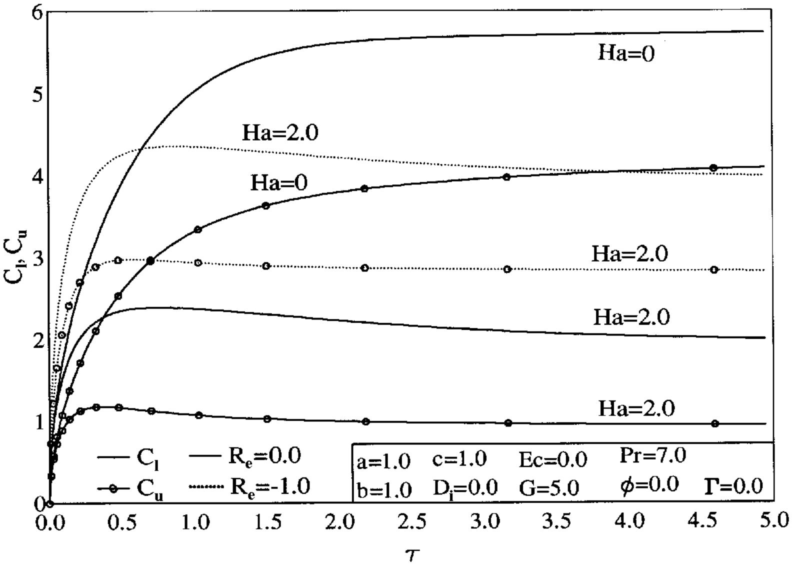

Figure 5. Effects of Ha on skin-friction coefficients

the Lorentz force and it has the tendency to slow down or suppress the movement of the fluid in the channel. In addition, the velocity gradients at both the upper and lower walls decrease. This has the direct effect of reducing the skin-friction coefficients at both walls of the channel. Furthermore, inspection of Figure 2 shows that the wall velocity gradients increase as time progresses. This causes both C_u and C_l to increase as τ increases. The increase in the fluid velocity as a result of the electric field as mentioned before is accompanied by decreases in the velocity gradients at the walls. This causes reductions in the values of both C_u and C_l. These and the previous behaviors are clearly evident in Figures 4 and 5.

Figures 6 and 7 show typical steady-state velocity profiles and time histories for the skin-friction coefficients at both the upper and lower walls of the channel (C_u and C_l), respectively for different combinations of the inverse Darcy number D_i and the porous medium inertia parameter Γ, respectively. The presence of a porous medium in the channel has the tendency to increase the resistance to flow. This causes the velocity in the channel to decrease. In addition, the presence of the porous media inertia effect ($\Gamma > 0$) provides further restriction to flow causing further reductions in the fluid velocity as depicted in Figure 6. As a result of this flow behavior, both C_u and C_l decrease as either D_i or Γ increases. The skin-friction coefficient at the lower wall is predicted to be higher than that at the upper wall.

Figures 8 and 9 depict the effects of the heat generation or absorption coefficient ϕ on the velocity and temperature profiles for both open and short circuit conditions, repectively. In the presence of heat generation (ϕ>0), increases in the value of ϕ have the tendency to increase the fluid temperature everywhere in the channel. This causes its velocity to decrease due to the

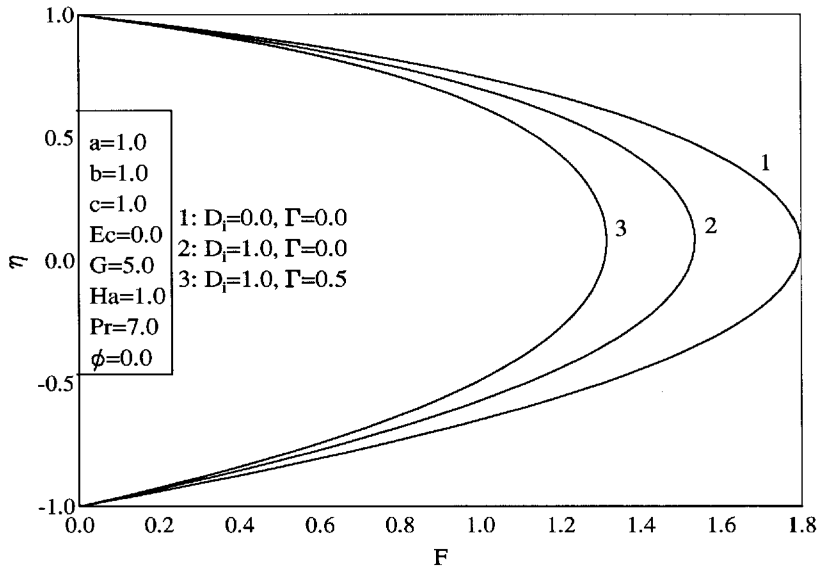

Figure 6.
Effects of D_i and Γ on steady-state velocity profiles

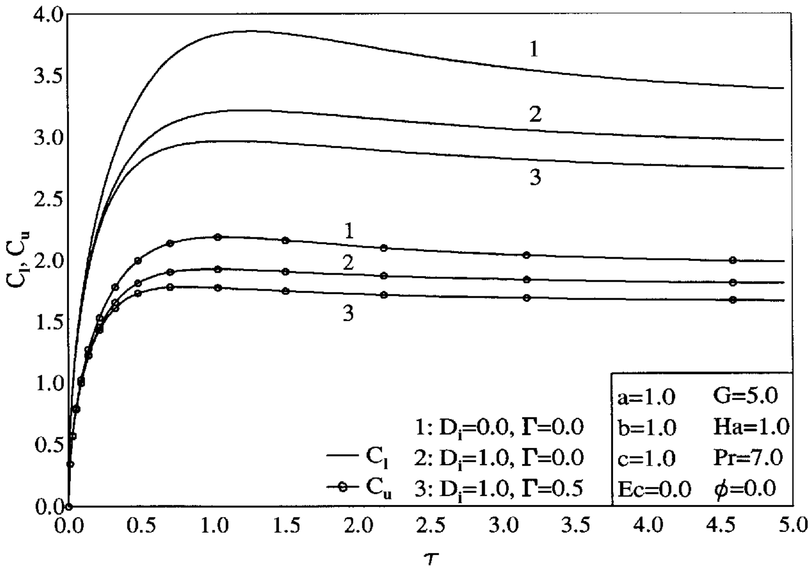

Figure 7.
Effects of D_i and Γ on skin-friction coefficients

dependence of the flow properties on temperature. However, in the presence of heat absorption ($\phi < 0$), the fluid temperature decreases in the upper part of the channel while it increases in most of its lower part. This behavior produces a net increase in the fluid velocity as shown in Figure 8.

In Figures 10 and 11, the effects of ϕ on the upper wall skin-friction coefficient C_u, the lower wall skin-friction coefficient C_l, and the lower wall heat

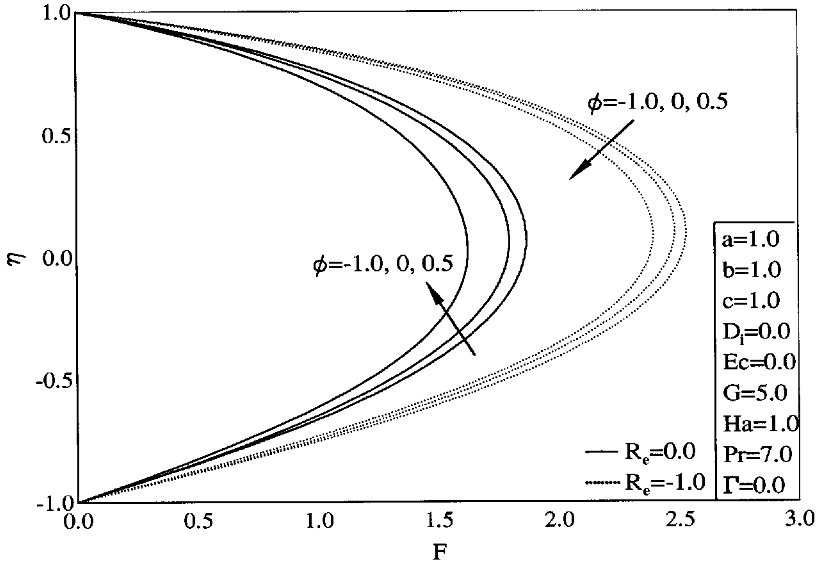

Figure 8. Effects of ϕ on steady-state velocity profiles

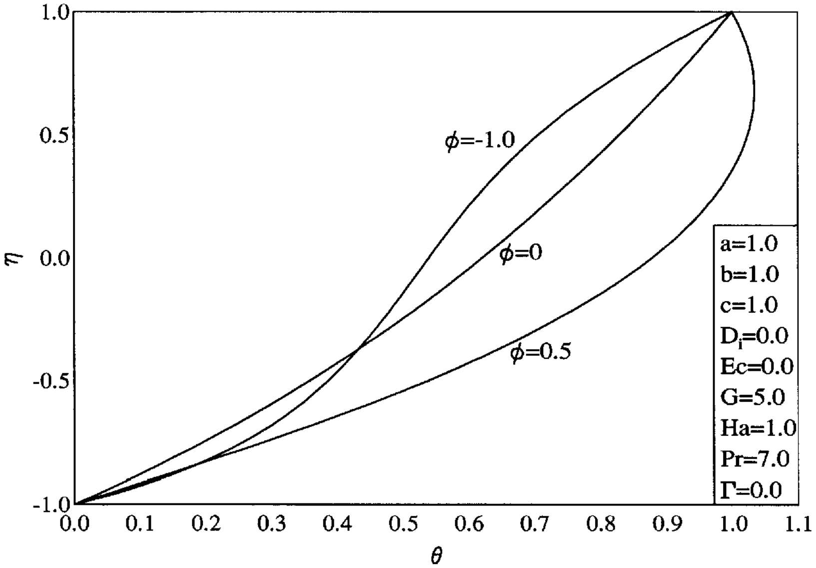

Figure 9. Effects of ϕ on steady-state temperature profiles

transfer Nu_l are shown. It is clearly observed that, for the parametric conditions used, the values of C_l increase due to heat generation while they decrease for heat absorption. Very little change in the values of C_u is observed. Again, the effect of the electric field is seen to increase the values of both C_l and C_u. The heat transfer at the lower wall of the channel is seen to increase due to heat absorption and to decrease due to heat generation during most of the time. However, at steady state, the value of Nu_l is higher for $\phi = 0.5$ than that for

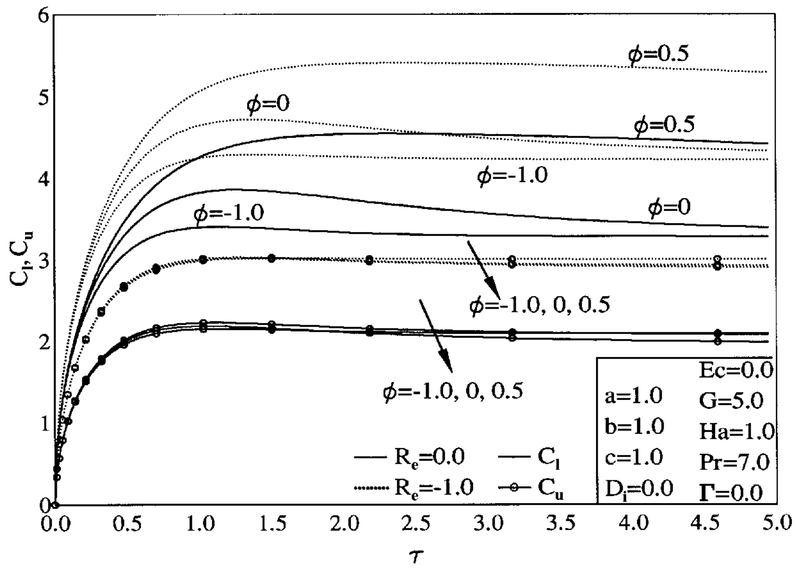

Figure 10.
Effects of ϕ on skin-friction coefficients

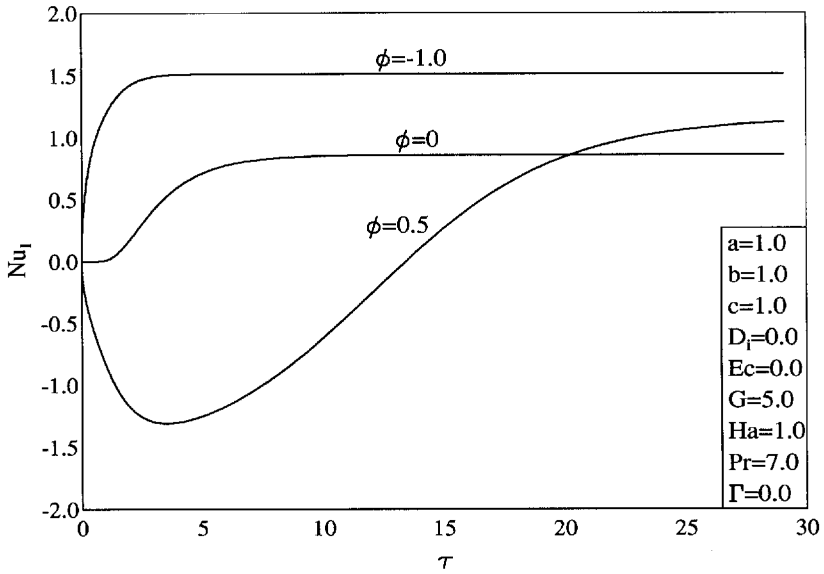

Figure 11.
Effects of ϕ on lower wall heat transfer

$\phi = 0$. This is associated with the shape of the steady-state temperature profile as shown in Figure 9.

The effects of the exponents "a" and "b" for the dimensionless viscosity and electrical conductivity coefficient on the steady-state velocity and the evolution of the skin-friction coefficients are presented in Figures 12-15, respectively. Increases in the values of "a" cause the fluid viscosity to decrease. This produces increases in the fluid velocity in the channel. As a result, the

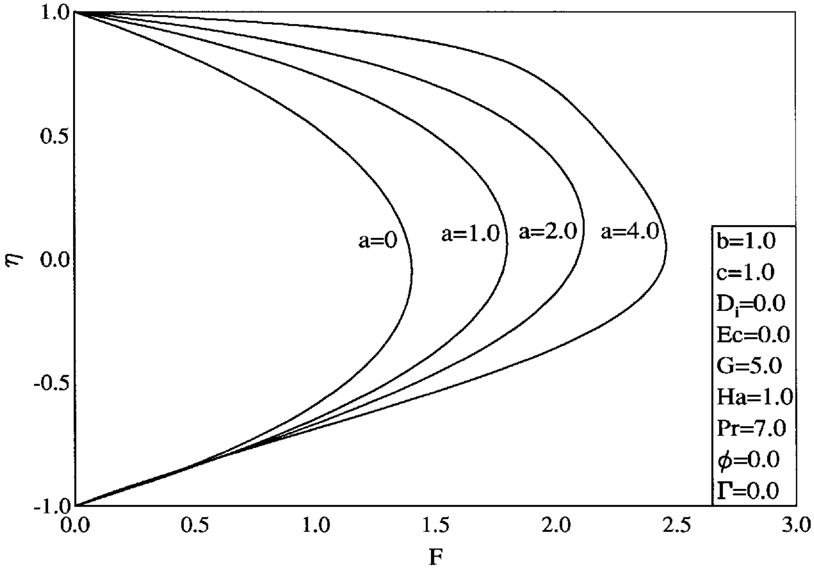

Figure 12.
Effects of a on steady-state velocity profiles

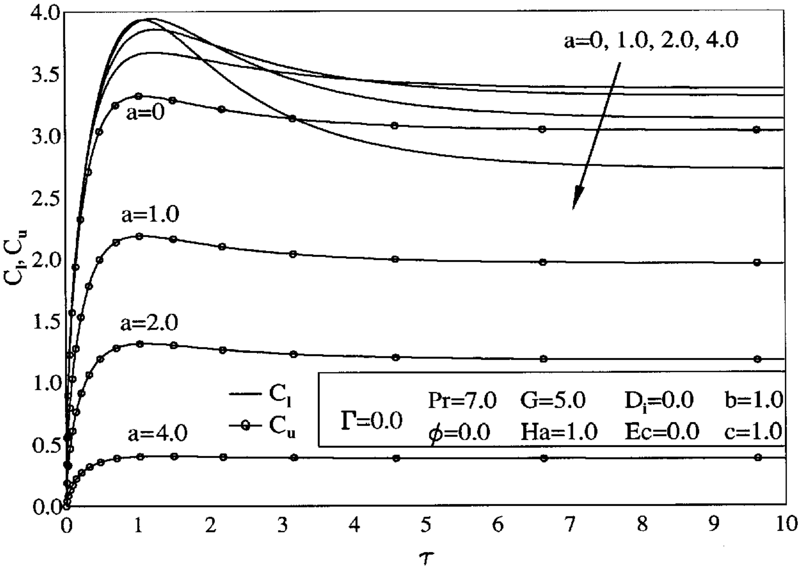

Figure 13.
Effects of an on skin-friction coefficients

steady-state values of the skin-friction coefficients decrease. However, in the initial stages of the flow, the values of C_l tend to increase as "a" increases. By increasing the value of "b", an interesting behavior in the fluid velocity is predicted whereby it decreases and more flow occurs in the lower part of the channel as is obvious from Figure 14. This is associated with the fact that the resistance due to the magnetic field increases as "b" increases. In addition,

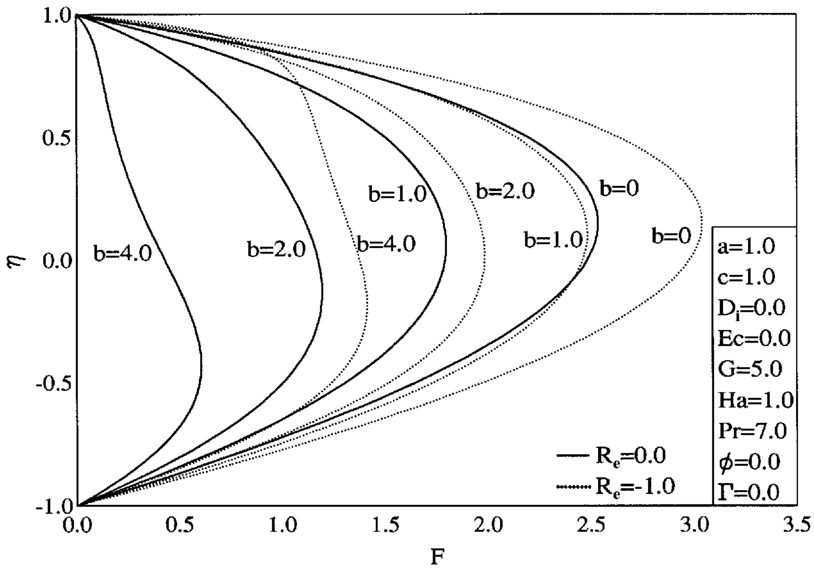

Figure 14.
Effects of b on steady-state velocity profiles

the values of both C_u and C_l are predicted to decrease as “b” increases. It is interesting to observe from Figure 15 the formation of a distinctive peak in the values of both C_u and C_l as “b” increases beyond the value of 2. The peak in C_u for b = 4.0 is seen to be eliminated for an open circuit condition ($R_e = -1.0$).

In Figures 16 and 17, the influence of the dimensionless effective thermal conductivity exponent “c” on the steady-state temperature profiles and the time history of the lower wall heat transfer is illustrated. Increases in the values of

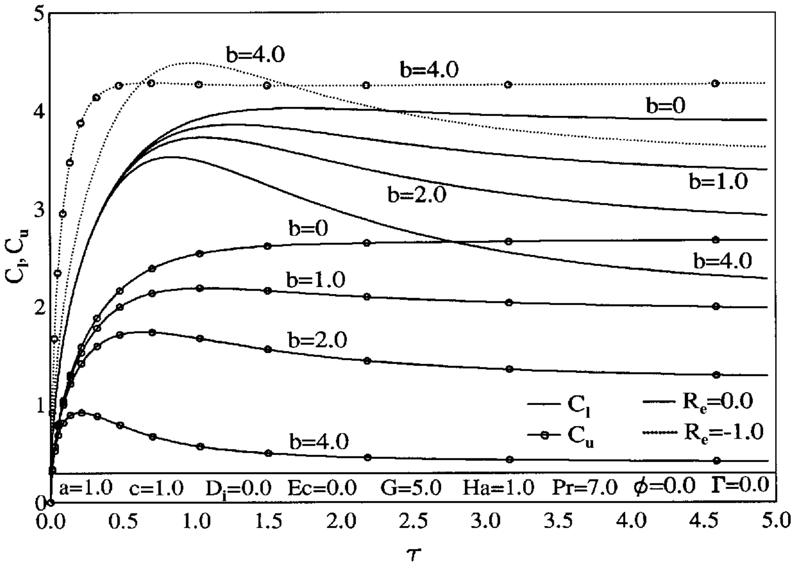

Figure 15.
Effects of b on skin-friction coefficients

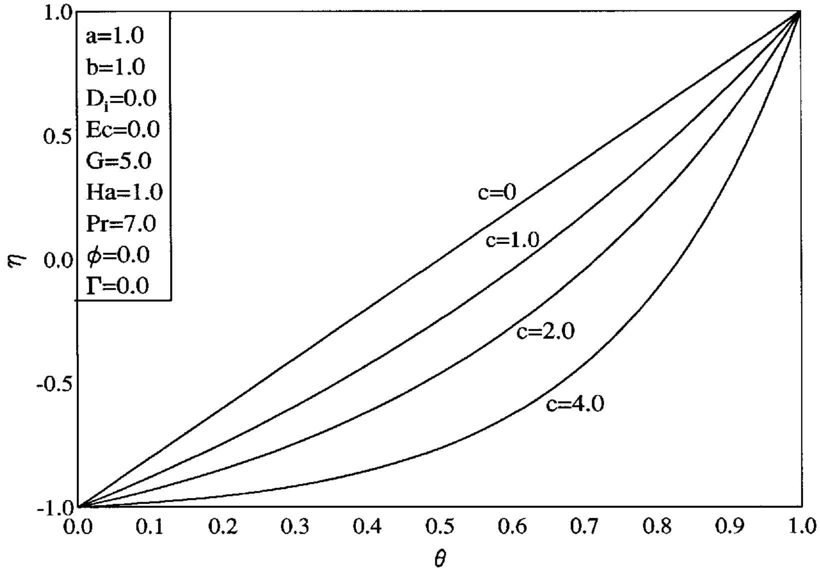

Figure 16.
Effects of c on steady-state temperature profiles

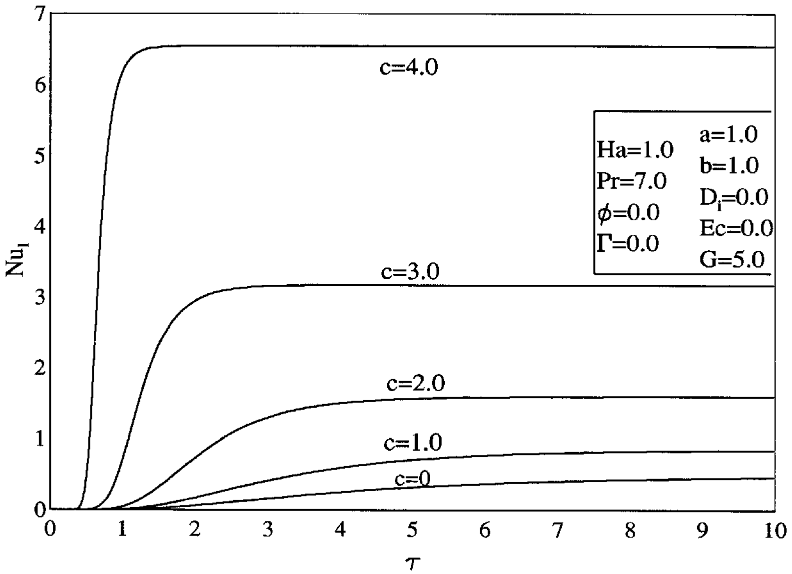

Figure 17.
Effects of c on lower wall heat transfer

"c" increase the thermal conductivity of the porous medium causing the fluid and the porous medium temperature as well as the lower wall heat transfer to increase. These behaviors are obvious from Figures 16 and 17. It is worth noting that increasing the value of "c" speeds up the approach to steady state as is clearly seen from Figure 17.

Figures 18-21 elucidate the effects of the Eckert number Ec on the steady-state velocity and temperature profiles as well as the lower wall skin-friction

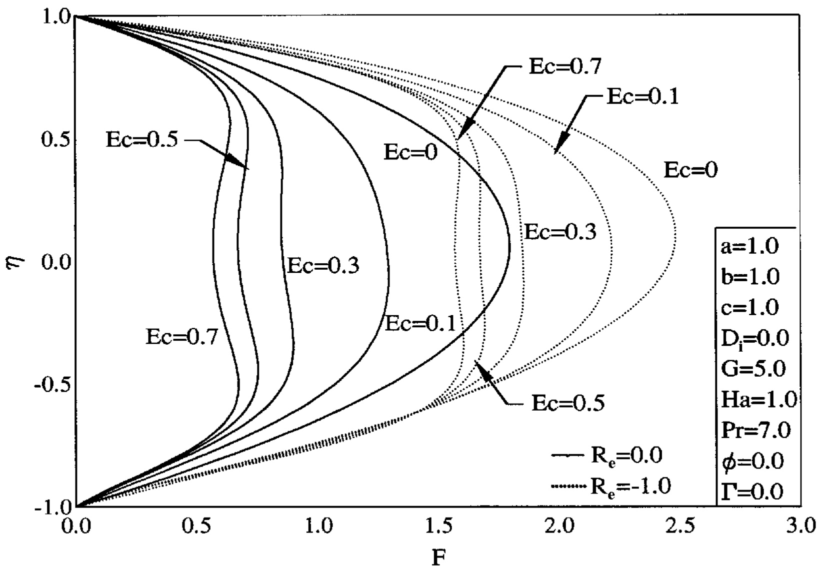

Figure 18.
Effects of Ec on steady-state velocity profiles

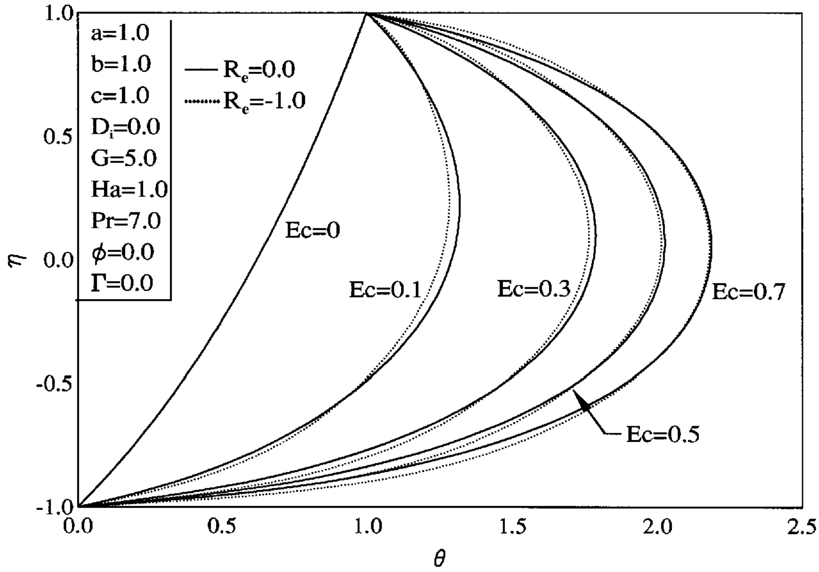

Figure 19.
Effects of Ec on steady-state temperature profiles

coefficient and heat transfer for both open and closed circuit conditions, respectively. In these figures, Ec represents all of the viscous dissipation, Joule heating and electrical dissipation effects (since Ha is taken to be finite). Increasing the Eckert number produces significantly higher steady-state fluid temperature distribution and lower fluid velocity distribution. In fact, the non-linearities in the temperature profiles increase and the parabolic velocity profile for Ec = 0 becomes more like an M-shape profile for Ec = 0.7. The presence of

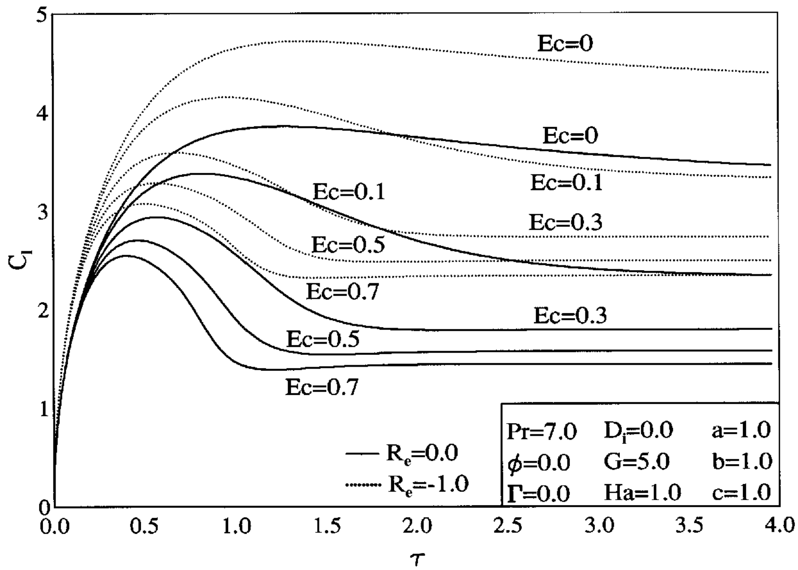

Figure 20. Effects of Ec on lower wall skin-friction coefficient

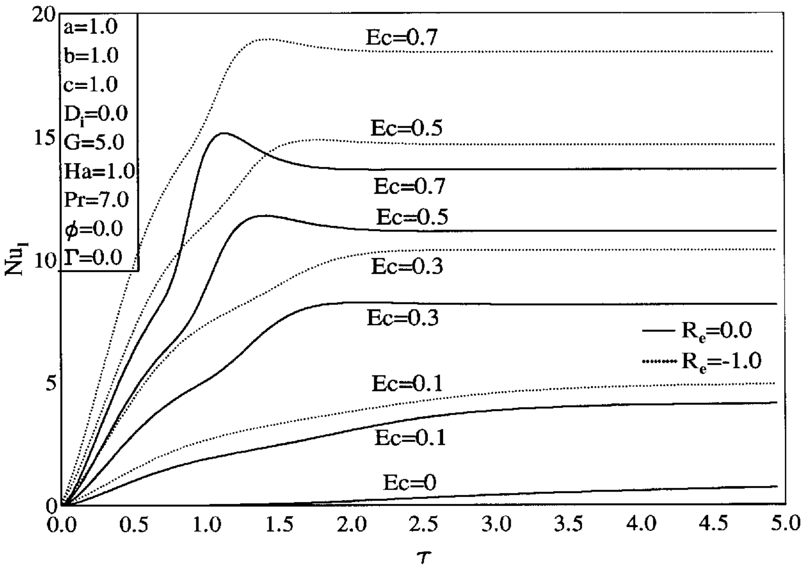

Figure 21. Effects of Ec on lower wall heat transfer

the electric field produces higher velocity values and slightly lower temperature values as displayed in Figures 18 and 19. Also, increasing the value of Ec causes the values of C_l to decrease and the values of Nu_l to increase. Similar to the effect of the thermal conductivity exponent "c", as Ec is increased, distinctive peaks in the time histories of both C_l and Nu_l are predicted. The results of C_l and Nu_l for Re=–1 are higher than those corresponding to $R_e = 0$ at the same conditions.

Conclusion

In this paper the problem of unsteady, fully-developed flow and heat transfer of an electrically-conducting fluid with variable properties in a porous medium channel with an applied transverse magnetic field was studied for the case of a constant pressure gradient. The governing equations for this investigation which included the effects of the magnetic field, electric field, porous medium inertia and heat generation or absorption were derived, nondimensionalized, and solved numerically by an implicit finite-difference method. Graphical results for the steady and transient velocity and temperature profiles as well as the skin-friction coefficients and the wall heat transfer coefficient at the lower wall were presented and discussed to illustrate the effects of the Hartmann number, electric field parameter, inverse Darcy number, inertia parameter, heat generation or absorption coefficient, exponents of variable properties and the Eckert number. It was found that both the magnetic field and the porous medium caused lower velocity distributions and skin-friction coefficients while the presence of the electric field produced higher velocity distributions and skin-friction coefficients. The presence of a heat-generating source resulted in higher steady-state temperatures and lower velocities due to variable properties. The lower wall heat transfer decreased due to heat generation for most of the transient stages while its steady-state value increased. On the other hand, a heat-absorption sink produced, in general, lower temperatures, and higher velocities and heat transfer at the lower wall. Increases in the variable viscosity exponent caused higher velocities and lower skin-friction coefficients. As the variable electrical conductivity exponent was increased, lower velocity and skin-friction coefficients were obtained. In addition, increasing the variable thermal conductivity exponent yielded higher temperatures and heat transfer rates at the lower wall of the channel. Finally, the fluid velocity and skin friction at the lower wall decreased while its temperature and heat transfer at the lower wall increased due to increases in the Eckert number. While favorable comparisons with previously published theoretical work on this problem were performed, no experimental data were found to check the validity of the assumptions made. It is hoped that the results reported herein will serve as a check for further theoretical modeling and a stimulus for experimental work on this problem.

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