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MHD FLOW OF A UNIFORMLY STRETCHED VERTICAL PERMEABLE SURFACE IN THE PRESENCE OF HEAT GENERATION/ABSORPTION AND A CHEMICAL REACTION

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ABSTRACT

Analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorbing fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and a first-order chemical reaction are reported. The solutions are obtained for all heat absorption conditions and restricted heat generation conditions. In the absence of heat generation/absorption and magnetic field effects are consistent with those previously reported in the literature. A parametric study is conducted and the results are presented and discussed.

Introduction

The study of flow and heat and mass transfer in the boundary layer induced by a surface moving with a uniform or non-uniform velocity in a quiescent ambient fluid is important in several manufacturing processes in industry which include the boundary layer along material handling conveyers, the extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath. Glass blowing, continuous casting and spinning of fibers also involve the flow due to a stretching surface. Sakiadis [1] studied the flow induced by a surface moving with a constant velocity in an ambient fluid. The corresponding heat transfer problem was considered theoretically by Tsou et al. [2] and Erickson et al. [3] and experimentally by Griffin and Throne [4]. Crane [5] studied the same problems as in [1], but assumed that the surface velocity U varies linearly with the stream-wise distance x. Dutta [6] analyzed heat transfer from a stretching sheet with uniform suction or blowing.

Early studies dealing with coupled heat and mass transfer include the works of Gebhart and Pera [7] on vertical plate, Pera and Gebhart, [8] and Chen and Yuh [9] on inclined plates. Gupta and Gupta [10] studied heat and mass transfer on a stretching sheet with suction or blowing.

The magnetohydrodynamics of electrically conducting fluids in the presence of a magnetic field is encountered in many important problems in geophysics and astrophysics. There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer aspects in various geometries due to the effect of magnetic fields on the flow control and on the performance of many systems using electrically conducting fluids such as liquid metals, water mixed with little acid and others. Chakrabarti and Gupta [11] considered hydromagnetic flow and heat and mass transfer over a stretching sheet. Vajravelu and Hadjinicolaou [12] reported on convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Other examples of studies dealing with hydromagnetic flows can be found in the papers by Gray [13], Michiyoshi et al. [14], and Fumizawa [15].

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions (see Vajravelu and Hadjinicolaou [16] and Vajravelu and Nayfeh [17]). In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretching of a surface. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first-order reaction in which the rate of reaction is directly proportional to the species concentration. Recently, Muthucumaraswamy [18] studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction.

The purpose of the present paper is to report analytical solutions for the problem of heat and mass transfer by steady flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field a first-order chemical reaction. This problem represents a generalization of the problem considered by Muthucumaraswamy [18] through the inclusion of heat generation/absorption and magnetic field effects.

Governing Equations

Consider coupled heat and mass transfer by hydromagnetic flow of a continuously moving vertical permeable surface in the presence of surface suction, heat generation/absorption effects, transverse magnetic field effects and a first-order chemical reaction. The flow is assumed steady, laminar and two-dimensional and the surface is maintained at a uniform temperature and concentration species and is assumed to be infinitely long, i.e. the dependent variables are not dependent on the vertical or axial coordinate. It is also assumed that the applied transverse magnetic field is uniform and that the magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected. All thermophysical properties are assumed constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation. Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T (T - T_\infty) + g\beta_c (c - c_\infty) - \frac{\sigma B_0{}^2}{\rho} u \tag{2}$$

$$\rho c_p v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + Q_o (T - T_\infty) \tag{3}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - \gamma c \tag{4}$$

where y is the horizontal or transverse coordinate, u is the axial velocity, v is the transverse velocity, T is the fluid temperature, c is the species concentration, T_∞ is the ambient temperature, c_∞ is the ambient concentration and ρ, g, β_T, β_c, μ, σ, B_0, Q_0, D and γ are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation/absorption coefficient, mass diffusion coefficient and the chemical reaction parameter, respectively.

The physical boundary conditions for the problem are

$$\begin{aligned} &u(0) = u_w, \quad v(0) = -v_w, \quad T(0) = T_w, \quad c(0) = c_w \\ &y \to \infty, \quad u \to 0, \quad T \to T_\infty, \quad c \to c_\infty \end{aligned} \tag{5}$$

where u_w (a constant), $v_w>0$, T_w, and c_w are the surface velocity, suction velocity, surface temperature and concentration, respectively.

The solution for Eq.(1) subject to Eq. (5) is $v = -v_w$. Using this, Eqs. (2) through (4) can be non-dimensionalized using the variables

$$Y = \frac{y v_w}{\nu}, \quad U = \frac{u}{u_w}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{c - c_\infty}{c_w - c_\infty} \tag{6}$$

Thus, one gets

$$\frac{d^2 U}{dY^2} + \frac{dU}{dY} + Gr_T \theta + Gr_c C - M^2 U = 0 \tag{7}$$

$$\frac{d^2 \theta}{dY^2} + \Pr \frac{d\theta}{dY} + \phi\theta = 0 \tag{8}$$

$$\frac{d^2 C}{dY^2} + Sc\frac{dC}{dY} - KScC = 0 \tag{9}$$

where

$$Gr_T = \frac{g\beta_T \nu (T_w - T_\infty)}{u_w v_w^2}, Gr_c = \frac{g\beta_c \nu (c_w - c_\infty)}{u_w v_w^2}$$

$$\Pr = \frac{\mu c_p}{k}, Sc = \frac{\nu}{D}, K = \frac{\gamma \nu}{v_w^2}, M^2 = \frac{\sigma B_0^2 \nu}{\rho v_w^2}, \phi = \frac{\nu Q_0}{\rho c_p v_w^2} \tag{10}$$

The dimensionless boundary conditions (5) become

$$\begin{aligned} &U(0) = 1, \quad \theta(0) = 1, \quad C(0) = 1 \\ &Y \to \infty, \quad U \to 0, \quad \theta \to 0, \quad C \to 0 \end{aligned} \tag{11}$$

The solutions of Eqs. (8) and (9) subject to Eqs. (11) are uncoupled from Eq. (7). These equations can be shown to have the following solutions

$$\theta = \exp(-m_1 Y) \ , \ C = \exp(-m_2 Y) \tag{12a,b}$$

where

$$m_1 = \frac{1}{2}\left[\Pr + (\Pr^2 + 4\Pr\phi)^{1/2}\right], \quad m_2 = \frac{1}{2}\left[Sc + (Sc^2 + 4ScK)^{1/2}\right] \tag{13}$$

Equation (13) is the same analytical solution given by Muthucumaraswamy [18].

Substitution of the solutions for θ and C given by Eqs. (12) into Eq. (7) and then solving analytically for U subject to Eqs. (11) obtains the following solution:

$$U = C_1 \exp(-\lambda_1 Y) + A\exp(-m_1 Y) + B\exp(-m_2 Y) \tag{14}$$

where

$$\lambda_1 = \frac{1}{2}\left[1 + (1 + 4M^2)^{1/2}\right] \tag{15}$$

and the constants C_1, A and B are given by

$$A = \frac{-Gr_T}{m_1^2 - m_1 - M^2}, \ B = \frac{-Gr_c}{m_2^2 - m_2 - M^2}, \ C_1 = 1 - A - B \tag{16}$$

The skin-friction coefficient, Nusselt number and the Sherwood number are important physical parameters for this problem. These can be defined as

$$C_f = \frac{\tau_f}{\rho u_w v_w} = \frac{dU}{dY}(0)\,; \quad \tau_f = \mu \left.\frac{du}{dy}\right|_{y=0} \tag{17}$$

$$Nu = \frac{q_w \nu}{(T_w - T_\infty) k v_w} = -\frac{d\theta}{dY}(0)\,; \quad q_w = -k \left.\frac{dT}{dy}\right|_{y=0} \tag{18}$$

$$Sh = \frac{J_w \nu}{(c_w - c_\infty) D v_w} = -\frac{d\phi}{dY}(0)\,; \quad J_w = -D \left.\frac{dc}{dy}\right|_{y=0} \tag{19}$$

According to the analytical solutions reported before, C_f, Nu, and Sh take on the respective forms:

$$C_f = -\lambda_1 C_1 - m_1 A - m_2 B\,,\ Nu = m_1\,,\ Sh = m_2 \tag{20}$$

Results and Discussion

Figure 1 presents representative axial velocity profiles for various combination of the parameters K, Pr, Sc and ϕ in the absence of a magnetic field (M=0) and in the presence of both thermal and concentration buoyancy effects. It should be mentioned that K>0 indicates a destructive chemical reaction while K<0 corresponds to a generative chemical reaction. Also, ϕ<0 indicates heat generation while ϕ>0 corresponds to heat absorption. Also, K=0 and ϕ=0 indicate no chemical reaction and no heat generation/absorption effects, respectively. It can be clearly observed that for a destructive chemical reaction (K=2.0) with Sc=2.0, increasing

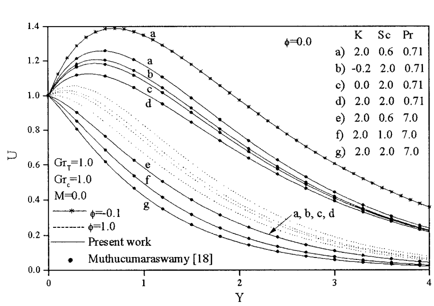

FIG. 1
Effects of various parameters on the velocity profiles.

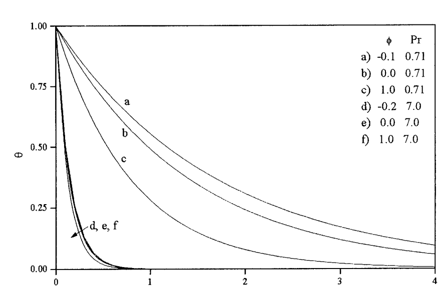

FIG. 2
Effects of Pr and ϕ on the temperature profiles.

Pr produces lower fluid velocities. Also, for lower values of Pr (Pr=0.71) irregardless of the value of K, a distinctive peak in the velocity profile is predicted. However, this does not occur for relatively higher values of Pr (Pr=7.0). The presence of the peak indicates that the maximum value of velocity occurs in the body of fluid close to the surface and not at the surface. In addition, it can also be seen from this figure that increases in the values of Sc results in reduced flow velocities. Furthermore, it can be seen that increases in the values of K cause reductions in the fluid velocities. Similarly, increases in the value of ϕ cause the fluid to move at a slower rate. It can be also observed that the effect of heat generation ($\phi<0$) on the velocity profile is more pronounced than that of heat absorption.

In Fig. 2, the effects of both Pr and ϕ on the temperature profiles are illustrated. It can be seen that as either of Pr or ϕ increases, the fluid temperature decreases. Also, the effects of increasing or decreasing ϕ for Pr=0.71 is much more pronounced than that corresponding to Pr=7.0.

Figure 3 displays the effects of both K and Sc on the species concentration profiles. As expected, the presence of a chemical reaction significantly affects the concentration profile. In fact, as K increases, considerable reduction in the concentration is predicted. Also, increasing the value of Sc produces lower concentration values.

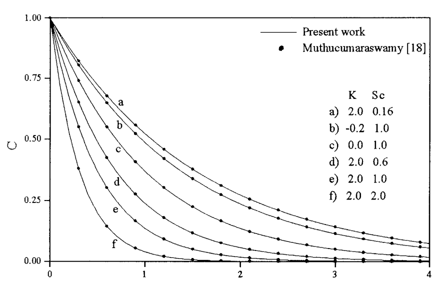

FIG. 3
Effects of K and Sc on the concentration profiles.

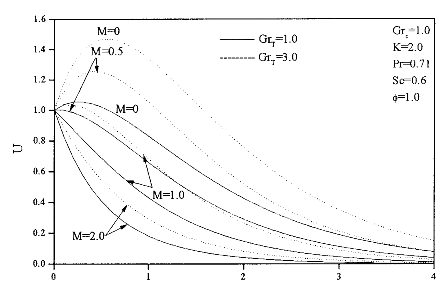

FIG. 4
Effects of M and Gr_T on the velocity profiles.

Figure 4 depicts the influence of the magnetic field and the thermal buoyancy effect on the velocity profiles. Obviously, increasing the thermal buoyancy effects aids the flow caused by the stretching of the surface. This produces higher fluid velocities. This is depicted in the increases in the values of U as Gr_T increases from 1 to 3. It is interesting to note that this increase in the velocity is accompanied by a greater increase in the peak value mentioned before. In addition, application of a magnetic field to an electrically conducting fluid produces a drag-like force called the Lorentz force. This force causes reduction in the fluid velocity. This is clear from the decreases in the fluid velocity as M increases. It is interesting to observe that for a relatively strong magnetic field, the distinctive peak in the velocity profile can be removed where the maximum velocity will be that of the surface.

Table 1 presents the effects of changing Gr_c, Gr_T, K, M, Pr, Sc and ϕ on the values of the skin-friction coefficient C_f, the Nusselt number Nu, and the Sherwood number Sh. It can be seen that increasing either of Gr_c or Gr_T causes C_f to increase while increasing either of K, M, Pr, Sc or ϕ produces lower values of C_f. On the other hand, as expected, Nu is enhanced by increases in either Pr or ϕ and Sh is increased by increases in either K or Sc.

TABLE 1

Effects of various parameters on C_f, Nu and Sh

Gr_c	Gr_T	K	M	Pr	Sc	ϕ	C_f	Nu	Sh
1	1	0	0	0.71	0.6	0	2.0751	0.7100	0.6000
1	1	2	0	0.71	0.6	0	1.1049	0.7100	1.4358
1	1	2	0	0.71	0.6	1	0.4843	1.2693	1.4358
1	1	2	0	0.71	0.6	-0.1	1.3926	0.5896	1.4358
1	1	0	0	7.0	0.6	0	0.8095	7.0000	0.6000
1	1	2	0	0.71	2.0	0	0.7175	0.7100	3.2361
1	1	0	0.5	7.0	0.6	0	0.1706	7.0000	0.6000
1	1	2	0.5	0.71	0.6	1	0.0789	1.2693	1.4358
1	3	2	0.5	0.71	0.6	1	1.4335	1.2693	1.4358
3	1	2	0.5	0.71	0.6	1	1.2962	1.2693	1.4358

Concluding Remarks

Analytical solutions for heat and mass transfer by steady, laminar flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field and a first-order chemical reaction were reported. Based on the obtained graphical results, the following conclusions were deduced:

1. The fluid velocity decreased as either of the Prandtl number, the Schmidt number or the strength of the magnetic field was increased and increased as either of the thermal or concentration buoyancy effects were increased.
2. The fluid velocity increased during a generative chemical reaction and decreased during a destructive one. Also, the presence of heat generation effects increased the fluid velocity while the presence of heat absorption effects decreased it.
3. The skin-friction coefficient increased as either of the thermal or concentration buoyancy effects were increased and decreased as either of the chemical reaction parameter, Hartmann number, Prandtl number, Sherwood number or the heat generation/absorption was increased.

4. The Nusselt number increased as either of the Prandtl number or the heat generation/absorption was increased while the Sherwood number increased as either of the Sherwood number or the chemical reaction parameter was increased.

Nomenclature

B_0	magnetic induction
c	concentration
c_p	specific heat at constant pressure
C	dimensionless concentration
C_f	skin-friction coefficient
C_w	wall concentration
D	mass diffusion coefficient
g	acceleration due to gravity
Gr_c	mass Grashof number
Gr_T	thermal Grashof number
k	fluid thermal conductivity
K	dimensionless chemical reaction parameter
M	Hartmann number
Nu	Nusselt number
Pr	Prandtl number
Q_0	heat generation/absorption coefficient
Sc	Schmidt number
Sh	Sherwood number
T	fluid temperature
T_w	wall temperature
u	fluid axial velocity
u_w	velocity of the vertical surface
U	dimensionless axial velocity
v	fluid transverse velocity

v_w suction velocity

x axial or vertical coordinate

y transverse or horizontal coordinate

Y dimensionless transverse coordinate

Greek Symbols

β_T coefficient of thermal expansion

β_c coefficient of concentration expansion

ϕ dimensionless heat generation/absorption coefficient

γ chemical reaction parameter

κ vortex viscosity

μ fluid dynamic viscosity

ν fluid kinematic viscosity

ρ fluid density

σ fluid electrical conductivity

θ dimensionless temperature

Subscripts

w conditions on the wall

∞ ambient condition

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