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Hydromagnetic free convection of a particulate suspension from a permeable inclined plate with heat absorption for non-uniform particle-phase density

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Abstract The problem of steady, laminar, free convection flow of a particulate suspension over an infinite, permeable, inclined, and isothermal flat plate in the presence of a transverse magnetic field and fluid heat absorption effects is studied numerically. The problem accounts for particulate viscous effects which are absent from most two-phase models. An analytical solution is developed for the particle-phase density distribution and numerical solutions for the velocity and temperature profiles of both phases are obtained by using an implicit and iterative finite-difference method. A parametric study illustrating the influence of the magnetic field, heat absorption effects and particle loading is conducted. The obtained results for velocity, temperature and skin-friction coefficients for both phases as well as the Nusselt number are illustrated graphically to show the features of the solution.

Nomenclature

B_o	Magnetic induction
c	Fluid-phase specific heat
C_f	Fluid-phase skin coefficient of friction
D	Diffusion coefficient
f	Any dependent variable
g	Gravitational acceleration
Gr	Grashof number
H	Dimensionless gravitational acceleration
k	Fluid-Phase thermal conductivity
L	Characteristic length
M	Hartmann number
N	Interphase momentum transfer coefficient
N_T	Interphase heat transfer coefficient
Nu	Nusselt number
P	Fluid-phase pressure
Pr	Fluid-phase Prandtl number
q_o	Dimensional heat absorption coefficient
Q_p	Dimensionless particle-phase density
Rv	Wall suction velocity
S	Dimensionless heat absorption coefficient
Sc	Inverse Schmidt number
T	Fluid-phase temperature
u	Fluid-phase x-component of velocity
U	Fluid-phase dimensionless tangential velocity
v	Fluid-phase y-component of velocity
V	Fluid-phase dimensionless normal velocity
x, y	Cartesian coordinates
Y	Dimensionless normal distance

Greek symbols

α	Velocity inverse Stokes number
β	Particle-phase to fluid viscosity ratio
$\bar{\beta}$	Volume expansion coefficient
γ	Specific heats ratio
ε	Temperature inverse Stokes number
θ	Fluid-phase dimensionless temperature
κ	Particle loading
μ	Fluid-phase dynamic viscosity
ν	Fluid-phase kinematic viscosity
ρ	Fluid-phase density
σ_o	Fluid-phase electrical conductivity
ϕ	Tilt angle
ω	Particle-phase wall slip coefficient

Subscripts

$( )_n$	Numerical scheme grid position index
$( )_p$	Particle phase
$( )_w$	Plate wall
$( )_\infty$	Very large distance away from the plate surface (ambient condition)

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1 Introduction

Hydromagnetic flows are found in many industrial applications. Moreau [1] listed some of these numerous industrial processes. Gebhart et al. [4] indicated that early interest in such flows arose in astrophysics, geophysics, and controlled nuclear fusion. Many authors and researchers have reported a large bulk of solutions and results for such flows [1–9]. The study of heat absorption effects in moving fluids is also gaining wide attention in view of several physical problems faced in industry such as those dealing with chemical reactions and those concerned dissociating fluids. Example of this type of problems can be found in the paper by Chamkha [10].

The common ground for most of these studies is that they are solved and analyzed by assuming a pure fluid with no contaminants that will effect the heat transfer or the resultant type of fluid flow. While this assumption approximates the reality in many cases quite well especially for low contamination levels, it is not, however, valid in a lot of other cases in which the contaminants in the fluid play a major role in altering the resultant flow and heat transfer characteristics. A survey of the technical literature concluded that no work has been done on the problem of hydromagnetic free or natural convection heat transfer from surfaces for a particulate (fluid-particle) suspension. Recently, the present authors [11] have reported analytical solutions for free convection flow of a particulate suspension past an infinite permeable vertical plate. In those solutions the particle-phase density distribution was assumed to be uniform across the domain of interest. This assumption greatly simplified the governing equations which in turn facilitated the development of the analytical solutions. The above referenced work has been extended further [12] by assuming the particle-phase density distribution to be a variable and considering the effect of plate's inclination. These assumption resulted in a set of nonlinear ordinary differential equations from which only the particle-phase density distribution possessed an analytical solution. The remaining equations had to be solved numerically. Furthermore, the present authors [13] have reported analytical solutions for hydromagnetic free convection of a particulate suspension from an infinite permeable inclined plate with heat absorption for uniform particle phase density. The present work is a continuation to the above effort. In the present work, the particle-phase density distribution is assumed to be a variable in the presence of a transverse magnetic field with a heat absorbing fluid. This assumption will result in a set of nonlinear ordinary differential equations, from which only the particle-phase density distribution possess an analytical solution. The remaining equations must be solved numerically.

2
Problem description and governing equations

Consider steady, laminar, hydromagnetic free convection flow of a fluid-particle suspension over an isothermal and permeable infinitely long inclined flat plate. Uniform fluid-phase suction is imposed at the plate surface. In addition, a uniform magnetic field is applied normal to the flow direction (or the plate surface) at all times. Far from the plate wall, the fluid phase is stagnant and is maintained at a constant temperature T_∞ while the particle phase is having a uniform density distribution and is affected by the gravitational force (see Fig. 1). The plate is allowed to tilt about its lower base in the range of ±60° measured from the plate's vertical position. The temperature of the wall T_w will be assumed to be greater than that of the ambient fluid T_∞ at all times.

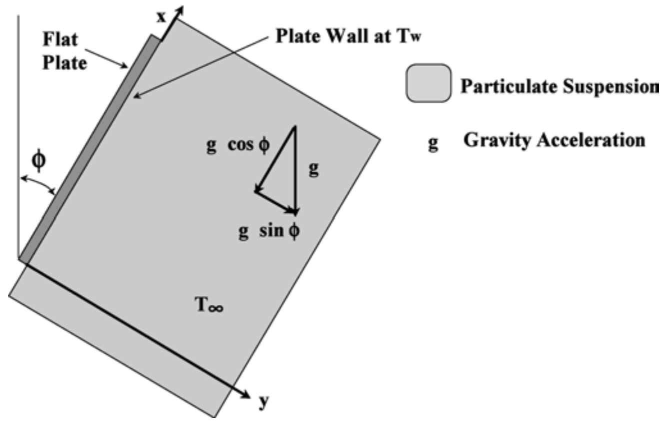

Fig. 1. The general configuration of the problem

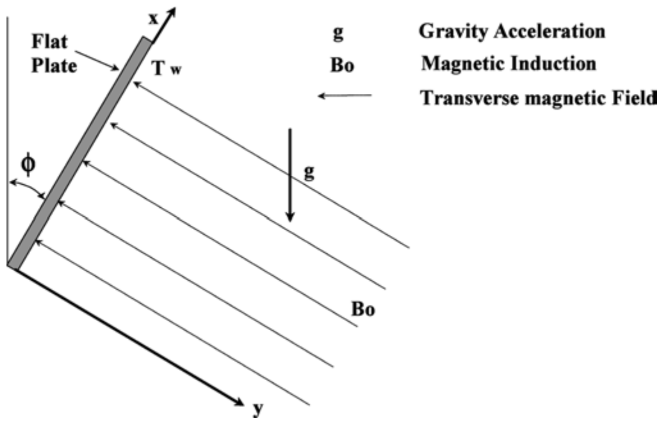

Fig. 2. The transverse magnetic field orientation

The fluid is assumed to be Newtonian, electrically conducting and heat absorbing and has constant properties in the range of the study conditions except for the density in the buoyancy term. As mentioned before, the magnetic field is assumed to be normal to the plate surface regardless of the plate orientation as shown in Fig. 2. The induced magnetic and electrical fields as well as the Hall effect of magnetohydrodynamics are all assumed to be negligible and the magnetic Reynolds number, electrical displacement and convection are all assumed to be small. Only the fluid phase will be affected by the presence of the magnetic field since it is assumed to be electrically conducting. However, the particle phase will not be affected by that field directly since it is assumed to be electrically non-conducting (insulator). Nevertheless, it will be influenced indirectly by the magnetic field due to the interphase drag mechanism between the phases.

In this study, the particle phase is assumed to be made of non-uniformly distributed spherical particles all having one size and is endowed by a constant viscosity and diffusivity. The particle-phase viscosity can be thought of as a natural consequence of the averaging processes involved in representing a discrete system of particles as a continuum (see, for example, Drew [14] and Drew and Segal [15]). Also, the particle-phase viscous effects can be used to model particle-particle interaction and particle-wall interaction in relatively dense suspensions. These effects have been investigated previously by many authors such as Tsuo and Gidaspow [16] and Gadiraju et al. [17]. Both the fluid and the particle phases are modelled as interacting continua exchanging both momentum and heat transfer as discussed by Marble [18]. The volume fraction of suspended particles is considered small compared to that of the fluid phase.

The governing equations for the problem under consideration are based on the balance laws of mass, linear momentum, and energy for both the fluid and the particle phases. Since the plate is assumed to be infinite, all dependent variables will be functions of the independent variable y only. Taking all the previous assumptions into consideration, the governing equations can be written as

$$\frac{dv}{dy} = 0 \tag{1}$$

$$\mu\frac{d^2u}{dy^2} - \rho v\frac{du}{dy} - \frac{dP}{dx} - \rho_p N(u - u_p) - \rho g\cos\phi - \sigma_o B_o^2 u = 0 \tag{2}$$

$$k\frac{d^2T}{dy^2} - \rho c v\frac{dT}{dy} + \rho_p c_p N_T(T_p - T) + q_o(T - T_\infty) = 0 \tag{3}$$

$$D_p\frac{d^2\rho_p}{dy^2} - \frac{d(\rho_p v_p)}{dy} = 0 \tag{4}$$

$$\nu_p\frac{d}{dy}\left(\rho_p\frac{du_p}{dy}\right) - \rho_p v_p\frac{du_p}{dy} + \rho_p N(u - u_p) - \rho_p g\cos\phi = 0 \tag{5}$$

$$2\nu_p\frac{d}{dy}\left(\rho_p\frac{dv_p}{dy}\right) - \rho_p v_p\frac{dv_p}{dy} + \rho_p N(v - v_p) + \rho_p g\sin\phi = 0 \tag{6}$$

$$\frac{k_p}{\rho_p}\frac{d}{dy}\left(\rho_p\frac{dT_p}{dy}\right) - \rho_p c_p v_p\frac{dT_p}{dy} - \rho_p c_p N_T(T_p - T) = 0 \tag{7}$$

The boundary conditions for this problem are

$$\begin{aligned}
&v(0) = -v_w\\
&u(0) = 0, \quad u(\infty) = 0\\
&T(0) = T_w, \quad T(\infty) = T_\infty\\
&\rho_p(0) = \rho_{pw}, \quad \rho_p(\infty) = \rho_{p\infty}\\
&u_p(0) = \omega_s\frac{du_p}{dy}\Big|_{y=0}, \quad u_p(\infty) = -\frac{g}{N}\cos\phi\\
&T_p(0) = T_w, \quad T_p(\infty) = T_\infty
\end{aligned} \tag{8}$$

where ω_s is the dimensional particle-phase wall slip coefficient. To date, the exact form of boundary conditions to be satisfied by a particle phase at a given surface is unknown. Since the particle phase may resemble a rarefied gas and undergoes slip at a boundary, then a boundary condition borrowed from rarefied gas dynamics as done by previous authors (Soo [19] and Chamkha [20]) will be employed in this study. It should be mentioned that, in writing Equations (1) through (7), both viscous dissipation and Joule heating are neglected.

The hydrostatic gradient pressure in Equation (2) is approximated as

$$\frac{dP}{dx} = \rho_{p_\infty} N u_{p_\infty} - \rho_\infty g\cos\phi \tag{9}$$

where

$$u_{p_\infty} = -\frac{g\cos\phi}{N} \tag{10}$$

This approximations can be easily obtained by evaluating Equations (1) through (7) at $y = \infty$. Using Boussinesq approximation [21] to couple the fluid momentum equation to the temperature field and substituting Equations (9) and (10), Equation (2) can be written as

$$\begin{aligned}
&\mu\frac{d^2u}{dy^2} - \rho_\infty v\frac{du}{dy} - \rho_p N(u - u_p) + \rho_{p_\infty} g\cos\phi\\
&\quad + \rho_\infty g\bar{\beta}(T - T_\infty)\cos\phi - \sigma_o B_o^2 u = 0
\end{aligned} \tag{11}$$

where $\bar{\beta}$ is the volume expansion coefficient.

Equations (1), (3) through (7), and (11) constitute the governing equations of the problem. These equations represent a generalization of the dusty-gas equations discussed by Marble [18] to include particle-phase viscosity and diffusivity, and fluid-phase hydromagnetic, absorption, and buoyancy effects.

To nondimensionalize the above governing equations, the following variables and parameters are used

$$\begin{aligned}
&Y = \frac{yGr^{1/4}}{L}, \quad U = \frac{uL}{\nu Gr^{1/2}}, \quad V = \frac{vL}{\nu Gr^{1/4}}\\
&U_p = \frac{u_pL}{\nu Gr^{1/2}}, \quad V_p = \frac{v_pL}{\nu Gr^{1/4}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}\\
&\theta_p = \frac{T_p - T_\infty}{T_w - T_\infty}, \quad Q_p = \frac{\rho_p}{\rho_{p_\infty}}, \quad S = \frac{q_oL^2}{\mu c Gr^{1/2}}\\
&\kappa = \frac{\rho_{p_\infty}}{\rho_\infty}, \quad \alpha = \frac{NL^2}{\nu Gr^{1/2}}, \quad H = \frac{gL^3}{\nu^2 Gr}\\
&\gamma = \frac{c_p}{c}, \quad \varepsilon = \frac{N_TL^2}{\nu Gr^{1/2}}, \quad \beta = \frac{\nu_p}{\nu}\\
&M^2 = \frac{\sigma_o B_o^2 L^2}{\mu Gr^{1/2}}, \quad Pr = \frac{\mu c}{k}, \quad Pr_p = \frac{\mu_p c_p}{k_p}\\
&Sc = \frac{D_p}{\nu}, \quad Gr = \frac{g\bar{\beta}(T_w - T_\infty)L^3}{\nu^2}
\end{aligned} \tag{12}$$

When the above defined quantities are substituted in Equations (1), (3) through (7), and (11) and after simplifying the following dimensionless equations will result

$$\frac{dV}{dY} = 0 \tag{13}$$

$$\frac{d^2U}{dY^2} - V\frac{dU}{dY} - \alpha\kappa Q_p(U - U_p) - M^2U + (\kappa H + \theta)\cos\phi = 0 \tag{14}$$

$$Pr^{-1}\frac{d^2\theta}{dY^2} - V\frac{d\theta}{dY} + \gamma\varepsilon\kappa Q_p(\theta_p - \theta) + S\theta = 0 \tag{15}$$

$$Sc\frac{d^2Q_p}{d^2Y} - V_p\frac{dQ_p}{dY} = 0 \tag{16}$$

$$\beta Q_p\frac{d^2U_p}{dY^2} + \beta\frac{dQ_p}{dY}\frac{dU_p}{dY} - V_pQ_p\frac{dU_p}{dY} + \alpha Q_p(U - U_p) - HQ_p\cos\phi = 0 \tag{17}$$

$$2\beta\frac{d}{dY}\left(Q_p\frac{dV_P}{dY}\right) - Q_pV_p\frac{dV_p}{dY} + \alpha Q_p(V - V_P) + HGr^{\frac{1}{4}}Q_p\sin\phi = 0 \tag{18}$$

$$\beta Pr_p^{-1}Q_p\frac{d^2\theta_p}{dY^2} + \beta Pr_p^{-1}\frac{dQ_p}{dY}\frac{d\theta_p}{dY} - V_pQ_p\frac{d\theta_p}{dY} + \varepsilon Q_p(\theta - \theta_p) = 0 \tag{19}$$

The dimensionless boundary conditions become as follows

$$\begin{aligned}
&V(0) = -Rv\\
&U(0) = 0, \quad U(\infty) = 0\\
&\theta(0) = 1, \quad \theta(\infty) = 0\\
&Q_p(0) = Q_{po}, \quad Q_p(\infty) = 1\\
&U_p(0) = \omega\frac{dU_p}{dY}|_{y=0}, \quad U_p(\infty) = \frac{H}{\alpha}\cos\phi\\
&\theta_p(0) = 1, \quad \theta_p(\infty) = 0
\end{aligned} \tag{20}$$

where $Rv = v_wL/(vGr^{1/4})$ and $\omega = \omega_s\ Gr^{1/4}/L$ are the dimensionless fluid-phase suction velocity and the particle-phase slip coefficient. It should be noted that Rv, Q_{po}, and ω are all assumed to be constant.

The fluid-phase skin friction coefficient C_f, the particle-phase skin friction coefficient C_{fp} and the Nusselt number Nu are defined, respectively, as

$$C_f = \frac{\mu\frac{du}{dy}|_{y=0}}{\frac{1}{2}\rho\left(\frac{\nu}{L}\right)^2Gr^{3/4}} = 2\frac{dU}{dY}|_{Y=0} \tag{21}$$

$$C_{fp} = \frac{\mu_p\frac{du_p}{dy}|_{y=0}}{\frac{1}{2}\rho\left(\frac{\nu}{L}\right)^2Gr^{3/4}} = 2\kappa\beta\frac{dU_p}{dY}|_{Y=0} \tag{22}$$

$$Nu = \frac{hL}{kGr^{1/4}} = -\frac{d\theta}{dY}|_{Y=0} \tag{23}$$

3
Particle-phase density analytical solution

Integrating Equation (13) and applying the boundary condition for V yields

$$V = -Rv \tag{24}$$

Assuming that V_p is constant throughout the domain of interest, and substituting Equation (24) into (18) and rearranging gives

$$V_p = \frac{H}{\alpha}Gr^{\frac{1}{4}}\sin\phi - Rv = \Pi \tag{25}$$

Obviously, for a vertical plate ($\phi = 0$) both V and V_p will have the same wall suction velocity Rv.

Substituting Equation (25) into (16) yields

$$Sc\frac{d^2Q_p}{d^2Y} - \Pi\frac{dQ_p}{dY} = 0 \tag{26}$$

The above equation is a linear, second order, ordinary differential equation which is uncoupled from all other equations. Therefore, Equation (26) can be solved independently from the other system of equations. The solution of Equation (26) subject the required boundary conditions can be shown to be

$$Q_p = 1 + (Q_{p0} - 1)\exp\left(\frac{\Pi}{Sc}Y\right) \tag{27}$$

This solution has been reported previously by the present authors [12]. However, it is repeated here because Equation (27) is a corner stone in the solution of the present problem. An important observation in the above solution needs further elaboration. A physically acceptable solution for Q_p requires that Pi be always negative. If Pi is allowed to attain a positive value, the exponential in Equation (27) will grow without limits towards infinity, which is, obviously, not a valid solution. Physically, the requirement that Pi be negative means that the particle-phase normal velocity must be negative, i.e. towards the plate, to ensure a continuous flow of particles to compensate for the lost particles through the suction and to sustain the requirement of constant particle-phase density at the wall surface. If V_p is allowed to be positive, i.e. away from the plate surface, all the particles will be cleaned away from the vicinity of the plate and the wall boundary condition for Q_p can not be met. The requirement of Pi to be always negative or at most zero was also required in the analytical and numerical solutions reported previously by the current authors (see Ramadan and Chamkha [12, 13]).

4
Numerical technique

After substituting all of the results obtained in the previous section into the governing equations, the following equations will result

$$\frac{d^2U}{dY^2} + Rv\frac{dU}{dY} - \alpha\kappa Q_p(U - U_p) - M^2U + (\kappa H + \theta)\cos\phi = 0 \quad (28)$$

$$Pr^{-1}\frac{d^2\theta}{dY^2} + Rv\frac{d\theta}{dY} + \gamma\varepsilon\kappa Q_p(\theta_p - \theta) + S\theta = 0 \quad (29)$$

$$\beta Q_p\frac{d^2U_p}{dY^2} + \beta\frac{dQ_p}{dY}\frac{dU_p}{dY} - \Pi Q_p\frac{dU_p}{dY} + \alpha Q_p(U - U_p) - HQ_p\cos\phi = 0 \quad (30)$$

$$\beta Pr_p^{-1}Q_p\frac{d^2\theta_p}{dY^2} + \beta Pr_p^{-1}\frac{dQ_p}{dY}\frac{d\theta_p}{dY} - \Pi Q_p\frac{d\theta_p}{dY} + \varepsilon Q_p(\theta - \theta_p) = 0 \quad (31)$$

where

$$\frac{dQ_p}{dY} = \frac{\Pi}{Sc}(Q_{p0} - 1)\exp\left(\frac{\Pi}{Sc}Y\right), \quad (32)$$

Q_p is given by Equation (27) and Pi is given by Equation (25). The above set of equations is obviously coupled and nonlinear and possess no closed-form solution. Therefore, they must be solved numerically for the flow and heat transfer variables.

To solve the above set of ordinary differential equations, a finite difference approximation is adopted. Equations (28) through (31) have the general form of

$$\tau_1 f'' + \tau_2 f' + \tau_3 f + \tau_4 = 0 \quad (33)$$

where f is any dependent variable, a prime denotes a differentiation with respect to Y, and the τ'$_s$ are constants and/or functions of the independent variable. To approximate f′ and f″, a three-point central difference representation is used with constant step size throughout the computational domain. This yields

$$f' = \frac{1}{2\Delta Y}(f_{n+1} - f_{n-1}) \quad (34)$$

$$f'' = \frac{1}{(\Delta Y)^2}(f_{n-1} - 2f_n + f_{n+1}) \quad (35)$$

where ΔY is the step size, and the subscript n corresponds to the n^{th} point in the Y direction, as shown in Fig. 3.

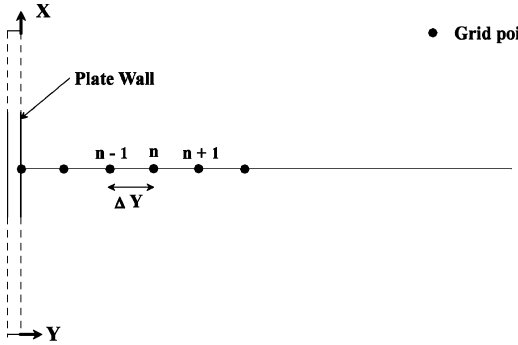

Fig. 3. Finite difference grid representation

Substituting Equations (34) and (35) into Equation (33) and rearranging yields

$$Af_{n-1} + Bf_n + Cf_{n+1} = D \quad (36)$$

where

$$A = \frac{1}{\Delta Y}\left(\frac{\tau_1}{\Delta Y} - \frac{\tau_2}{2}\right) \quad (37)$$

$$B = \tau_3 - \frac{2\tau_1}{(\Delta Y)^2} \quad (38)$$

$$C = \frac{1}{\Delta Y}\left(\frac{\tau_1}{\Delta Y} + \frac{\tau_2}{2}\right) \quad (39)$$

$$D = -\tau_4 \quad (40)$$

When Equation (36) is repeated for every grid point in the domain, a tridiagonal system of algebraic equations will result. The Thomas algorithm is used to solve these algebraic equations (see Chapra and Canale [22]).

The solution starts by guessing an initial solution for each of the variables U, U_p, θ, and θ_p. These solutions are used to produce new solutions by substituting these initial guesses in the respective equations and solving the resulting tridiagonal matrix for each variable in a row. The new solution for each variable is then used as an initial guess and the above procedure is repeated for a further modified solution until convergence is achieved.

5
Results and discussion

The effects of the square of the Hartmann number, M^2, are shown in Fig. 4 through Fig. 6. Increasing the Hartmann number has the effect of damping the fluid and the particle phases velocity profiles. This is because the application of a transverse magnetic field normal to the flow direction will result in a resistive force (Lorentz force) similar to the drag force which tends to resist the fluid flow and thus reducing its velocity. In the absence of viscous dissipation and Joule heating, it appears that the Hartmann number has no effect on the thermal layers and the temperature profiles of both the fluid and the particle phases as shown in Fig. 6. Thus, the Hartmann number has no effect on the Nusselt number.

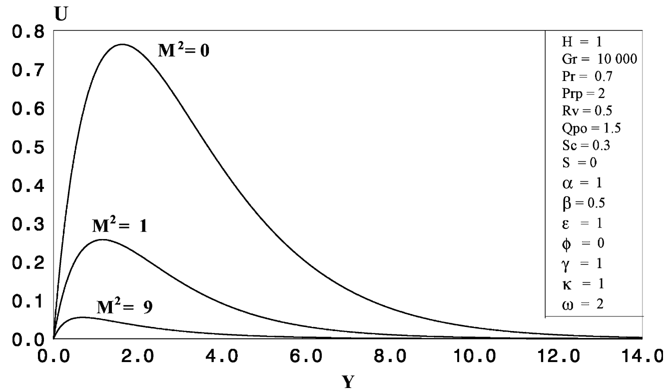

Fig. 4. Effect of Hartmann number on the fluid velocity profiles

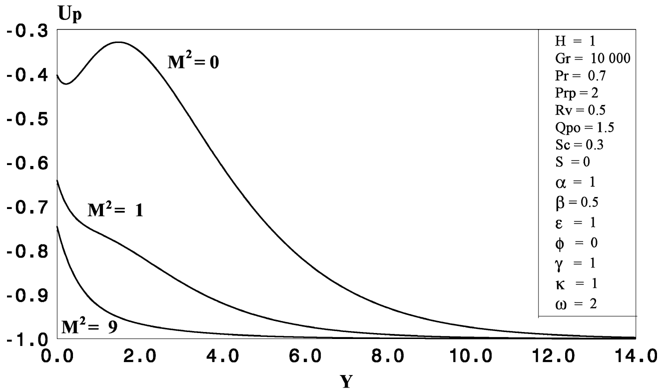

Fig. 5. Effect of Hartmann number on the particle-phase velocity profiles

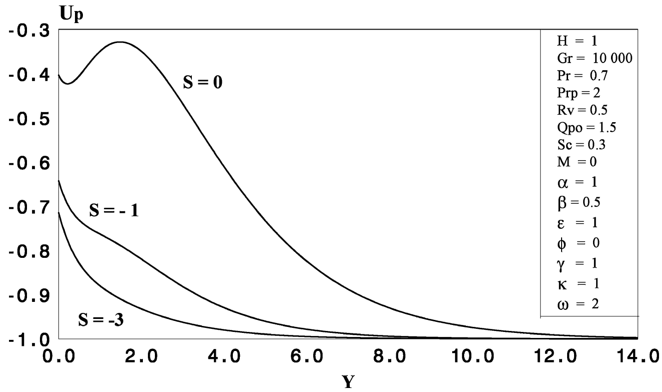

Fig. 8. Effect of heat absorption on the particle-phase velocity profiles

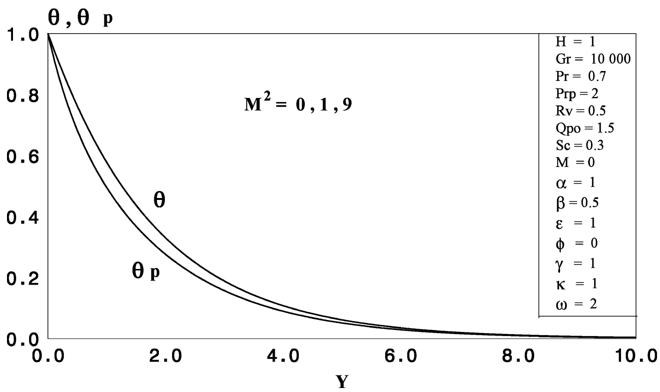

Fig. 6. Effect of Hartmann number on the fluid- and particle-phase temperature profiles

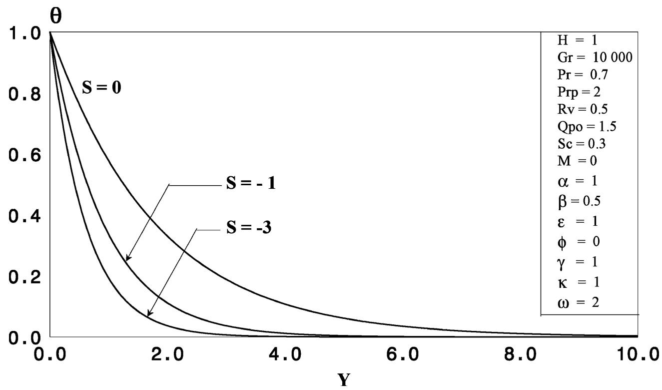

Fig. 9. Effect of heat absorption on the fluid temperature profiles

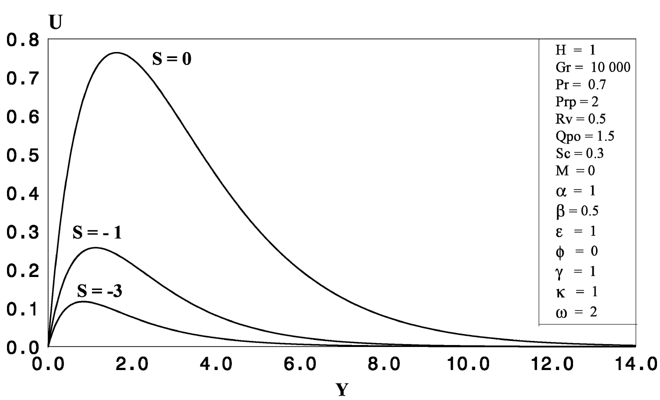

Fig. 7. Effect of heat absorption on the fluid velocity profiles

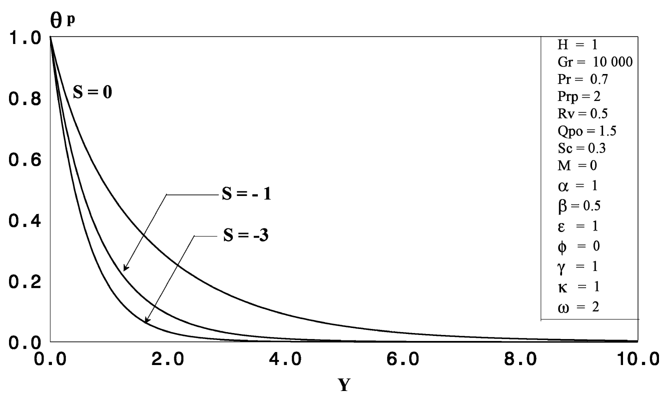

Fig. 10. Effect of heat absorption on the particle-phase temperature profiles

The effects of the heat absorption coefficient, S, are shown in Fig. 7 through Fig. 10. In general, the effect of heat absorption is to damp the flow and heat transfer characteristics of both phases and to reduce the thickness of both the velocity and the thermal layers. This result is expected since, with heat absorption, less energy is used to enhance the buoyancy effects while the majority of the energy is absorbed by the fluid. These behaviors are clearly illustrated in the decreases in V, U_p, and θ as $|S|$ increases shown in Fig. 7 through Fig. 10, respectively.

The effects of the ratio of the particle-phase density to the fluid density or the particle loading, κ, are shown in Fig. 11 through Fig. 14. As the value of κ is increased, the amount of drag experienced by the fluid from the particle phase increases, and vice versa. This increase in interphase drag tends to reduce the velocities of both phases as shown in Fig. 11 and Fig. 12. The temperature profiles are also affected in the sense that a given amount of energy is distributed on a larger mass of particles as the value of κ becomes bigger and vice versa. This explains the decrease

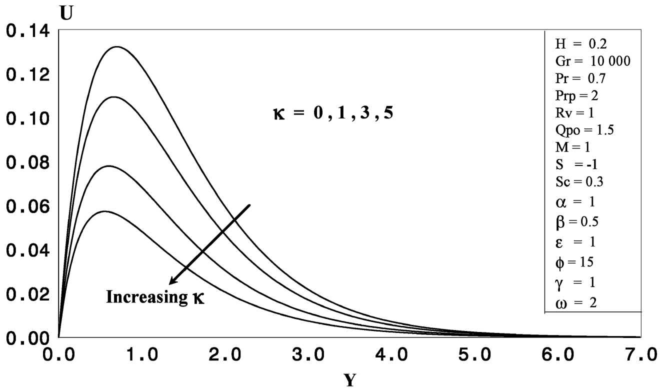

Fig. 11. Effect of κ on the fluid-phase velocity profiles

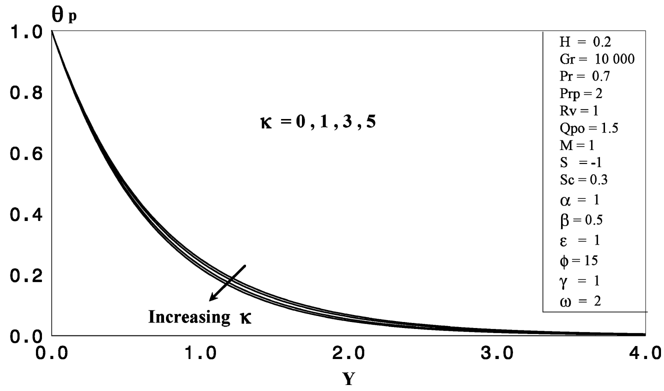

Fig. 14. Effect of κ on the particle-phase temperature profiles

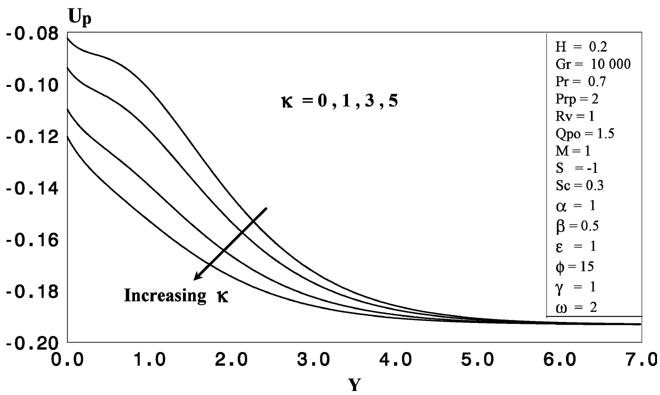

Fig. 12. Effect of κ on the particle-phase velocity profiles

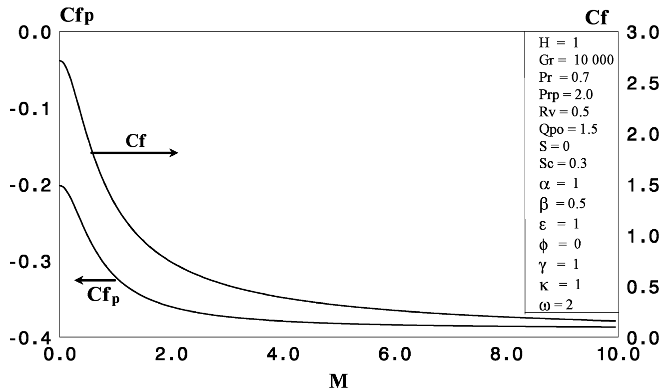

Fig. 15. Effects of Hartmann number on the fluid- and particle-phase skin friction coefficients

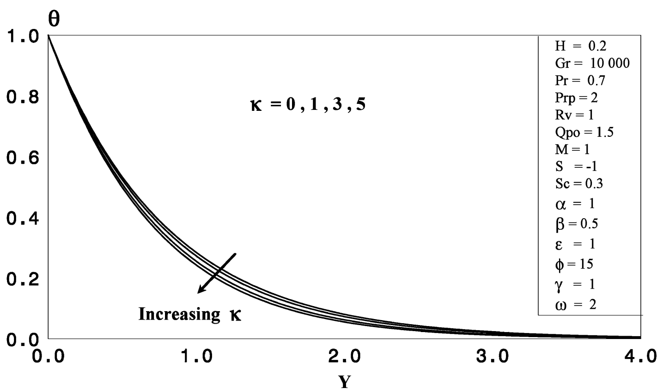

Fig. 13. Effect of κ on the fluid-phase temperature profiles

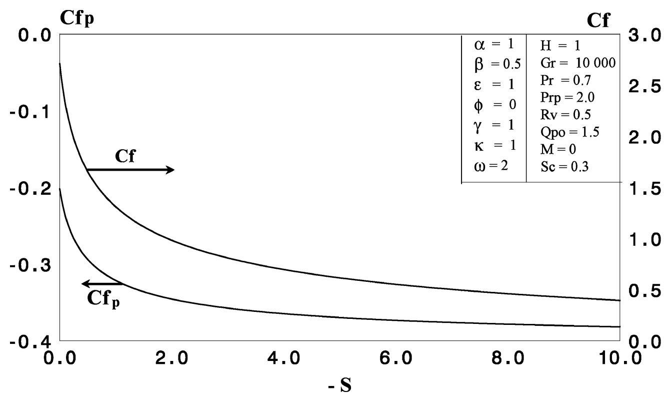

Fig. 16. Effects of heat absorption on the fluid- and particle-phase skin friction coefficients

in the profiles of both θ and θ_p as the particle loading increases shown in Fig. 13 and Fig. 14, respectively.

The combined effects of the magnetic field and heat absorption on the coefficients of friction of both phases are shown in Fig. 15 and Fig. 16, respectively. The observed reduction of C_f with the increase of both of these effects is due to the damping effect that they produce on the flow. The same damping effect on the fluid will impose a smaller opposing force to the particle motion due to the gravity force. This has the effect of increasing C_{fp} as these damping effects are increased.

Finally, the effect of heat absorption on the Nusselt number is shown in Fig. 17. The Nusselt number increases with the increase of the heat absorption. This is due to the fact that as the heat absorption increases, the plate tends to supply more energy through its wall to maintain the constant temperature of the wall, and hence the Nusselt number increases.

6 Conclusion

The mathematical modeling for free convection flow of a particulate suspension over an infinite, inclined, permeable, and isothermal plate in the presence of both magnetic

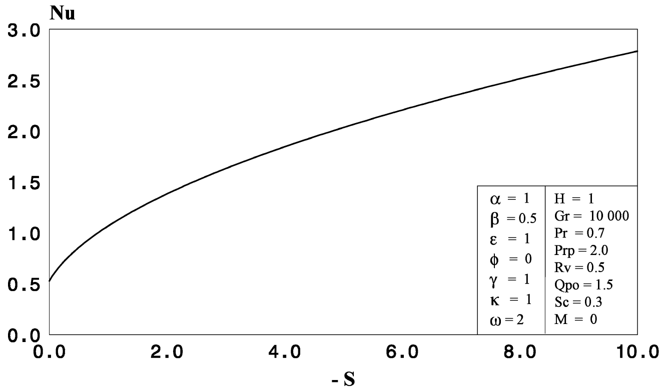

Fig. 17. Effect of heat absorption on the Nusselt number

field and fluid heat absorption effects was studied. The employed model accounted for both particle viscous and diffusive effects. The particle-phase density distribution was assumed to be variable in the domain of interest. An analytical solution was developed for the particle-phase density and a numerical solution was obtained for the remaining variables. The general effects of the magnetic field and the heat absorbing fluid were found to damp the flow and the thermal profiles. However, the Nusselt number was found to enhance with the heat absorption effects. While the fluid-phase skin friction coefficient decreased with the increase of the magnetic field strength or the heat absorption effects, the particle-phase skin friction coefficient was increased. This was due to the damping effect that both the magnetic and heat absorption effects introduced on the flow.

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