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International Journal of Engineering Science 42 (2004) 217–230
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption

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Received 13 November 2002; accepted 6 January 2003

Abstract

The problem of unsteady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects is considered. The plate is assumed to move with a constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. Time-dependent wall suction is assumed to occur at the permeable surface. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The obtained analytical results reduce to previously published results on a special case of the problem. Numerical evaluation of the analytical results is performed and some graphical results for the velocity, temperature and concentration profiles within the boundary layer and tabulated results for the skin-friction coefficient, Nusselt number and the Sherwood number are presented and discussed.

1. Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport.

Cheng and Minkowycz [1] have presented similarity solutions for free thermal convection from a vertical plate in a fluid-saturated porous medium. The problem of combined thermal convection

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0020-7225/$ - see front matter
doi:10.1016/S0020-7225(03)00285-4

Nomenclature

A	suction velocity parameter
B_0	magnetic induction
c	concentration
c_p	specific heat at constant pressure
C	dimensionless concentration
C_f	skin-friction coefficient
D	mass diffusion coefficient
G_c	solutal Grashof number
G_T	thermal Grashof number
g	acceleration due to gravity
K	permeability of the porous medium
k	thermal conductivity
M	magnetic field parameter
N	dimensionless material parameter
n	dimensionless exponential index
Nu	Nusselt number
Pr	Prandtl number
Q_0	heat absorption coefficient
Re_x	local Reynolds number
Sc	Schmidt number
Sh	Sherwood number
T	temperature
t	dimensionless time
U_0	scale of free stream velocity
u, v	components of velocities along and perpendicular to the plate, respectively
V_0	scale of suction velocity
x, y	distances along and perpendicular to the plate, respectively

Greek symbols

α	fluid thermal diffusivity
β_c	coefficient of volumetric concentration expansion
β_T	coefficient of volumetric thermal expansion
ε	scalar constant ($\ll 1$)
η	dimensionless normal distance
ϕ	dimensionless heat absorption coefficient
σ	fluid electrical conductivity
ρ	fluid density
μ	fluid dynamic viscosity
ν	fluid kinematic viscosity
τ	friction coefficient
θ	dimensionless temperature

Superscripts
$'$ differentiation with respect to y
$*$ dimensional properties

Subscripts
p plate
w wall condition
∞ free stream condition

from a semi-infinite vertical plate in the presence or absence of a porous medium has been studied by many authors (see, for example [2–5]). Nakayama and Koyama [4] have studied pure, combined and forced convection in Darcian and non-Darcian porous media. Lai and Kulacki [6] has investigated coupled heat and mass transfer by mixed convection from an isothermal vertical plate in a porous medium. Hsieh et al. [5] has presented non-similar solutions for combined convection in porous media. Chamkha [7] has investigated hydromagnetic natural convection from a isothermal inclined surface adjacent to a thermally stratified porous medium.

There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary-layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow has attracted the interest of many investigators in view of its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. For example, Rapits et al. [8] have analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [9] have studied mixed convection from a vertical plate embedded in a porous medium in the presence of a magnetic field. Gribben [10] has studied boundary-layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtl numbers using the method of matched asymptotic expansion. Takhar and Ram [11] have studied the effects of Hall currents on hydromagnetic free convection boundary-layer flow via a porous medium past a plate using harmonic analysis. Takhar and Ram [12] have also studied the MHD free porous convection heat transfer of water at 4 °C through a porous medium. Soundalegkar [13] obtained approximate solutions for two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity, the difference between the temperature of the plate and the free steam is moderately large causing the free convection currents. Raptis and Kafousias [14] have studied the influence of a magnetic field on steady free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [15] has studied mathematically the case of time-varying two-dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Bian et al. [16] have reported on the effect of an electromagnetic field on natural convection in an inclined porous medium. Buoyancy-driven convection in a rectangular enclosure with a transverse magnetic field has been considered by Garandet et al. [17] and Khanafer and Chamkha [18]. A great number of Darcian porous MHD studies have been performed examining the effects of magnetic field on hydrodynamic flow without heat transfer in various configurations, e.g., in channels and past plates, wedges, etc. [19,20].

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and/or endothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by such authors as Acharya and Goldstein [21], Vajravelu and Nayfeh [22] and Chamkha [23,24].

The objective of this paper is to consider unsteady simultaneous convective heat and mass transfer flow along a vertical permeable plate embedded in a fluid-saturated porous medium in the presence of mass blowing or suction, magnetic field effects, and absorption effects. Most of previous works assumed that the semi-infinite plate is at rest. In the present work, it is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. It is also assumed that the free stream to consist of a mean velocity and temperature over which are superimposed an exponentially varying with time.

2. Problem formulation

Consider unsteady two-dimensional flow of a laminar, incompressible, viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects (see Fig. 1). It is assumed that there is no applied voltage which implies the absence of an electrical field. The transversely applied magnetic

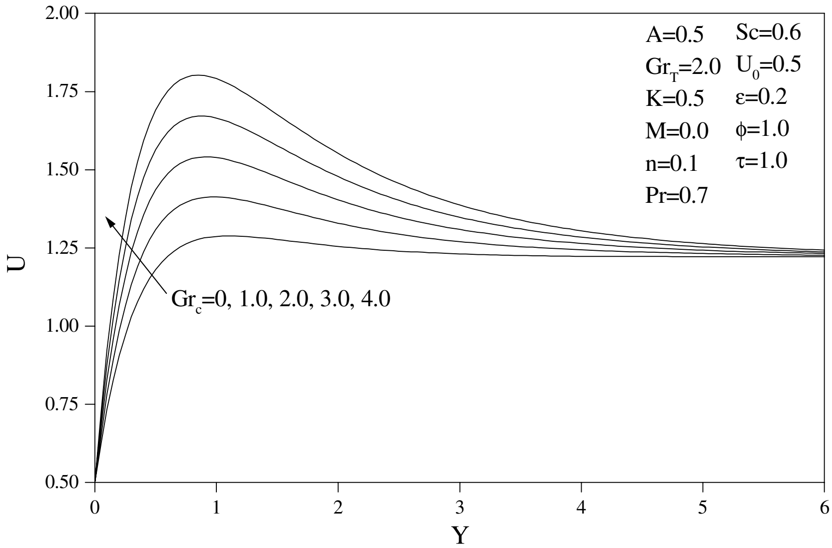

Fig. 1. Effects of Gr_c on velocity profiles.

field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible [25]. A consequence of the small magnetic Reynolds number is the uncoupling of the Navier–Stokes equations from Maxwell's equations [25]. The governing equations for this investigation are based on the balances of mass, linear momentum, energy and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + v\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_{\mathrm{T}}(T - T_\infty) + g\beta_{\mathrm{c}}(c - c_\infty) - v\frac{u^*}{K^*} - \frac{\sigma}{\rho}B_0^2 u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha\frac{\partial^2 T}{\partial y^{*2}} - \frac{Q_0}{\rho c_p}(T - T_\infty) \tag{3}$$

$$\frac{\partial c}{\partial t^*} + v^* \frac{\partial c}{\partial y^{*2}} = D\frac{\partial^2 c}{\partial y^{*2}} \tag{4}$$

where x^*, y^*, and t^* are the dimensional distances along and perpendicular to the plate and dimensional time, respectively. u^* and v^* are the components of dimensional velocities along x^* and y^* directions, respectively, ρ is the fluid density, v is the kinematic viscosity, c_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, T is the dimensional temperature, Q_0 is the dimensional heat absorption coefficient, c is the dimensional concentration, α is the fluid thermal diffusivity, D is the mass diffusivity, g is the gravitational acceleration, and β_{T} and β_{c} are the thermal and concentration expansion coefficients, respectively. The magnetic and viscous dissipations are neglected in this study. The third and fourth terms on the RHS of the momentum Eq. (2) denote the thermal and concentration buoyancy effects, respectively. Also, the last term of the energy Eq. (3) represents the heat absorption effects. It is assumed that the permeable plate moves with a constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time.

Under these assumptions, the appropriate boundary conditions for the velocity, temperature and concentration fields are

$$u^* = u_{\mathrm{p}}^*, \quad T = T_{\mathrm{w}} + \varepsilon(T_{\mathrm{w}} - T_\infty)\mathrm{e}^{n^*t^*}, \quad c = c_{\mathrm{w}} + \varepsilon(c_{\mathrm{w}} - c_\infty)\mathrm{e}^{n^*t^*} \text{ at } y^* = 0 \tag{5}$$

$$u^* \to U_\infty^* = U_0(1 + \varepsilon\mathrm{e}^{n^*t^*}), \quad T \to T_\infty, \quad c \to c_\infty \text{ as } y^* \to \infty \tag{6}$$

where u_{p}^*, c_{w} and T_{w} are the wall dimensional velocity, concentration and temperature, respectively. U_∞^*, c_∞ and T_∞ are the free stream dimensional velocity, concentration and temperature, respectively. U_0 and n^* are constants.

It is clear from Eq. (1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form:

$$v^* = -V_0(1 + \varepsilon A e^{n^* t^*}) \tag{7}$$

where A is a real positive constant, ε and εA are small less than unity, and V_0 is a scale of suction velocity which has non-zero positive constant. Outside the boundary layer, Eq. (2) gives

$$-\frac{1}{\rho}\frac{d\rho^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\nu}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 U_\infty^* \tag{8}$$

It is convenient to employ the following dimensionless variables:

$$\begin{aligned} &u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad \eta = \frac{V_0 y^*}{\nu}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad U_p = \frac{u_p^*}{U_0}, \\ &t = \frac{t^* V_0^2}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{c - c_\infty}{c_w - c_\infty}, \quad n = \frac{n^* \nu}{V_0^2}, \\ &K = \frac{K^* V_0^2}{\nu^2}, \quad Pr = \frac{\nu \rho c_p}{k} = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \\ &G_T = \frac{\nu \beta_T g (T_w - T_\infty)}{U_0 V_0^2}, \quad G_c = \frac{\nu \beta_c g (c_w - c_\infty)}{U_0 V_0^2}, \\ &\phi = \frac{\nu Q_0}{\rho c_p V_0^2} \end{aligned} \tag{9}$$

In view of Eqs. (7)–(9), Eqs. (2)–(4) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial \eta} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial \eta^2} + G_T\theta + G_c C + N(U_\infty - u) \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} - \phi\theta \tag{11}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial C}{\partial \eta} = \frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2} \tag{12}$$

where

$$N = \left(M + \frac{1}{K}\right)$$

and G_c, G_T, Pr, ϕ and Sc are the solutal Grashof number, thermal Grashof number, Prandtl number, dimensionless heat absorption coefficient, and the Schmidt number, respectively. By setting G_c and ϕ equal to zero and ignoring Eq. (12), Eqs. (10) and (11) reduce to those reported by Kim [26].

The dimensionless form of the boundary conditions (5) and (6) become

$$u = U_{\mathrm{p}}, \quad \theta = 1 + \varepsilon \mathrm{e}^{nt}, \quad C = 1 + \varepsilon \mathrm{e}^{nt} \ \text{ at } \eta = 0 \tag{13}$$

$$u \to U_{\infty}, \quad \theta \to 0, \quad C \to 0 \ \text{ as } \eta \to \infty \tag{14}$$

3. Problem solution

Eqs. (10)–(12) represent a set of partial differential equations that can not be solved in closed-form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

$$u = f_0(\eta) + \varepsilon \mathrm{e}^{nt} f_1(\eta) + \mathrm{O}(\varepsilon^2) + \cdots \tag{15}$$

$$\theta = g_0(\eta) + \varepsilon \mathrm{e}^{nt} g_1(\eta) + \mathrm{O}(\varepsilon^2) + \cdots \tag{16}$$

$$C = h_0(\eta) + \varepsilon \mathrm{e}^{nt} h_1(\eta) + \mathrm{O}(\varepsilon^2) + \cdots \tag{17}$$

Substituting Eqs. (15)–(17) into Eqs. (10)–(12), equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of $\mathrm{O}(\varepsilon^2)$, one obtains the following pairs of equations for (f_0, g_0, h_0) and (f_1, g_1, h_1).

$$f_0'' + f_0' - N f_0 = -N - G_{\mathrm{T}} g_0 - G_{\mathrm{c}} h_0 \tag{18}$$

$$f_1'' + f_1' - (N + n) f_1 = -(N + n) - A f_0' - G_{\mathrm{T}} g_1 - G_{\mathrm{c}} h_1 \tag{19}$$

$$g_0'' + Pr g_0' - Pr\phi g_0 = 0 \tag{20}$$

$$g_1'' + Pr g_1' - nPr g_1 - Pr\phi g_1 = -APr g_0' \tag{21}$$

$$h_0'' + Sc h_0' = 0 \tag{22}$$

$$h_1'' + Sc h_1' - nSc h_1 = -ASc h_0' \tag{23}$$

where a prime denotes ordinary differentiation with respect to η. The corresponding boundary conditions can be written as

$$f_0 = U_{\mathrm{p}}, \quad f_1 = 0, \quad g_0 = 1, \quad g_1 = 1, \quad h_0 = 1, \quad h_1 = 1 \ \text{ at } \eta = 0 \tag{24}$$

$$f_0 = 1, \quad f_1 = 1, \quad g_0 \to 0, \quad g_1 \to 0, \quad h_0 \to 0, \quad h_1 \to 0 \ \text{ as } \eta \to \infty \tag{25}$$

Without going into detail, the solutions of Eqs. (18)–(23) subject to Eqs. (24) and (25) can be shown to be

$$f_0 = 1 + C_3 e^{-\lambda_1 \eta} + B_1 e^{-m_1 \eta} + C_1^* e^{-Sc\eta} \tag{26}$$

$$f_1 = 1 + C_4 e^{-\lambda_3 \eta} + B_3 e^{-m_3 \eta} + D_3 e^{-m_1 \eta} + E_3 e^{-\lambda_2 \eta} + F_3 e^{-Sc\eta} + G_3 e^{-\lambda_1 \eta} \tag{27}$$

$$g_0 = e^{-m_1 \eta} \tag{28}$$

$$g_1 = (1 - A_2) e^{-m_3 \eta} + A_2 e^{-m_1 \eta} \tag{29}$$

$$h_0 = e^{-Sc\eta} \tag{30}$$

$$h_1 = e^{-\lambda_2 \eta} + ASc(e^{-\lambda_2 \eta} - e^{-Sc\eta})/n \tag{31}$$

where

$$m_1 = \frac{Pr + \sqrt{Pr^2 + 4Pr\phi}}{2} \tag{32}$$

$$m_3 = \frac{Pr + \sqrt{Pr^2 + 4(\phi + n)}}{2} \tag{33}$$

$$\lambda_1 = \frac{1 + \sqrt{1 + 4N}}{2} \tag{34}$$

$$\lambda_3 = \frac{1 + \sqrt{1 + 4(N + n)}}{2} \tag{35}$$

$$\lambda_2 = \frac{Sc + \sqrt{Sc^2 + 4nSc}}{2} \tag{36}$$

and

$$C_3 = U_p - 1 - B_1 - C_1^* \tag{37}$$

$$B_1 = \frac{G_T}{-m_1^2 + m_1 + N} \tag{38}$$

$$C_1^* = \frac{G_c}{-Sc^2 + Sc + N} \tag{39}$$

$$A_2 = \frac{m_1}{m_1^2 - Prm_1 - (\phi + n)} \tag{40}$$

$$C_4 = -(1 + B_3 + D_3 + E_3 + F_3 + G_3) \tag{41}$$

$$B_3 = \frac{-G_T(1 - A_2)}{m_3^2 - m_3 - (N + n)} \tag{42}$$

$$D_3 = \frac{-G_T A_2 + A m_1 B_1}{m_1^2 - m_1 - (N+n)} \tag{43}$$

$$E_3 = \frac{-G_c(1 + ASc/n)}{h_3^2 - h_3 - (N+n)} \tag{44}$$

$$F_3 = \frac{G_c ASc/n + AScC_1^*}{Sc^2 - Sc - (N+n)} \tag{45}$$

$$G_3 = \frac{A\lambda_1 C_3}{\lambda_1^2 - \lambda_1 - (N+n)} \tag{46}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(\eta,t) = 1 + C_3 e^{-\lambda_1\eta} + B_1 e^{-m_1\eta} + C_1^* e^{-Sc\eta} + \varepsilon e^{nt}(1 + C_4 e^{-\lambda_3\eta} + B_3 e^{-m_3\eta} + D_3 e^{-m_1\eta} + E_3 e^{-\lambda_2\eta} + F_3 e^{-Sc\eta} + G_3 e^{-\lambda_1\eta}) \tag{47}$$

$$\theta(\eta,t) = e^{-m_1\eta} + \varepsilon e^{nt}((1 - A_2)e^{-m_3\eta} + A_2 e^{-m_1\eta}) \tag{48}$$

$$C(\eta,t) = e^{-Sc\eta} + \varepsilon e^{nt}(e^{-\lambda_2\eta} + ASc(e^{-\lambda_2\eta} - e^{-Sc\eta})/n) \tag{49}$$

The skin-friction coefficient, the Nusselt number and the Sherwood number are important physical parameters for this type of boundary-layer flow. These parameters can be defined and determined as follows:

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} = 1 - \lambda_1 C_3 - m_1 B_1 - ScC_1^* + \varepsilon e^{nt}(1 - \lambda_3 C_4 - m_3 B_3 - m_1 D_3 - \lambda_2 E_3 - ScF_3 - \lambda_1 G_3) \tag{50}$$

$$Nu = x\frac{\partial T/\partial y^*|_{y^*=0}}{T_w - T_\infty} \Rightarrow NuRe_x^{-1} = \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} = -m_1 + \varepsilon e^{nt}(-m_3(1 - A_2) - m_1 A_2) \tag{51}$$

$$Sh = x\frac{\partial c/\partial y^*|_{y^*=0}}{c_w - c_\infty} \Rightarrow ShRe_x^{-1} = \left.\frac{\partial C}{\partial \eta}\right|_{\eta=0} = -Sc + \varepsilon e^{nt}(-\lambda_2 + ASc(-\lambda_2 + Sc)/n) \tag{52}$$

where $Re_x = V_0 x/v$ is the local Reynolds number. It should be mentioned that in the absence of the concentration buoyancy and heat absorption effects, all of the flow and heat transfer solutions reported above are consistent with those reported earlier by Kim [26].

4. Results and discussion

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically in Figs. 1–5. These results are obtained to illustrate the influence of the solutal Grashof number Gr_c, the heat absorption coefficient ϕ and

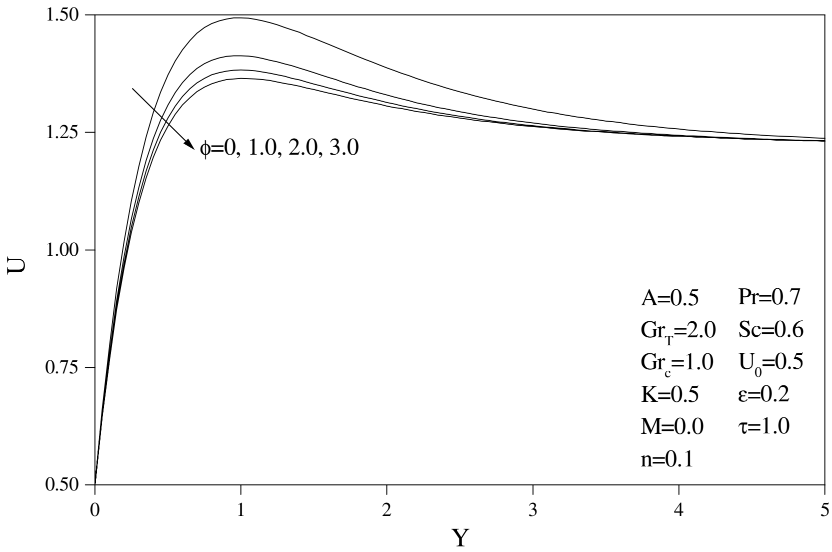

Fig. 2. Effects of ϕ on velocity profiles.

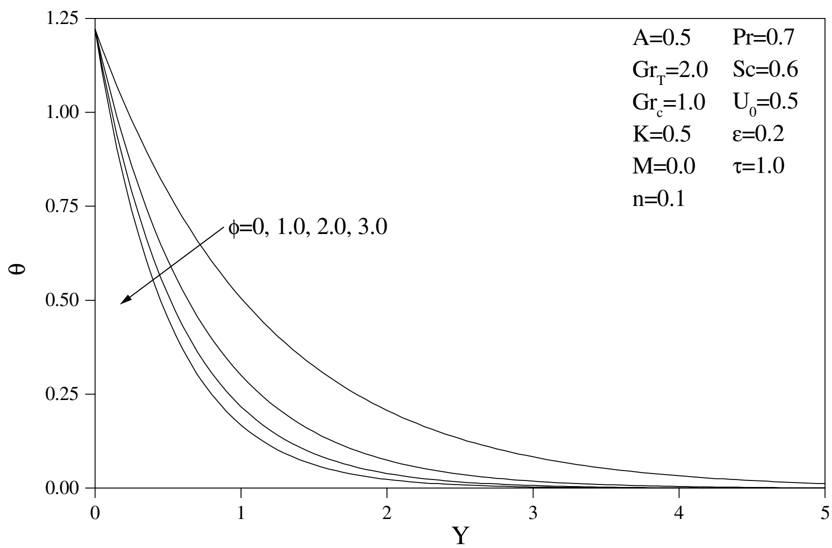

Fig. 3. Effects of ϕ on temperature profiles.

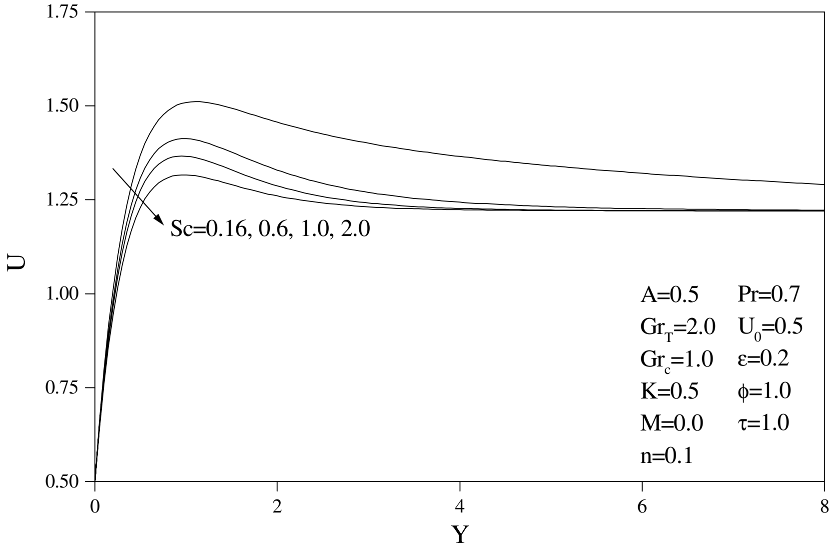

Fig. 4. Effects of *Sc* on velocity profiles.

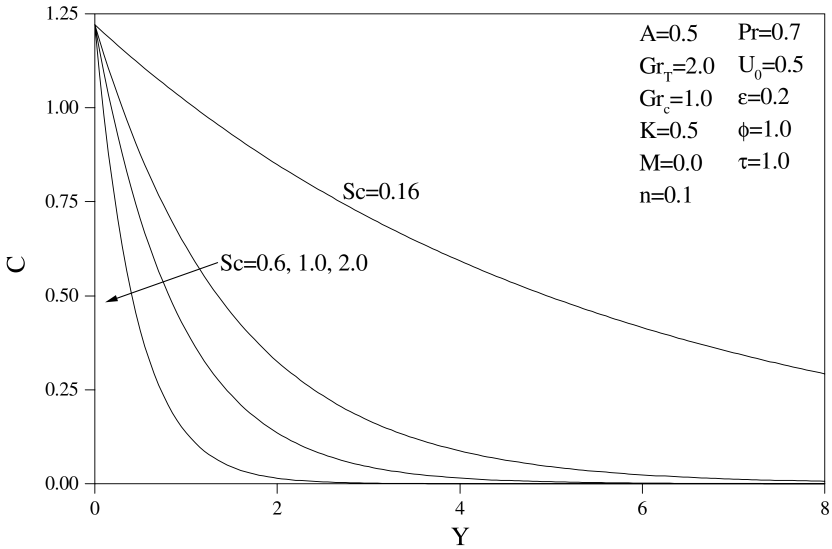

Fig. 5. Effects of *Sc* on concentration profiles.

the Schmidt number *Sc* on the velocity, temperature and the concentration profiles. The effects of the other physical parameters on the solutions were reported previously by Kim [26] and, therefore, will not be repeated herein.

Fig. 1 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gr_c while all other parameters are kept at some fixed values with $\tau = 1$. The velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then decreases properly to approach the free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increases in the concentration buoyancy effects represented by Gr_c. This is evident in the increases of U as Gr_c increases in Fig. 1.

Figs. 2 and 3 illustrate the influence of the heat absorption coefficient ϕ on the velocity and temperature profiles at $\tau = 1$, respectively. Physically speaking, the presence of heat absorption (thermal sink) effects has the tendency to reduce the fluid temperature. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity. These behaviors are clearly obvious from Figs. 2 and 3 in which both the velocity and temperature distributions decrease as ϕ increases. It is also observed that both the hydrodynamic (velocity) and the thermal (temperature) boundary layers decrease as the heat absorption effects increase.

Figs. 4 and 5 display the effects of the Schmidt number Sc on the velocity and concentration profiles at $\tau = 1$, respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clearly shown in Figs. 4 and 5.

Table 1
Effects of Gr_c on C_f, Nu/Re_x and Sh/Re_x for the reference values in Fig. 1

G_c	C_f	Nu/Re_x	Sh/Re_x
0	2.7200	−1.7167	−0.8098
1	3.2772	−1.7167	−0.8098
2	3.8343	−1.7167	−0.8098
3	4.3915	−1.7167	−0.8098
4	4.9487	−1.7167	−0.8098

Table 2
Effects of ϕ on C_f, Nu/Re_x and Sh/Re_x for the reference values in Fig. 2

ϕ	C_f	Nu/Re_x	Sh/Re_x
0	3.4595	−1.0699	−0.8098
1	3.2772	−1.7167	−0.8098
2	3.1933	−2.1193	−0.8098
3	3.1378	−2.4388	−0.8098

Table 3
Effects of Sc on C_f, Nu/Re_x and Sh/Re_x for the reference values in Fig. 4

Sc	C_f	Nu/Re_x	Sh/Re_x
0.16	3.4328	−1.7167	−0.2231
0.6	3.2772	−1.7167	−0.8098
1	3.1847	−1.7167	−1.3425
2	3.0481	−1.7167	−2.6741

Tables 1–3 depict the effects of the solutal Grashof number Gr_c, the heat absorption coefficient ϕ and the Schmidt number Sc on the skin-friction coefficient C_f, Nusselt number Nu and the Sherwood number Sh, respectively. It is observed from these tables that as Gr_c increases, the skin-friction coefficient increases whereas the Nusselt and Sherwood numbers remain unchanged. However, as the heat absorption effects increase, both the skin-friction coefficient and the Nusselt number decrease whereas the Sherwood number remains unaffected. Also, increases in the Schmidt number cause reductions in the skin-friction coefficient and the Sherwood number while the Nusselt number remains constant.

5. Conclusions

The governing equations for unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat absorption was formulated. The plate velocity was maintained at a constant value and the flow was subjected to a transverse magnetic field. The resulting partial differential equations were transformed into a set of ordinary differential equations using two-term series and solved in closed-form. Numerical evaluations of the closed-form results were performed and some graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some of the physical parameters. It was found that when the solutal Grashof number increased, the concentration buoyancy effects were enhanced and thus, the fluid velocity increased. However, the presence of heat absorption effects caused reductions in the fluid temperature which resulted in decreases in the fluid velocity. Also, when the Schmidt number was increased, the concentration level was decreased resulting in a decreased fluid velocity. In addition, it was found that the skin-friction coefficient increased due to increases in the concentration buoyancy effects while it decreased due to increases in either of the heat absorption coefficient or the Schmidt number. However, the Nusselt number decreased as the heat absorption coefficient was increased and the Sherwood number decreased as the Schmidt number was increased.

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