NATURAL CONVECTION MHD FLOW ON A CONTINUOUS MOVING INCLINED SURFACE EMBEDDED IN A NON-DARCIAN HIGH-POROSITY MEDIUM

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The flow and heat transfer characteristics of the steady laminar incompressible boundary layer induced by an inclined surface that moves with non-uniform velocity in a high-porosity ambient medium have been studied. We have considered the effects of the buoyancy forces, the magnetic field and the non-Darcy parameter in our analysis. The coupled nonlinear parabolic partial differential have been solved numerically by using an implicit finite-difference scheme. The effects of inertia and porosity on the skin friction and heat transfer are found to be more pronounced than that of the permeability parameter. The buoyancy force significantly affects the heat transfer. The magnetic field increases the skin friction, but reduces the heat transfer.

Key Words: MHD Flow; Heat Transfer; Porosity

1. INTRODUCTION

The flow and heat transfer on the boundary layer induced by a surface moving with uniform or non-uniform velocity in an otherwise ambient fluid have many applications in the field of metallurgy and chemical engineering. Sakkiadis1 was the first to study the flow due to a solid surface moving with a constant velocity in an ambient fluid. The corresponding heat transfer problem was studied theoretically and experimentally by Tsou et al.2 and theoretically by Erickson et al.3, and experimentally by Griffin and Thorne4. Since then several investigators5-16 have considered various aspects of this problem such as heat transfer with prescribed wall temperature or heat flux, mass transfer, non-uniform wall velocity, magnetic field, suction or injection and parallel free stream velocity. In almost all these cases, self-similar solutions were obtained. However, Jeng et al.17, Chiam18 and Takhar et al.19 have obtained non-similar solutions. The effect of the buoyancy force on the flow and heat transfer on an inclined surface moving with a constant velocity in an ambient fluid was studied by Moutsoglou and Chen20.

Natural convection flow over vertical surfaces immersed in porous media occurs in many engineering problems such as those in the design of pebble-bed nuclear reactors, catalytic reactors and compact heat exchangers, in geothermal energy conversion, in petroleum reservoirs, in use of fibrous materials in the thermal insulation of buildings, in nuclear wastes etc. Excellent reviews of the natural convection flows in porous media have been given by Cambarious and Bories21, Bejan22,23 and Tien and Vafai24. The natural convection flow over a vertical heated surface in a
porous medium has been investigated by Bejan and Khair, and Nakayama and Koyama. The non-Darcy effect on the natural convection boundary layer on an isothermal vertical flat plate embedded in a high-porosity medium was considered by Chen et al. Some recent studies on this topic were carried out by a number of investigators.

The aim of this analysis is to study the natural convection boundary layer flow with a magnetic field induced by an inclined surface that moves with non-uniform velocity \( U = U_0 (1 + x/L) \), where \( x \) is the streamwise distance measured from the leading edge of the surface, \( U_0 \) is the surface velocity at \( x = 0 \) and \( L \) is the characteristic length, in an otherwise ambient fluid. The buoyancy forces arise due to the temperature difference between the fluid and the wall. Both the constant wall temperature and the constant heat flux conditions have been included in the analysis. The coupled nonlinear parabolic partial differential equations have been solved numerically by using a finite-difference scheme similar to that of Blottner. For a uniform medium (without porosity), the results have been compared with the theoretical and experimental results of Tsou et al., the theoretical results of Erickson et al., the experimental results of Griffin and Thorne, and the theoretical results of Jeng et al., and Moutsoglou and Chen.

2. ANALYSIS

Let us consider the steady laminar natural convection flow of an incompressible viscous and electrically conducting fluid on a heated inclined plate maintained at a constant temperature \( T_w \). The plate is inclined at an acute angle \( \Omega \) from the vertical and it is placed in a high-porosity ambient fluid. The plate is moving with velocity \( U = U_0 (1 + x/L) \) along the x-direction. Fig. 1 shows the...
physical model and the coordinate systems. The fluid properties are assumed to be constant except
the density changes which give rise to the buoyancy forces. The magnetic field \( B \) is applied normal
to the plate which is electrically non-conducting. The viscous and Ohmic dissipation terms have been
neglected in the energy equation. It is assumed that the magnetic Reynolds number \( Re_m = \mu_0 \sigma V L \)
\( \ll 1 \), where \( \mu_0 \) and \( \sigma \) are, respectively, the magnetic permeability and the electrical conductivity,
and \( V \) and \( L \) are the characteristic velocity and length, respectively. Under this condition, it is possible
to neglect the induced magnetic field in comparison to the applied magnetic field. Since there is no
applied or polarized voltage imposed on the flow field, the electric field \( \mathbf{E} = 0 \). Hence only the
applied magnetic field contributes to the Lorentz force which acts in the \( x \)-direction along the plate.
Since the plate is inclined, both the streamwise pressure term and the buoyancy force term exist,
but they have different magnitudes depending on the inclination angle. The buoyancy induced
streamwise pressure gradient term can be neglected in comparison to the buoyancy force term if
\( \Omega \leq \Omega_0 \). Under the above assumptions, the equations of continuity, momentum and energy under
the boundary layer approximations governing the natural convection over a moving inclined plate in
a high-porosity non-Darcy medium can be expressed as\( ^{17,29,35} \).

\[
\begin{align*}
    u_x + v_y &= 0, \quad \text{(1)} \\
    \varepsilon^{-2} (u u_x + v u_y) &= \varepsilon^{-1} v v_y + \frac{g \beta}{\rho} (T - T_\infty) \cos \Omega - \sigma B^2 \frac{u}{\rho} \quad \text{(2)} \\
    u T_x + v T_y &= \alpha T_{xx} \quad \text{(3)}
\end{align*}
\]

The boundary conditions are the no-slip conditions at the wall and the ambient conditions
at the edge of the boundary layer and these can be expressed as

\[
\begin{align*}
    u(x, 0) &= U(x), \quad v(x, 0) = 0, \quad u(x, \infty) = 0, \quad T(x, \infty) = T_w, \\
    T(x, 0) &= T_w \quad \text{for the constant wall temperature case (CWT case),} \\
    \delta T(x, 0) / \delta y &= -q_w / K_1 \quad \text{for the constant heat flux case (CHF case),} \\
    U(0, y) &= 0, \quad T(0, y) = T_w, \quad y > 0. \quad \text{(4)}
\end{align*}
\]

Here \( x \) and \( y \) are the distances along and perpendicular to the surface, respectively; \( u \) and
\( v \) are the velocity components along \( x \) and \( y \) directions, respectively; \( \varepsilon \) is the porosity; \( \rho \) and \( \nu \) are
the density and kinematic viscosity, respectively; \( T \) is the temperature; \( g \) is the acceleration due to
gravity; \( \beta \) is the volumetric coefficient of thermal expansion; \( B \) is the magnetic field; \( K^* \) is the
permeability of the medium; \( C^* \) is the inertia coefficient; \( \alpha \) is the thermal diffusivity; \( q_w \) is the heat
flux at the surface; \( K_1 \) is the thermal conductivity; the subscripts \( t, x \) and \( y \) denote derivatives with
respect to \( t, x \) and \( y \), respectively; and the subscripts \( w \) and \( \infty \) denote conditions at the wall and in
the ambient fluid, respectively.

It is convenient to express equations (1) - (3) in dimensionless form and to reduce the
number of equations from three to two by introducing the stream function. The required
transformations are given by

\[ \eta = (2 \xi \nu)^{-1/2} U \eta, \quad \xi = x/L, \quad U = U_0 (1 + \xi), \quad u = \partial \Psi / \partial y. \]

\[ v = -\partial \Psi / \partial x, \quad \Psi (x, y) = (2 \nu \xi)^{1/2} F(\xi, \eta), \quad \xi = U_0 L \xi (1 + \xi/2). \]

\[ G(\xi, \eta) = (T(x, y) - T_w)/(T_w - T_{\infty}) \] for the CWT case,

\[ G(\xi, \eta) = (T(x, y) - T_w) U(q_w / K)^{1/2} \] for the CWT case,

\[ \lambda = Gr_{L} / \text{Re}_{L}^2, \quad Gr_{L} = g \beta (T_w - T_{\infty}) L^3 / \sqrt{\nu} \] for the CWT case,

\[ \lambda^* = Gr_{L}^{1/2} / \text{Re}_{L}^{1/2}, \quad Gr_{L} = g \beta q_w L^3 / (K \nu^3) \] for the CHF case,

\[ \text{Re}_{L} = U_0 L / \nu, \quad M = Ha / \text{Re}_{L}, \quad Ha = \sigma B^2 L^2 / \mu, \quad Pr = \nu / \alpha, \]

\[ K = (K^* / L^3) \text{Re}_{L}, \quad C = C^* L. \quad \ldots \ (5) \]

Using (5) in eqs. (1)-(3), we find that equation (1) is identically satisfied and equations (2) and (3) for the CWT case reduce to

\[ \epsilon^{-1} F'' + \epsilon^{-2} (F'' - s_1 F') + \lambda^* \gamma = \Omega - M \xi, \quad F'' = F' / \partial F / \partial \xi, \]

\[ - (K^{-1} s_2 F' + C_{R, 3} F') = \epsilon^{-3} s_3 (F' / \partial F / \partial \xi - F'' / \partial F / \partial \xi), \quad \ldots \ (6) \]

\[ Pr^{-1} G'' + FG' = s_3 (F' / \partial G / \partial \xi - G' / \partial F / \partial \xi), \quad \ldots \ (7) \]

with boundary conditions

\[ F(\xi, 0) = 0, \quad F'(\xi, 0) = 0, \quad G(\xi, 0) = 0, \quad F'(\xi, \infty) = G(\xi, \infty) = 0, \quad \ldots \ (8) \]

where

\[ s_1 = (2 \xi / U) (dU / d\xi) = 2 \xi (1 + \xi/2) (1 + \xi)^{-2}, \]

\[ s_2 = (2 \xi / U_0 L) (U / U_0)^2 = 2 \xi (1 + \xi/2) (1 + \xi)^{-2}, \]

\[ s_3 = (2 \xi / UL) = 2 \xi (1 + \xi/2) (1 + \xi)^{-2}. \quad \ldots \ (9) \]

The corresponding equations for CHF case are given by
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\[ \varepsilon^{-2} F^{\prime\prime\prime} + \varepsilon^{-2} (F F^{\prime\prime} - s_1 F^{\prime}) + \lambda^* s_2 G \cos \Omega - M s_2 F^{\prime} \]

\[ - (K^{-1} s_2 F^{\prime} + C s_3 F^{\prime}) = \varepsilon^{-2} s_3 (F^{\prime} \partial F^{\prime}/\partial \xi - F^{\prime\prime} \partial F/\partial \xi), \]

\[ Pr^{-1} G^{\prime} + F G^{\prime} - s_3 F^{\prime} G = s_3 (F^{\prime} \partial G/\partial \xi - G^{\prime} \partial F/\partial \xi), \]

with boundary conditions

\[ F(\xi, 0) = 0, F^{\prime}(\xi, 0) = 1, G^{\prime}(\xi, 0) = 1, F^{\prime}(\xi, \infty) = G(\xi, \infty) = 0, \]

where

\[ s_4 = (2 \xi/U_0 L)^{3/2} (U/U_0) \]

\[ s_5 = 1 - (2 \xi/L) (dU/d\xi) U^{-2} = 1 - s_1. \]

Here \( \xi \) and \( \eta \) are the transformed coordinates; \( \psi \) and \( F \) are the dimensional and dimensionless stream functions, respectively; \( G \) is the dimensionless temperature; \( F^{\prime} \) is the dimensionless velocity; \( Re_L \) is the Reynolds number; \( Pr \) is the Prandtl number; \( Gr_L \) and \( G^* r_L \) are the Grashof numbers for the CWT and CHF cases, respectively; \( \lambda \) and \( \lambda^* \) are the buoyancy parameters for the CWT and CHF cases, respectively; \( M \) is the magnetic parameter; \( Ha \) is the Hartmann number; \( K \) and \( C \) are the dimensionless permeability and inertia parameters, respectively; \( \mu \) is the coefficient of viscosity; \( s_i, i = 1, 2, ..., 5 \) are functions of \( \xi \) and prime denotes derivative with respect to \( \eta \).

It may be remarked that for \( \lambda = 0 \) (without buoyancy force) \( M = 0 \) (no magnetic field) \( \varepsilon = 1 \) (uniform medium), \( K^{-1} = C = 0 \) (without permeability and inertia parameters) \( \xi = 0 \) (self-similar flow) equations (6) - (8) reduce to those of Tsou et al.\(^2\) and Erickson et al.\(^5\) if we apply the transformations

\[ \eta = 2^{-1/2} \eta_1, F(\eta) = 2^{-1/2} F_1(\eta_1), G(\eta) = G_1(\eta_1). \]

to our equations. Also equations (6) - (8) for \( \lambda = M = K^{-1} = C = 0, \varepsilon = 1 \) reduce to those of Jeng et al.\(^7\). Further, equations (6) - (8) for \( M = K^{-1} = C = 0, \varepsilon = 1 \) are identical to those of Moutsoglu and Chen\(^20\) if we replace the surface velocity \( U = U_0 (1 + \xi) \) by \( U = U_0 \) and \( \lambda \xi \) by \( \xi_1 \) and use the transformations\(^14\).

The quantities of physical interest are the skin friction coefficient and the Nusselt number which are expressed as

\[ C_{fs} = -\mu (\partial U/\partial y)_{y=0} U^2 = -2^{-1/2} Re_x^{-1/2} (1 + \xi/2)^{-1/2} F^{\prime\prime}(\xi, 0). \]
\[ Nu_x = 2^{-1/2} Re_x^{-1/2} (1 + \xi)(1 + \xi/2)^{-1/2} G' (\xi, 0). \] 

For the CHF case, \( Nu_x \) is given by
\[
Nu_x = 2^{-1/2} Re_x^{-1/2} (1 + \xi)(1 + \xi/2)^{-1/2} G' (\xi, 0),
\]
where \( C_{f} \) is the local skin friction coefficient, \( Nu_x \) is the local Nusselt number and \( Re_x \) \( (= U_x u/v) \) is the local Reynolds number.

3. METHOD OF SOLUTION

The coupled nonlinear parabolic partial differential equations (6) - (7) under the conditions (8) have been solved by using an implicit, iterative tridiagonal finite-difference scheme similar to that of Blottner\textsuperscript{34}. All the first-order derivatives with respect to \( \xi \) are replaced by two-point backward difference formulae

\[
\frac{\partial R}{\partial \xi} = \frac{(R_{i,j} - R_{i-1,j})}{\Delta \xi}
\]

where \( R \) represents any dependent variable and \( i \) and \( j \) are the node locations along \( \xi \), and \( \eta \), directions, respectively. First the third-order partial differential equations (6) is converted into a second order by substituting \( F' = H \). Then the second-order partial differential equations for \( H \) and \( G \) are discretized by using three point central difference formulae and all the first-order partial differential equations are discretized by employing the trapezoidal rule. The nonlinear terms are evaluated at the previous iteration. At each time-step of constant \( \xi \), a system of algebraic equations are solved iteratively by using the well known Thomas algorithm (see Blottner\textsuperscript{34}). The same process is repeated for the next \( \xi \) value and the equations are solved line by line until the desired \( \xi \) value is reached. A convergence criterion based on the relative difference between the current and previous iterations is used. When this difference becomes \( 10^{-5} \), the solution is assumed to have converged and the iterative process is terminated.

In a similar manner, eqs. (10) and (11) under conditions (12) have been solved.

We have examined the effect of grid size \( \Delta \eta \) and \( \Delta \xi \), and the edge of the boundary layer \( \eta_{in} \) on the solution. Finally we have selected \( \Delta \eta = 0.05, \Delta \xi = 0.02, \eta_{in} = 12. \)

4. RESULTS AND DISCUSSION

Eqs. (6) and (7) under conditions (8), and eqs. (10) and (11) under conditions (12) have been solved by using the implicit finite-difference scheme described earlier. In order to assess the accuracy of our method, we have compared the velocity profile \( u/U_{0} = F' \) for \( \lambda = M = 0 \) (without buoyancy force and magnetic field), \( \xi = 0 \) (self-similar flow) with the theoretical and experimental results of Tsou et al.\textsuperscript{2} in Fig. 2. The velocity profile is found to be in very good agreement with the theoretical results. It also agrees well with the experimental values near the wall. Further the Nusselt number \( Nu_{x} \) for the CWT case when \( \lambda = M = \xi = 0 \) has been compared with the theoretical values of Erickson...
Fig. 2. Comparison of the velocity profile $u/U_0$ for $\zeta = \lambda = 0$ with the theoretical and experimental results of Tsou et al.\textsuperscript{2}.

Fig. 3. Comparison of the local Nusselt number, $N_u$, for $\zeta = \lambda = 0$ with that of Erickson et al.\textsuperscript{3} and Griffin and Thorne\textsuperscript{4}.
et al.\textsuperscript{3} and experimental values of Griffin and Thorne\textsuperscript{4}. This comparison is shown in Fig. 3 and the results are found to be in good agreement with the theoretical and experimental values when the wall velocity $U_0 \geq 8.92$. We have compared our skin friction and heat transfer results $(Re_{x}^{1/2} C_f, 2 Re_{x}^{-1/2} Nu_x)$ for $\lambda = M = K^{-1} = C = 0, \epsilon = 1$ with those of Jeng et al.\textsuperscript{17}. The results are found to be in good agreement. However, for $\xi > 0.5$, the results slightly differ (about 4\%) from those of Jeng et al.\textsuperscript{17}. This difference is attributed to the series solution method (an approximate method) used by Jeng et al.\textsuperscript{17}. Further, we have compared the surface shear stress ($F^{\prime \prime} (\xi, 0)$) and the surface heat transfer ($-G^{\prime} (\xi, 0)$) for $M = K^{-1} = C = 0, \epsilon = 1$ with those of Moutsoglou and Chen\textsuperscript{30}. For direct comparison, we have to divide our results by $2^{1/2}$ to take into account the scaling difference. The results are in excellent agreement. The comparison is shown in Table 1.

Figures 5 and 6 show the effect of the buoyancy parameter $\lambda$ on the skin-friction coefficient $(Re_{x}^{1/2} C_f)$ and the Nusselt number $(Re_{x}^{-1/2} Nu_x)$ for the CWT case when $C = 100, K = M = 1, Pr = 5.4, \epsilon = 0.9, \Omega = \pi/6$. Since the buoyancy parameter $\lambda$ is multiplied by $\xi$ (see eq. (6)), its effect increases with the streamwise distance $\xi$. However, the effect of $\lambda$ is more pronounced on the Nusselt number than on the skin friction. At $\xi = 2$, the Nusselt number increases by about 82\% as $\lambda$ increases from zero to 5, but the skin-friction coefficient decreases by about 5\%. The reason for this trend is that for large $Pr$ ($Pr = 5.4$), the density of the fluid is high which exhibits a lesser sensitivity to the buoyancy force effect, thereby causing a smaller change in the velocity gradient at the wall and, hence, in the wall shear results. For a fixed $\lambda$, the skin friction coefficient increases continuously with $\xi$, but the Nusselt number first decreases and attains a minimum value near the wall and then increases. This trend is due to the fact that the wall velocity increases with $\xi$ which


**Table 1**: Comparison of the surface shear stress $\tau'(\xi, 0)$ and the surface heat transfer $\theta_0(\xi, 0)$ for CWT case when $M = K = 0$, $\epsilon = 1$, $Pr = 7.0$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\tau'(\xi, 0)$</th>
<th>$\theta_0(\xi, 0)$</th>
<th>$\tau'(\xi, 0)$</th>
<th>$\theta_0(\xi, 0)$</th>
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</thead>
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<tr>
<td>0</td>
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<td>-0.44375</td>
<td>1.38702</td>
</tr>
<tr>
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<td>1.41322</td>
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<tr>
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<td>1.58517</td>
<td>0.99201</td>
<td>1.58510</td>
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</tbody>
</table>

Fig. 5: Effect of the buoyancy parameter $\lambda$ on the skin friction coefficient $Re^{1/2} C_f$, for the CWT case.
The results in higher velocity gradient at the wall and hence higher skin friction. The reason for the reduction in the Nusselt number in the range $0 < \xi < \xi_0$ and its subsequent increase is due to the competition between the parameters. Also for $\xi > 0$, the Nusselt number increases with the buoyancy parameter $\lambda$, but the skin friction coefficient slightly decreases. The reason for this behaviour can be explained as follows. Since the positive buoyancy force acts like a favourable pressure gradient, the fluid gets accelerated which results in thinner thermal boundary layer. Hence the Nusselt number increases with $\lambda$. On the other hand, for a fixed $\xi$ $(\xi > 0)$ the fluid velocity near the wall increases with $\lambda$. Hence the difference between the wall velocity and the fluid velocity near the wall decreases (both velocities act in the same direction) thus causing reduction in the surface skin friction with increasing $\lambda$. 

**Fig. 6.** Effect of the buoyancy parameter $\lambda$ on the Nusselt number, $Re_x^{-1/2} Nu_x$, for the CWT case.
The effect of the magnetic parameter $M$ on the surface skin friction coefficient and the Nusselt number $(Re_x^{1/2} C_{f_x}, Re_x^{-1/2} N_u)$ for the CWT case when $C = 100, K = 1, Pr = 5.4, \varepsilon = 0.9, \Omega = \pi/6, \lambda = 3$ is presented in Figs. 7 and 8. Since $M$ is multiplied by $\xi$, the effect of $M$ increases with $\xi$. For a fixed $\xi$, the skin friction increases with $M$, but the Nusselt number decreases. The reason for this trend is that the momentum boundary layer reduces with increasing $M$ due to the enhanced Lorentz force. Hence the velocity normal to the surface $F$ decreases. Hence the Nusselt number decreases with increasing $M$.

![Graph showing the effect of magnetic parameter $M$ on skin friction coefficient for CWT case.](image)

**Fig. 7.** Effect of the magnetic parameter $M$ on the skin friction coefficient, $Re_x^{1/2} C_{f_x}$, for the CWT case.

In Figs. 9 and 10, the effects of the inertia parameter $C$ and the permeability parameter $K$ on the skin friction and Nusselt number $(Re_x^{1/2} C_{f_x}, Re_x^{-1/2} N_u)$ for the CWT case when $\lambda = 3$, $C = 100, K = 1.0, Pr = 5.4, \varepsilon = 0.9, \Omega = \pi/6, \lambda = 3.0$.
Fig. 8. Effect of the magnetic parameter $M$ on the Nusselt number, $Re_x^{1/2} Nu_x$, for the CWT case.

$M = 0, 1.0, 3.0, 5.0$

$C = 100$
$K = 1.0$
$Pr = 5.4$
$\varepsilon = 0.9$
$\Omega = \pi/6$
$\lambda = 3.0$

$M = 1, Pr = 5.4, \varepsilon = 0.9, \Omega = \pi/6$ are presented. It can be seen that the effect of the inertia parameter $C$ is more pronounced than that of the permeability parameter $K$. The skin friction coefficient increases with $C$, but the Nusselt number decreases. Since the inertia parameter $C$ accelerates the flow field which results in thinner momentum boundary layer, the skin friction increases with $C$. As the momentum boundary layer reduces, the dimensionless normal velocity $F$ gets reduced and this in turn reduces the Nusselt number. For example, for $\zeta = 2$ the skin friction coefficient for $C = 200$ is about 20 times its value at $C = 0$. On the other hand, the Nusselt number decreases by about 70% as $C$ increases from zero to 200.
Fig. 9. Effect of the permeability parameter $K$ and the inertia parameter $C$ on the skin friction coefficient $Re^{1/2}C_f$. for the CWT case.

Fig. 11 display the effect of porosity parameter $\epsilon$ on the skin friction and Nusselt number $(Re^{1/2}C_f, Re^{-1/2}Nu_x)$ for the CWT case when $\lambda = 3$, $M = 1$, $Pr = 5.4$, $\epsilon = 0.9$, $\Omega = \pi/6$. The increase in $\epsilon$ implies less resistance offered by the medium. Hence the surface skin friction increases with $\epsilon$, but the Nusselt number decreases. For $\xi = 2$, the skin friction increases by about 38% as $\epsilon$ increases from 0.5 to 0.9, but the Nusselt number decreases by about 15%.

The effect of the inclination angle $\Omega$ on the skin friction and the Nusselt number $(Re^{1/2}C_f, Re^{-1/2}Nu_x)$ for the CWT case when $\lambda = 3$, $M = 1$, $C = 100$, $K = 1$, $Pr = 5.4$. $\epsilon$
$= 0.9$ is given in Fig. 12. It can be observed from the figure that the skin friction is very little affected by the inclination angle, but the Nusselt number decreases. As the inclination angle $\Omega$ increases, for a fixed $\xi$, the magnitude of the buoyancy force decreases, hence the thermal boundary layer is increased. Consequently, the Nusselt number is reduced.

Since the effect of $\lambda$, $M$, $C$, $K$, $\epsilon$ and $\Omega$ on the skin friction and the Nusselt number $(Re_x^{1/2} C_f, Re_x^{-1/2} Nu_x)$ for the CHF case are qualitatively similar to those of CWT case, only a few results are presented in Figs. 13-17, but they are not discussed here. The Nusselt number for the CHF case shows slightly different trend from that of the CWT case when $\xi \geq \xi_0$. For the CWT
Fig. 11. Effect of the porosity parameter $e$ on the skin friction coefficient and the Nusselt number, $Re_x^{1/2} C_{f}$ and $Re_x^{1/2} Nu_x$, for the CWT case.
Fig. 12. Effect of the inclination angle $\Omega$ on the skin friction coefficient and the Nusselt number, $Re_x^{1/2} C_{f_x}$ and $Re_x^{-1/2} Nu_x$, for the CWT case.
Fig. 13. Effect of the buoyancy parameter $\lambda$ on the skin friction coefficient $Re^{1/2} C_{f\lambda}$ for the CHF case.

In the CWT case the Nusselt number first decreases till $\xi \geq \xi_0$ and then increases, whereas for the CHF case it continuously decreases with increasing $\xi$. For $\xi \geq \xi_0$ the Nusselt number for the CHF case is greater than that of the CWT case, but for $\xi \geq \xi_0$ it is otherwise.

5. CONCLUSION

The skin friction coefficients for the constant wall temperature and constant heat flux cases increase continuously with the streamwise distance. The Nusselt number for the constant heat flux decreases with increasing streamwise distance, but the Nusselt number for the constant wall temperature case...
Fig. 14. Effect of the buoyancy parameter $\lambda$ on the Nusselt number, $Re_x^{1/2} Nu_x$, for the CHF case.

decreases for small streamwise distance and then it increases. The effects of the inertia and porosity parameters on the skin friction and Nusselt number are found to be more pronounced than that of the permeability parameter. The buoyancy force exerts a strong influence on the Nusselt number, but for large Prandtl number its effect on the skin friction is comparatively small. The magnetic field increases the skin friction, but reduces the Nusselt number.

REFERENCES

FIG. 15: Effect of the permeability parameter $K$ and the inertia parameter $C$ on the skin friction coefficient, $Re^{1/2} C_{fr}$, for the CHF case.

Fig. 16. Effect of the permeability parameter $K$ and the inertia parameter $C$ on the Nusselt number, $Re_x^{-1/2} Nu_x$, for the CHF case.


Fig. 17. Effect of the porosity parameter $\varepsilon$ on the skin friction coefficient and the Nusselt number, $Re_x^{1/2} C_f$, and $Re_x^{1/2} Nu_x$, for the CHF case.