Double-Diffusive Convective Flow of a Micropolar Fluid Over a Vertical Plate Embedded in a Porous Medium with a Chemical Reaction †

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The problem of steady, laminar, double-diffusive natural convection boundary-layer flow of a micropolar fluid over a vertical permeable semi-infinite plate embedded in a uniform porous medium in the presence of non-Darcian and thermal dispersion effects is investigated. Also, the model problem allows for possible heat generation or absorption and first-order chemical reaction effects. Both the wall temperature and wall concentration are assumed to have linear variations with the distance along the plate. Appropriate transformations are employed to transform the governing differential equations into a non-similar form that can be solved as an initial-value problem. The resulting equations are solved numerically by an efficient implicit, iterative, finite-difference scheme. The obtained results are checked against previously published work on special cases of the problem and are found to be in good agreement. A parametric study illustrating the influence of the microrotation material parameter, concentration to thermal buoyancy ratio, chemical reaction parameter, Schmidt number, heat generation or absorption and the surface suction or injection effects on the fluid velocity, microrotation, temperature and solute concentration as well as the local skin-friction coefficient, local wall microrotation coefficient and the local wall heat and mass transfer coefficients is conducted. The results of this parametric study are shown graphically and the physical aspects of the problem are highlighted and discussed.

* * *

Nomenclature

\[ A \] constant for wall temperature and concentration;
\[ B \] microrotation material constant \((B = \nu/(g_{\beta T}A^{1/2}))\);
\[ C \] concentration at any point in the flow field;
\[ C_f \] local skin-friction coefficient defined by Eq. (19);
\[ C_m \] local mass transfer coefficient defined by Eq. (22);

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$C_q$ local heat transfer coefficient defined by Eq. (21);  
$C_r^*$ local wall couple stress defined by Eq. (20);  
$C_r$ local wall microrotation coefficient ($C_r = \xi C_r^*$);  
$C_w$ concentration at the wall;  
$C_{\infty}$ concentration at the free stream;  
$D$ mass diffusivity;  
$D_a^*$ inverse Darcy number ($D_a^* = \nu/(K(g\beta_T A)^{1/2})$);  
$F$ inertia coefficient of the porous medium;  
$f$ dimensionless stream function ($f = \psi/((g\beta_T A\nu^2)^{1/4}x)$);  
$f_o$ dimensionless wall mass transfer coefficient ($f_o = V_w/(g\beta_T A\nu^2)^{1/4}$);  
$g$ gravitational acceleration;  
$j$ microrotation per unit mass;  
$K$ permeability of the porous medium;  
$K^*$ vortex viscosity;  
$k_e$ porous medium effective thermal conductivity;  
$N$ angular velocity or microrotation;  
$N_R$ buoyancy ratio ($N_R = \beta_C(C_w - C_{\infty})/(\beta_T(T_w - T_{\infty})$));  
$Pr$ Prandtl number ($Pr = \rho\nu c_p/k_e$);  
$Q_o$ heat generation or absorption coefficient;  
$R$ chemical reaction parameter;  
$S$ porous medium thermal dispersion parameter ($S = c_p\sigma d(g\beta_T A)^{1/2}/(g\beta_T \alpha)$);  
$S^*$ local porous medium thermal dispersion parameter ($S^* = S\xi$);  
$Sc$ Schmidt number ($Sc = \nu/D$);  
$T$ temperature at any point;  
$T_w$ wall temperature;  
$T_{\infty}$ free stream temperature;  
$u$ tangential or $x-$component of the Darcian velocity;  
$v$ normal or $y-$component of the Darcian velocity;  
$V_w$ dimensional wall mass transfer;  
$w$ dimensionless microrotation ($w = N/(g\beta_T A/\nu^{2/3})^{3/4}$);  
$x$ distance along the plate;  
$y$ distance normal to the plate.

Greek Symbols

$\Delta$ microrotation material parameter ($\Delta = K^*/(\rho\nu)$);  
$\Gamma$ dimensionless porous medium inertia coefficient ($\Gamma = Fc_p/(g\beta_T)$);  
$\alpha$ molecular thermal diffusivity;  
$\alpha_e$ effective thermal diffusivity of the porous medium;  
$\alpha_d$ thermal diffusivity of the porous medium due to thermal dispersion;  
$\beta_C$ concentration expansion coefficient;  
$\beta_T$ thermal expansion coefficient;  
$\delta$ dimensionless heat generation or absorption parameter ($\delta = Q_o/((\rho c_p(g\beta_T A)^{1/2})$);  
$\phi$ dimensionless concentration ($\phi = (C - C_{\infty})/(C_w - C_{\infty})$);  
$\gamma$ spin gradient viscosity;  
$\eta$ dimensionless distance normal to the plate ($\eta = (g\beta_T A/\nu^{2/3})^{1/4}$);  
$\kappa$ dimensionless chemical reaction parameter ($\kappa = R/(g\beta_T A)^{1/2}$);  
$\lambda$ microrotation material constant ($\lambda = \gamma/(\rho\nu j)$);  

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Introduction

Double-diffusive convection is referred to buoyancy-induced flow due to the combined effects of both temperature and concentration gradients. Double-diffusive convection from different geometries embedded in porous media has a wide range of engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. Most early studies on porous media have used the Darcy law which is a linear empirical relation between the Darcian velocity and the pressure drop across the porous medium and is limited to relatively slow flows. However, for relatively higher velocity flow situations, the Darcy law becomes inadequate for representing the flow behavior correctly since it does not account for the resulting inertia effects of the porous medium. In this situation, the pressure drop has a quadratic relationship with the volumetric flow rate. The high flow situation is established when the Reynolds number based on the pore size is greater than unity. Vafai and Tien (1981) [1] discussed the importance of inertia effects for high velocity flows in porous media.


Micropolar fluids are referred to those fluids that contain micro-constituents that can undergo rotation which affect the hydrodynamics of the flow. In the context, they can be distinctly non-Newtonian in nature. The basic continuum theory for this class of fluids was originally formulated by Eringen (1966) [13]. Eringen’s micropolar fluid theory has been employed to study a number of various flow situations such as the flow of low concentration suspensions, liquid crystals, blood,
and turbulent shear flows.


In the presence of a porous medium, a secondary effect on the flow arises as a result of mixing and recirculation of local fluid particles through tortuous paths formed by the porous medium solid particles. This effect is classified as thermal dispersion (see Amiri and Vafai (1994) [23]). Plumb (1983) [24] modeled thermal dispersion effects over a vertical plate as linear increases of a fluid thermal diffusivity with the increases in the tangential flow velocity. In their model, Amiri and Vafai (1994) [24] have shown that the thermal diffusivity of the fluid is also proportional to the free stream Reynolds number based on the porous medium pore diameter. Other works dealing with thermal dispersion effects in porous media can be found in the papers by Hunt and Tien (1988) [25] and Cheng and Vortmeyer (1988) [26].

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported previously by such authors as Acharya and Goldstein (1985) [27], Vajravelu and Nayfeh (1992) [28] and Chamkha (1997) [29].

The objective of this paper is to consider double-diffusive natural convection flow of a micropolar fluid along a vertical semi-infinite permeable plate embedded in a saturated porous medium in the presence of a first-order chemical reaction, wall suction or injection effects, heat generation or absorption effects, and porous medium inertia and thermal dispersion effects. This will be done for linear variations of both the wall temperature and concentration with the distance along the plate.

1. Problem Formulation

Consider steady, laminar, double-diffusive natural convection boundary-layer flow of a micropolar fluid over a permeable semi-infinite vertical plate embedded in a fluid-saturated porous medium in the presence of heat generation or absorption, first-order chemical reaction and viscous dissipation effects. The fluid is assumed to be viscous and has constant properties except the density in the buoyancy terms of the balance of linear momentum equation. Also, the porosity and permeability of the porous medium are assumed to be constant. The fluid and the porous medium are assumed to be in local thermal equilibrium. Both the wall temperature and concentration are assumed to have
linear variations with the plate’s vertical distance \( x \). The temperature and the concentration at the plate surface are always greater than their free stream values.

The governing boundary-layer equations which take into account the non-Darcian and thermal dispersion effects of the porous medium within the boundary layer and the presence of a chemical reaction in addition to the Boussinesq approximation may be written as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{1}{\varepsilon^2} \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{1}{\varepsilon} \left( \nu + \frac{K^*}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{K^*}{\rho} \frac{\partial N}{\partial y} + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty) - \frac{\nu}{K} u - Fu^2, \tag{2}
\]

\[
u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^2 N}{\partial y^2} + \frac{K^*}{\rho j} \left( 2N + \frac{\partial u}{\partial y} \right), \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_e}{\rho c_p} \frac{\partial T}{\partial y} + \frac{\nu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q_o}{\rho c_p} (T - T_\infty), \tag{4}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - R (C - C_\infty), \tag{5}
\]

where \( u, v, N, T, \) and \( C \) are the fluid \( x \)-component of the Darcian velocity, \( y \)-component of the Darcian velocity, angular velocity or microrotation, temperature, and concentration, respectively; \( \rho, \nu, c_p, \beta_T, \) and \( \beta_C \) are the fluid density, kinematic viscosity, specific heat at constant pressure, coefficient of thermal expansion, and coefficient of concentration expansion, respectively; \( \gamma, j, \) and \( K^* \) are the spin gradient viscosity, microrotation per unit mass and the vortex viscosity, respectively; \( g, Q_o, D, \) and \( R \) are the acceleration due to gravity, heat generation (>0) or absorption (<0) coefficient, mass diffusivity, and the rate of chemical reaction parameter, respectively; \( F, K, \alpha_e, \) and \( \varepsilon \) are the porous medium inertia coefficient, permeability, effective thermal diffusivity, and porosity, respectively. The last term in Eq. (5) is employed according to Rahman and Mulolani (2000) [30] for a first-order chemical reaction.

The boundary conditions for this problem can be written as:

\[
y = 0, \quad u = 0, \quad v = V_w, \quad N = 0, \quad T = T_w(x), \quad C = C_w(x), \tag{6}
\]

\[
y \to \infty, \quad u \to 0, \quad N \to 0, \quad T \to T_\infty, \quad C \to C_\infty, \tag{7}
\]

where both the wall temperature \( T_w \) and wall concentration \( C_w \) vary along the plate according to:

\[
T_w - T_\infty = Ax, \quad C_w - C_\infty = Ax. \tag{8}
\]

In Eqs (6) through (8), \( V_w \) is the surface mass transfer parameter; \( T_\infty \) and \( C_\infty \) are the free stream temperature and concentration, respectively, and \( A \) is a constant.

In situations of fluid flow and heat transfer in porous media, the effective thermal diffusivity is modeled by Yagi et al. (1964) [31] and later by Plumb (1983) [24] according to:

\[
\alpha_e = \alpha + \alpha_d. \tag{9}
\]
where $\alpha$ and $\alpha_d$ are the molecular and thermal dispersion thermal diffusivities of the fluid and the porous medium, respectively. Yagi et al. (1964) [31] and Plumb (1983) [24] represented $\alpha_d$ as a linear relation of the fluid velocity as follows:

$$\alpha_d = \sigma ud,$$

(10)

where $d$ is the mean particle diameter and $\sigma$ is a constant. Gorla et al. (1996) [32] have also used the same model.

Defining the dimensional stream function in the usual way such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and using the following dimensionless variables (El-Hakiem (2000) [33]):

$$\xi = \frac{g\beta T x}{c_p} , \quad \eta = \left(\frac{g\beta T A}{\nu^2}\right)^{1/4} y , \quad f(\xi, \eta) = \frac{\psi}{(g\beta T A\nu^2)^{1/4} x} , \quad \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} , \quad w(\xi, \eta) = \frac{N\beta T A/\nu^2}{3} , \quad \phi(\xi, \eta) = \frac{C - C_\infty}{C_w - C_\infty} ,$$

(11)

results in the following non-similar equations:

$$\begin{align*}
(1 + \Delta) f''' + f' + \frac{f'''}{\varepsilon^2} - \text{Da}^* f' - \left(\frac{1}{\varepsilon^2} + \Gamma\xi\right) f' + \Delta w' &+ \theta + N_R \phi = \frac{\xi}{\varepsilon^2} \left( f' \frac{\partial f}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right), \\
\lambda w'' + f w' - \Delta B (2w + f'') - f' w &+ \xi \left( f' \frac{\partial w}{\partial \xi} - w' \frac{\partial f}{\partial \xi} \right), \\
\frac{1}{\text{Pr}} (1 + S^* f') \theta'' + \left( \frac{S^*}{\text{Pr}} f'' + f \right) \theta' + (\delta - f') \theta &+ \xi \left( f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} - f' \frac{\partial^2 f}{\partial \xi^2} \right), \\
\frac{1}{\text{Sc}} \phi'' + f \phi' - (\kappa + f') \phi &+ \xi \left( f' \frac{\partial \phi}{\partial \xi} - \phi' \frac{\partial f}{\partial \xi} \right),
\end{align*}$$

(12)

(13)

(14)

(15)

where

$$\begin{align*}
\text{Da}^* &= \frac{\nu}{K(g\beta T A)^{1/2}} ; \quad \Gamma = \frac{F c_p}{g\beta T} ; \quad N_R = \frac{\beta C(C_w - C_\infty)}{\beta T(T_w - T_\infty)} ; \quad \text{Pr} = \frac{\rho \nu c_p}{k_e} ; \quad \\
\lambda &= \frac{\gamma}{\rho \nu j} ; \quad \Delta = \frac{K^*}{\nu} ; \quad B = \frac{\nu}{(g\beta T A)^{1/2}} ; \quad \text{Sc} = \frac{\nu}{D} ; \quad \\
\delta &= \frac{Q_o \rho c_p}{\rho c_p (g\beta T A)^{1/2}} ; \quad S^* = \frac{\sigma d (g\beta T A)^{1/2} x}{\alpha} = \frac{S \xi}{\alpha} ; \quad \kappa = \frac{R}{(g\beta T A)^{1/2}} ,
\end{align*}$$

(16)

are the inverse Darcy number, dimensionless porous medium inertia coefficient, buoyancy ratio, Prandtl number, three microrotation material constants, Schmidt number, dimensionless internal heat generation or absorption parameter, and the local porous medium thermal dispersion parameter, and dimensionless chemical reaction parameter, respectively.

The dimensionless form of the boundary conditions becomes:

$$\begin{align*}
\eta = 0 : & \quad f + \frac{\partial f}{\partial \xi} = f_0, \quad f' = 0, \quad w = 0, \quad \theta = 1, \quad \phi = 1,
\end{align*}$$

(17)
\( \eta \to \infty : \quad f' \to 0, \quad w \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad (18) \)

where \( f_o = V_w / (g_\beta T A \nu^2)^{1/4} \) is the dimensionless wall mass transfer coefficient such that \( f_o > 0 \) indicates fluid wall suction and \( f_o < 0 \) indicates fluid wall blowing or injection.

The local skin-friction coefficient, local wall couple stress and the local heat and mass transfer coefficients are important physical parameters for this flow and heat transfer situation. These are defined as follows:

\[
C_f = \frac{(\mu + K^*)(\partial u/\partial y)|_{y=0} + K^* N|_{y=0}}{\nu^{-1/2}(g_\beta T A)^{1/4} c_p(g_\beta T)^{-1}} = \xi (1 + \Delta) f''(\xi, 0),
\]

\[
C_r^* = -\frac{(\partial N/\partial y)|_{y=0}}{\nu^{-1}(g_\beta T A)} = -u'(\xi, 0),
\]

\[
C_q = \frac{-k_e (\partial T/\partial y)|_{y=0}}{k_e \nu^{-1/2}(g_\beta T A)^{1/4} c_p(g_\beta T)^{-1}} = -\xi \theta'(\xi, 0),
\]

\[
C_m = \frac{-D(\partial C/\partial y)|_{y=0}}{D \nu^{-1/2}(g_\beta T A)^{1/4} c_p(g_\beta T)^{-1}} = -\xi \phi'(\xi, 0).
\]

A local wall microrotation coefficient \( C_r \) will be defined as being equal to \( \xi C_r^* \). This is done for convenience and behavior consistency with the other coefficients and will be used in the presentation of results.

2. Numerical Method

The implicit finite-difference method discussed by Blottner (1970) [34] has proven to be adequate and accurate for the solution of differential equations similar to Eqs (12) through (15). For this reason, it is employed in the present work. These equations have been linearized and then discretized using three-point central difference quotients with variable step sizes in the \( \eta \)-direction. A backward-difference approximation is used for all first derivatives with respect to \( \xi \). Constant step sizes in the \( \xi \) direction are employed. The resulting equations form a tri-diagonal system of algebraic equations that can be solved line by line at each line of constant \( \xi \) by the well-known Thomas algorithm (see Blottner (1970) [34]). The problem is solved as an initial-value problem with \( \xi \) playing the role of time. The initial conditions are obtained by setting \( \xi = 0 \) in Eqs (12) through (15) and solving the resulting set of ordinary differential equations. Due to the nonlinearities of the equations, an iterative solution is required. For convergence, the maximum absolute error between two successive iterations was taken to be \( 10^{-7} \). A starting step size of 0.001 in the \( \eta \) direction with an increase of 1.0375 times the previous step size and a step size of 0.01 in the \( \xi \) direction were found to give accurate results. The total number of points in the \( \xi \) and \( \eta \) directions was taken to be 101 and 199, respectively. The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by El-Hakiem (2000) [33]. It should be mentioned that El-Hakiem (2000) [33] investigated hydromagnetic free convection flow over a non-isothermal surface in a micropolar fluid. The presence of a porous medium without the inertia effect produces the same effect as that produced by a magnetic field. By adding a term simulating the magnetic dissipation effect in the present equations, the results can be compared with
those reported by El-Hakiem (2000) [33]. Tables 1 and 2 present comparisons for the wall slopes of velocity, temperature and microrotation profiles for various inverse Darcy numbers Da* and thermal dispersion parameters S. These comparisons show good agreement between the results. The deviations between the results are probably associated with the fact that the results of El-Hakiem (2000) [33] do not appear to approach the free stream values accurately as obvious from the velocity deviations between the results. The peak value decreases significantly close to the wall while the velocity away from the wall increases. As expected, for \( \Delta = 0 \) the microrotation vanishes. However, for finite values of \( \Delta \), the microrotation profiles show the existence of a negative minimum close to the wall followed by a positive maximum away from the wall. These extreme values

### Table 1

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3. Results and Discussion

Figs 1 and 2 present representative velocity \( (f') \) and microrotation \( (\omega) \) profiles at \( \xi = 1 \) for various values of the microrotation parameter \( \Delta \) and buoyancy ratio \( N_R \), respectively. When \( \Delta = 0 \), the linear momentum equation (12) is uncoupled from the angular momentum equation (13). For this case, the fluid velocity profile exhibits a distinctive peak close to the wall. However, for finite values of \( \Delta \), these equations are coupled. As \( \Delta \) increases, the coupling increases causing a net decrease in the fluid velocity. The peak value decreases significantly close to the wall while the velocity away from the wall increases. As expected, for \( \Delta = 0 \) the microrotation vanishes. However, for finite values of \( \Delta \), the microrotation profiles show the existence of a negative minimum close to the wall followed by a positive maximum away from the wall. These extreme values
The present work demonstrates that the fluid velocity, microrotation, and temperature profiles are affected by the fluid velocity increasing. This reduction in the fluid velocity as \( \kappa \) increases results in less induced flow along the plate. As a consequence, the solute concentration in the boundary layer decreases, causing the buoyancy due to concentration to decrease. This reduction in the fluid velocity, microrotation, and temperature profiles is observed in Figs 1 and 2.

Comparisons of Table 2 illustrate the effects of \( \Delta \) and \( N_R \) on the distributions of the local skin-friction coefficient \( C_f \) and the local wall microrotation coefficient \( C_r \) along the plate, respectively. Inspection of these figures shows that both the wall slopes of the fluid velocity and the microrotation increase as either \( \Delta \) or \( N_R \) increases. This results in increases in the values of both of \( C_f \) and \( C_r \) as either \( \Delta \) or \( N_R \) increases. Obviously, both \( C_f \) and \( C_r \) increase with increasing values of \( \xi \) since they are both defined as directly proportional to \( \xi \).

Figs 5 through 8 present typical velocity \( (f') \), microrotation \( (w) \), temperature \( (\theta) \) and concentration \( (\phi) \) profiles at \( \xi = 1 \) for various values of the chemical reaction parameter \( \kappa \) and the Schmidt number \( Sc \), respectively. The presence of a first-order chemical reaction has the tendency to decrease the solute concentration in the boundary layer. This causes the buoyancy due to concentration gradient to decrease resulting in less induced flow along the vertical plate. As a consequence, the fluid velocity decreases. This reduction in the fluid velocity as \( \kappa \) increases produces decreases in the microrotation profiles and increases in the temperature profiles. In addition, increasing the Schmidt number \( Sc \) causes the solute concentration to decrease as well. This causes the exact same effect on the velocity, microrotation and temperature profiles as increasing the value of \( \kappa \) for the
Fig. 1. Effects of $\Delta$ and $N_R$ on velocity profiles.

Fig. 2. Effects of $\Delta$ and $N_R$ on microrotation profiles.
Fig. 3. Effects of $\Delta$ and $N_R$ on local skin-friction coefficient.

Fig. 4. Effects of $\Delta$ and $N_R$ on microrotation coefficient.
Fig. 5. Effects of $\text{Sc}$ and $\kappa$ on velocity profiles.

Fig. 6. Effects of $\text{Sc}$ and $\kappa$ on microrotation profiles.
**Fig. 7.** Effects of $Sc$ and $\kappa$ on temperature profiles.

**Fig. 8.** Effects of $Sc$ and $\kappa$ on concentration profiles.
same reasons mentioned above. These trends are clearly depicted in Figs 5 through 8.

Figs 9 through 12 illustrate the influence of increasing either of $\kappa$ or $Sc$ on the local skin-friction coefficient $C_f$, the local wall microrotation coefficient $C_r$, the local wall heat transfer coefficient $C_q$ and the local wall mass transfer coefficient $C_m$ along the plate, respectively. Increases in either of $\kappa$ or $Sc$ causes the wall slopes of the velocity and microrotation and the negative wall slopes of the temperature profiles to decrease and the negative wall slopes of the concentration profiles to increase. This has the direct effect of decreasing the values of $C_f$, $C_r$, and $C_q$ and increasing the values of $C_m$ as shown in Figs 9 through 12.

Figs 13 through 16 display the effect of the wall mass transfer parameter $f_o$ and the heat generation or absorption coefficient $\delta$ on the velocity, microrotation, temperature and concentration profiles at $\xi = 1$, respectively. For a fixed value of $\delta$, imposition of fluid wall suction ($f_o > 0$) tends to decrease all of the fluid velocity, microrotation, temperature and solute concentration as well as their boundary-layer thicknesses. On the other hand, injection or blowing of fluid at the plate surface ($f_o < 0$) produces the exact opposite effect namely increases in the fluid velocity, microrotation, temperature and solute concentration and their boundary-layer thicknesses. These behaviors are obvious from Figs 13 through 16. In addition, it is seen from Fig. 14 that as $f_o$ increases, the location of the negative minimum and positive maximum values of the microrotation moves in the direction towards the wall. Also, for a fixed value of $f_o$, the presence of heat generation effects ($\delta > 0$) has the tendency to increase the temperature of the fluid. This causes the thermal buoyancy effects to increase resulting in more induced flow along the plate. Through the coupling of the linear and angular momentum equations, this increase in the fluid velocity produces increases in the fluid microrotation extremum values as well. In addition, the solute concentration decreases while both of the hydrodynamic and thermal boundary layers increase as $\delta$ increases. It is also interesting to note that for $\delta > 0$ a distinctive peak in the temperature profile greater than that of the wall occurs in the vicinity of the plate surface. On the other hand, the opposite behavior of all of the velocity, microrotation, temperature and concentration profiles is obtained due to the presence of heat absorption effects ($\delta < 0$). The peak temperature value in this case is that of the wall. All of these trends are clearly seen from Figs 13 through 16.

Finally, Figs 17 through 20 depict the influence of both $f_o$ and $\delta$ on the values of $C_f$, $C_r$, $C_q$ and $C_m$ along the plate, respectively. It is predicted that, in general, imposition of fluid suction ($f_o > 0$) at the wall causes the wall slope of the fluid velocity, the negative wall slopes of temperature and concentration to increase. This yields increases in the local skin-friction coefficient $C_f$, the local wall heat transfer coefficient $C_q$ and the local mass transfer coefficient $C_m$. On the other hand, injection or blowing ($f_o < 0$) of fluid into the boundary layer from the plate surface produces the opposite effect namely, reductions in all of $C_f$, $C_q$ and $C_m$. The variations of $C_r$ along the plate as a result of varying $f_o$ do not follow a consistent trend for different values of $\delta$. The values of $C_m$ seem to increase for both suction and injection conditions for $\delta = -1$ (heat absorption) and $\delta = 0$ while they tend to decrease for both suction and injection conditions for $\delta = 1$ (heat generation). Furthermore, it is predicted that increases in the values of $\delta$ produce increases in the values of all of $C_f$, $C_r$ and $C_m$ and decreases in the values of $C_q$. It is also predicted that $C_q$ becomes negative along the whole plate for $\delta = 1$. This is associated with the existence of the peak in the temperature profiles for $\delta = 1$ (see Fig. 15) which causes its wall slope to change its sign.
**Fig. 9.** Effects of $Sc$ and $\kappa$ on skin-friction coefficient.

**Fig. 10.** Effects of $Sc$ and $\kappa$ on microrotation coefficient.
Fig. 11. Effects of $Sc$ and $\kappa$ on heat transfer coefficient.

Fig. 12. Effects of $Sc$ and $\kappa$ on local mass transfer coefficient.
Fig. 13. Effects of $f_o$ and $\delta$ on velocity profiles.

Fig. 14. Effects of $f_o$ and $\delta$ on microrotation profiles.
Fig. 15. Effects of $f_0$ and $\delta$ on temperature profiles.

Fig. 16. Effects of $f_0$ and $\delta$ on concentration profiles.
Fig. 17. Effects of \( f_o \) and \( \delta \) on local skin-friction coefficient.

Fig. 18. Effects of \( f_o \) and \( \delta \) on local wall microrotation coefficient.
Fig. 19. Effects of $f_o$ and $\delta$ on local wall transfer coefficient.

Fig. 20. Effects of $f_o$ and $\delta$ on local mass transfer coefficient.
Conclusions

The problem of steady, laminar, double diffusive natural convection boundary-layer flow of a viscous, micropolar and heat generating or absorbing fluid over a vertical permeable plate embedded in a uniform porous medium with non-Darcian, thermal dispersion and first-order chemical reaction effects was considered. Both the wall temperature and wall concentration were assumed to vary linearly with the vertical distance along plate. The governing equations for this problem were developed and transformed using appropriate non-similarity transformations. The resulting transformed equations were then solved numerically by an implicit, iterative, finite-difference scheme. The obtained results for special cases of the problem were compared with previously published work and found to be in good agreement. It was found that, in general, the local skin-friction coefficient increased as either of the buoyancy ratio, microrotation parameter, wall suction effects, or the heat generation effects increased and it decreased as a result of increasing either of the chemical reaction parameter, the Schmidt number, the surface blowing or injection effects, or the heat absorption effects. In addition, the local heat transfer coefficient was predicted to increase due to increases in either of the buoyancy ratio, suction effects, or the heat absorption effects while it decreased as either of the chemical reaction parameter, the wall injection effects or the Schmidt number increased. Furthermore, the local mass transfer coefficient was predicted to increase as a result of increasing either of the buoyancy ratio, the chemical reaction parameter, the wall fluid suction or the Schmidt number. Finally, the local wall microrotation coefficient was increased as either of the buoyancy ratio, the microrotation parameter or the heat generation or absorption parameter increased and decreased as either of the chemical reaction parameter or the Schmidt number increased. It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

REFERENCES


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