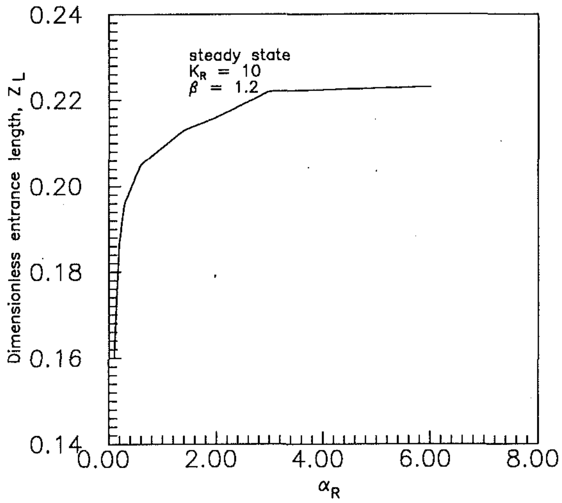

Fig. 4 Effect of the thermal diffusivity ratio on the thermal entrance length

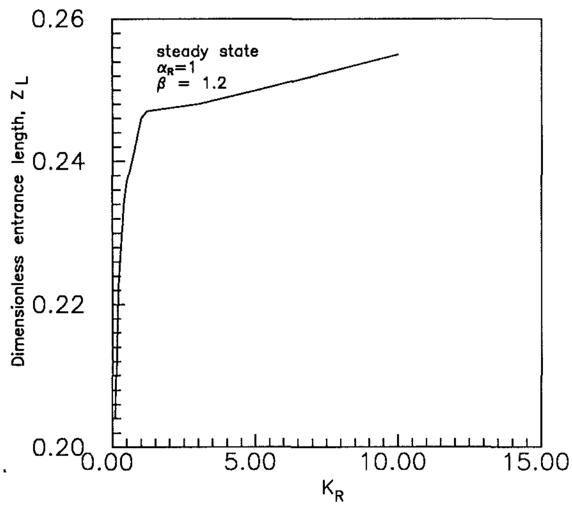

Fig. 5 Effect of the thermal conductivity ratio on the thermal entrance length

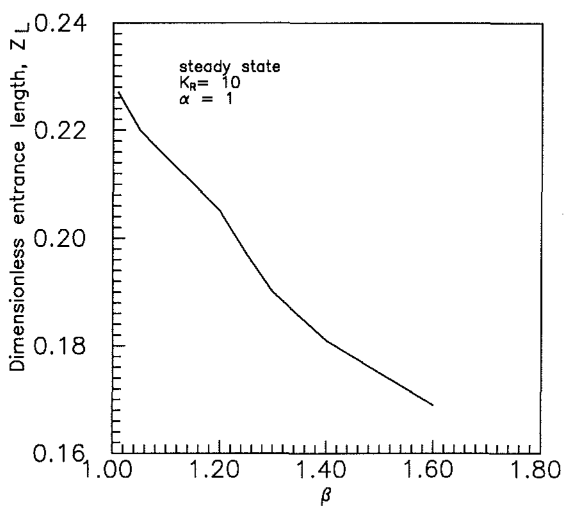

Fig. 6 Effect of the radius ratio on the thermal entrance length

of the outer surface of the pipe wall. To verify the adequacy of the numerical scheme used in this work, the hydrodynamic fully developed case is solved and the results are compared with previous work. It is found that the hydrodynamic history of the flow in the entrance length of the pipe has important effects on its thermal behavior. The solid-fluid parameters that affect the thermal behavior of the pipe flow are found to be conductivity ratio, diffusivity ratio, and radius ratio. The effect of these parameters on the temperature distribution, in both fluid and solid domains and on the wall heat flux, is studied. It is found that increasing the thermal conductivity ratio will increase the wall heat flux. Also, the effect of K_R, α_R, and β on the thermal entrance length of the tube is studied. It is concluded that increasing the conductivity and the diffusivity ratios will increase the thermal entrance length of the tube. On the other hand, increasing the radius ratio will reduce the thermal entrance length.

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Effects of Particulate Diffusion on the Thermal Flat Plate Boundary Layer of a Two-Phase Suspension

A. J. Chamkha[1]

Nomenclature

- c = fluid-phase specific heat at constant pressure
- d = particle diameter
- D = diffusion coefficient
- E_c = fluid-phase Eckert number
- $\mathbf{e}_x$, $\mathbf{e}_y$ = unit vectors in x and y directions, respectively
- F = nondimensionalized fluid-phase tangential velocity
- G = nondimensionalized fluid-phase normal velocity
- H = nondimensionalized fluid-phase temperature
- H_0 = nondimensionalized fluid-phase wall temperature
- $\underline{\underline{I}}$ = unit tensor
- k = fluid-phase thermal conductivity
- P = fluid-phase pressure
- Pr = fluid-phase Prandtl number
- ϱ_p = particle-phase density
- $\dot{q}_w$ = wall heat transfer
- S_e = interphase heat transfer rate per unit volume to the particle phase
- S_I = interphase force per unit volume acting on the particle phase

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Contributed by the Heat Transfer Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Heat Transfer Division February 1993; revision received May 1993. Keywords: Forced Convection, Multiphase Flows, Numerical Flows. Associate Technical Editor: J. H. Kim.

S_m = source term
T = fluid-phase temperature
V = fluid-phase velocity
x, y = Cartesian coordinate variables
γ = specific heat ratio
δ = inverse Schmidt number
ϵ = temperature inverse Schmidt number
κ = particle loading
μ = fluid-phase dynamic viscosity
ν = fluid-phase kinematic viscosity $=\mu/\rho$
ξ, η = transformed coordinates
ρ = fluid-phase density
$\underline{\underline{\sigma}}$ = fluid-phase stress tensor
τ_T = temperature relaxation time $=\rho_s d^2 c_p/(12k)$
τ_v = momentum relaxation time $=\rho_s d^2/(18\mu)$
∇ = gradient operator
∇^2 = Laplacian operator

Subscripts

∞ = free stream
p = particle phase
s = particulate material

Superscripts

T = transpose of a second-order tensor

Introduction

The Eulerian description of particulate suspensions has been employed extensively in modeling processes involving such suspensions. This approach treats the two phases (fluid and particle) as interacting continua (see, for instance, Marble, 1970). Because the solid phase consists of discrete particles of different sizes and shapes, a representation of the particles variables by averaged continuous functions leads to excess source terms in the governing equations of motion (see Drew and Segal, 1971). The source term resulting from the particle-phase continuity equation is modeled herein as diffusive in nature. It is of interest in the present paper to study the effects of particulate diffusivity on the thermal boundary layer of a two-phase particulate suspension past a semi-infinite impermeable plate. This is a classical problem in fluid mechanics and heat transfer, which has not been fully investigated.

Related work on this problem can be found in the papers by Soo (1968), Osiptsov (1980), Prabha and Jain (1980), and Chamkha (1992). It has been reported in all these papers that if the original dusty-gas equations, which do not account for particulate diffusivity, are used in solving this problem, a singularity in the particle-phase density is predicted. However, when the model is modified to include particulate diffusion, a singularity-free solution is possible (see Chamkha and Peddieson, 1989, who considered the hydrodynamic aspects of this problem). This work considers the thermal aspects of the problem. The fluid phase is assumed incompressible and has constant properties. The particle phase is assumed to consist of small solid spherical particles of uniform size with no mutual collision and radiative heat transfer and the particle-phase volume fraction is assumed to be small.

Governing Equations

Let the plate occupy the half of the x, z plane corresponding to $x>0$ with the y axis being normal to the plate, and let the flow far from the plate be a uniform stream in the x direction parallel to the plate with both phases in equilibrium. The governing equations for this problem are based on the balance laws of mass, linear momentum, and energy for both the fluid and particulate phases (see, for instance Marble, 1970). These can be written as

$$\nabla\cdot(\rho\mathbf{V})=S_m,\quad \nabla\cdot(\rho_p\mathbf{V}_p)=S_{pm},\quad \rho\mathbf{V}\cdot\nabla\mathbf{V}=\nabla\cdot\underline{\underline{\sigma}}+\mathbf{S}_l,$$
$$\rho_p\mathbf{V}_p\cdot\nabla\mathbf{V}_p=\nabla\cdot\underline{\underline{\sigma}}_p+\mathbf{S}_{pl},$$
$$\rho c\mathbf{V}\cdot\nabla T=k\nabla^2T+\underline{\underline{\sigma}}:\nabla\mathbf{V}+(\mathbf{V}-\mathbf{V}_p)\cdot\mathbf{S}_l+S_e,$$
$$\rho_p c_p\mathbf{V}_p\cdot\nabla T_p=\underline{\underline{\sigma}}_p:\nabla\mathbf{V}_p+S_{pe}\qquad(1)$$

where

$$S_m=0,\quad S_{pm}=D_p\nabla^2\rho_p,\quad \underline{\underline{\sigma}}=-p\underline{\underline{I}}+\mu(\nabla\mathbf{V}+\nabla\mathbf{V}^T),\quad \underline{\underline{\sigma}}_p=0$$
$$\mathbf{S}_{pl}=-\mathbf{S}_l=\rho_p(\mathbf{V}-\mathbf{V}_p)/\tau_v,\quad S_e=-S_{pe}=\rho_p c_p(T_p-T)/\tau_T\qquad(2)$$

It is useful to nondimensionalize Eqs. (1), and transform the computational domain from semi-infinite ($0\le x<\infty$) to finite ($0\le\xi\le1$). This is achieved by substituting the modified Blasius transformations

$$x=V_\infty\tau_v\xi/(1-\xi),\quad y=(2\nu\tau_v\xi/(1-\xi))^{1/2}\eta$$
$$\mathbf{V}=\mathbf{e}_xV_\infty F(\xi,\eta)+\mathbf{e}_y(\nu(1-\xi)/(2\tau_v\xi))^{1/2}(G(\xi,\eta)+\eta F(\xi,\eta))$$
$$\mathbf{V}_p=\mathbf{e}_xV_\infty F_p(\xi,\eta)+\mathbf{e}_y(\nu(1-\xi)/(2\tau_v\xi))^{1/2}(G_p(\xi,\eta)+\eta F_p(\xi,\eta))$$
$$\rho_p=\rho_{p\infty}Q_p(\xi,\eta),\quad T=T_\infty H(\xi,\eta),\quad T_p=T_\infty H_p(\xi,\eta)\qquad(3)$$

into the boundary-layer form of the governing equations to give

$$\partial_\eta G+F+2\xi(1-\xi)\partial_\xi F=0,$$
$$\delta\partial_{\eta\eta}Q_p-(\partial_\eta(Q_pG_p)+Q_pF_p+2\xi(1-\xi)\partial_\xi(Q_pF_p))=0,$$
$$\partial_{\eta\eta}F-G\partial_\eta F-2\xi(1-\xi)F\partial_\xi F+2\xi\kappa Q_p(F_p-F)/(1-\xi)=0$$
$$G_p\partial_\eta F_p+2\xi(1-\xi)F_p\partial_\xi F_p+2\xi(F_p-F)/(1-\xi)=0$$
$$G_p\partial_\eta G_p+\eta(G_p\partial_\eta F_p-F_p^2)+2\xi(1-\xi)F_p(\partial_\xi G_p+\eta\partial_\xi F_p)$$
$$+2\xi(G_p-G+\eta(F_p-F))/(1-\xi)=0$$
$$\partial_{\eta\eta}H-\Pr G\partial_\eta H+\Pr E_c(\partial_\eta F)^2-2\xi(1-\xi)\Pr F\partial_\xi H$$
$$+2\xi\kappa\Pr Q_p(\gamma\epsilon(H_p-H)+E_c(F_p-F)^2)/(1-\xi)=0$$
$$G_p\partial_\eta H_p+2\xi(1-\xi)F_p\partial_\xi H_p-2\xi\epsilon(H-H_p)/(1-\xi)=0\qquad(4)$$

where

$$\delta=D_p/\nu,\quad \kappa=\rho_{p\infty}/\rho,\quad \Pr=(\mu c)/k,\quad E_c=V_\infty^2/(cT_\infty),$$
$$\gamma=c_p/c,\quad \epsilon=\tau_v/\tau_T.\qquad(5)$$

are the inverse Schmidt number, particle loading, fluid-phase Prandtl number, Eckert number, specific heat ratio, and the ratio of momentum relaxation time to the temperature relaxation time, respectively. It should be pointed out that the use of the modified Blasius transformations eliminates the singularities associated with the sharp leading edge of the plate.

The boundary and matching conditions used in obtaining the solution of this problem are

$$F(\xi,0)=0,\quad G(\xi,0)=0,\quad H(\xi,0)=H_0,\quad \partial_\eta Q_p(\xi,0)=0,$$
$$F(\xi,\infty)\to1,\quad F_p(\xi,\infty)\to1,\quad G_p(\xi,\infty)\to G(\xi,\infty),$$
$$Q_p(\xi,\infty)\to1,\quad H(\xi,\infty)\to1,\quad H_p(\xi,\infty)\to1\qquad(6)$$

where H_0 is a dimensionless wall temperature. It should be noted that the fourth equation in Eqs. (6) allows the particle-phase diffusivity effects to vanish at the wall.

The wall heat transfer coefficient is an important physical thermal property, which can be used as a measure of the wall heat gain or loss that can be brought about by physical changes in the flow field. It can be defined in dimensionless form as

$$\tilde{q}_w(\xi)=-\partial_\eta H(\xi,0)/(\Pr E_c).\qquad(7)$$

Results

The boundary-layer equations, Eqs. (4), are solved subject to the boundary and matching conditions, Eqs. (6), using a fully implicit finite-difference scheme similar to that described by Blottner (1970). It is a well-known fact that the flow and temperature fields experience large velocity and temperature gradients near the plate surface and the leading edge. For this reason, a variable mesh (with initial step size $\Delta\eta_1=0.001$ and

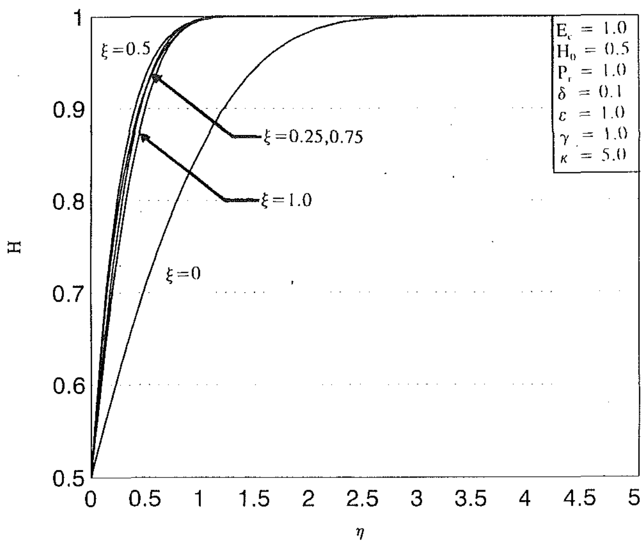

Fig. 1 Fluid-phase temperature profiles

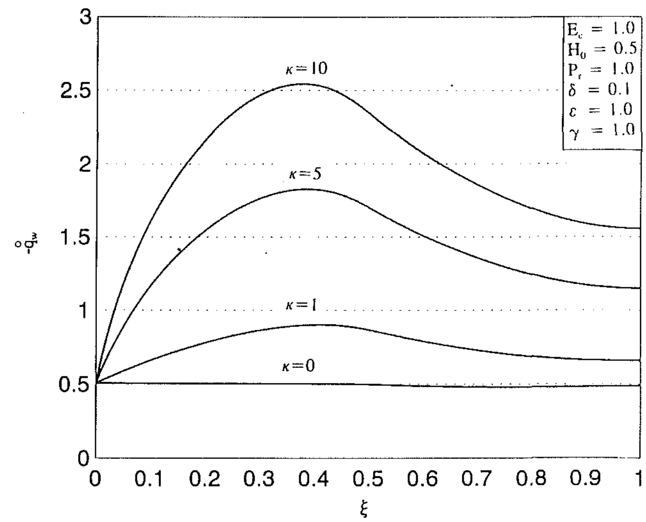

Fig. 3 Wall heat transfer coefficient versus position

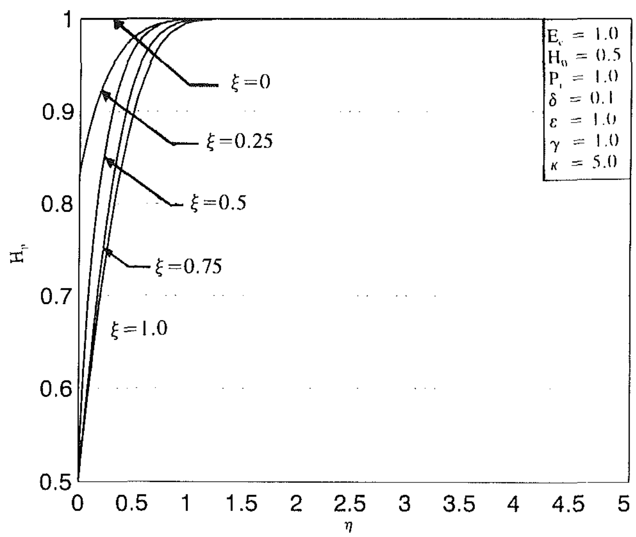

Fig. 2 Particle-phase temperature profiles

a growth factor of 1.03) is placed in the direction normal to the flow (η direction) to minimize the number of nodes and yet provide a sufficient number of nodes in the immediate vicinity of the wall where viscous effects are dominant. Fine meshes are used near the surface, while coarser ones are employed far from the plate as the free stream conditions are approached. Uniform fine meshes (with step size $\Delta\xi = 0.001$) are used in the flow direction parallel to the plate (ξ direction) in order to capture any discontinuities or singularities that may occur along the plate.

The first-order derivatives with respect to ξ are approximated by two-point backward difference quotients, whereas equations with second-order derivatives with respect to η are approximated by three-point central difference quotients. Differencing of first-order equations in η is accomplished by the trapezoidal rule. The solution is started at $\xi = 0$ and marched forward toward $\xi = 1$. At each line of constant ξ, a tridiagonal matrix of algebraic equations representing the partial differential equations is solved. Because of the nonlinear nature of the governing equations, iteration is used until convergence of the desired solution is reached. Since the solution of the hydrodynamic problem is independent from the thermal problem, the balance laws of mass and linear momentum for both phases are solved first. Once the hydrodynamic solution is obtained, it is used to solve the energy equations of both phases. At each line of constant ξ, iteration is continued until the desired convergence criterion between the current and the previous iteration (10^{-5} in this case) is satisfied.

Many numerical results were obtained through the course of this work. For brevity, a few are shown graphically in Figs. 1–5 to illustrate the influence of the particle loading κ and the inverse Schmidt number δ on the solutions.

Figures 1 and 2 present the fluid-phase temperature H and the particle-phase temperature H_p at various ξ locations along the plate, respectively. It is apparent that unlike the dusty-gas results (without diffusion), continuous solutions exhibiting the proper transition from thermally frozen conditions to thermally equal conditions exist. Figure 3 shows the development of the wall heat transfer coefficient $\tilde{q}_w$ along the plate for various particle loading conditions. It can be seen from this figure that the wall heat transfer increases as the particle loading increases. This is due to the increase in the interaction between the two phases in which the fluid gains kinetic and thermal energy from the particles. Figures 4 and 5 depict the effect of the inverse Schmidt number δ on the particle-phase density at the wall $Q_p(\xi, 0)$ and the wall heat transfer coefficient $\tilde{q}_w$, respectively. It can be seen that while δ appears to have a significant effect on the behavior of the particle-phase wall density, it seems to have little effect on the wall heat transfer. Figure 4 shows a situation in which a limiting process by a gradual reduction in the value of δ produces a closer and closer approach to the original dusty-gas solution. This is an indication that the singularity in the particle-phase density observed in the original dusty-gas model is real and not an artifact of the numerical procedure. It should be noted that the peaks in $\tilde{q}_w$ are expected and they are well-known features of relaxation type problems. Since the value of ϵ used to produce the numerical results is taken to be unity, the approach of the velocity and temperature profiles from frozen to equilibrium conditions occurs at the same rate.

Conclusion

Equations governing boundary-layer flow of a particulate suspension exhibiting small volume fraction with particulate diffusivity past a semi-infinite flat plate were developed. A fully implicit finite difference scheme was employed to solve the governing equations, and graphic results for the temperature profiles of both the fluid and particle phases, the particle-phase wall density, and the rate of heat transfer to the plate surface were presented and discussed. These results were used

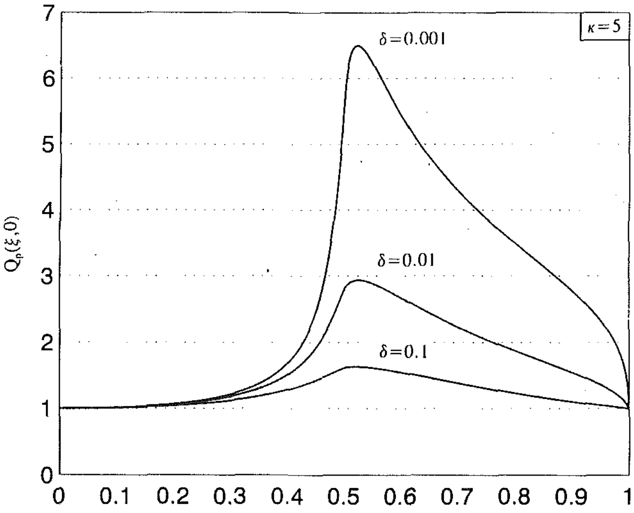

Fig. 4 Particle-phase wall density versus position

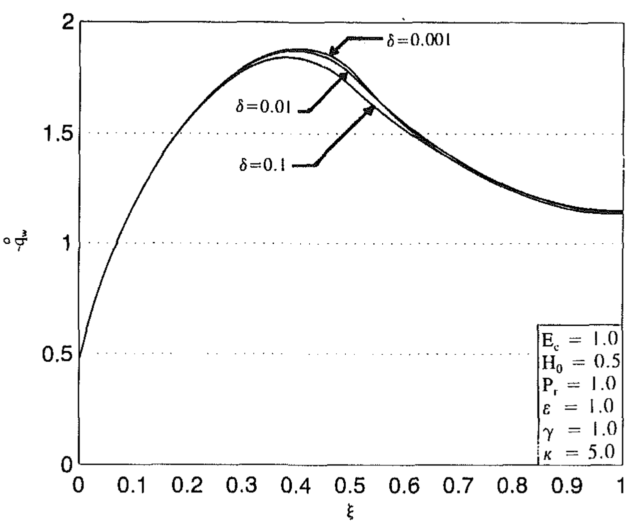

Fig. 5 Wall heat transfer coefficient versus position

to show the influence of the particle loading and the inverse Schmidt number on the thermal aspects of this problem. It was found that when the dusty-gas model is modified to include such effects as diffusion of particles, a singularity-free solution exists. The presence of the particle-phase diffusivity in the model provides enough smoothing to prevent a singularity from forming in the particle-phase density as predicted by the diffusionless dusty-gas equations. This is an example where a small change in the physical model causes significant changes in predictions. It was also found that while particle-phase diffusion affects the particle-phase density greatly, it has slight effect on the wall heat transfer. It should be pointed out that comparison of the numerical results with experimental data is not possible at present due to the absence of such data.

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Transport Phenomena at Entrance Regions of Rotating Heated Channels With Laminar Throughflow

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Nomenclature

a = channel height in the y direction, m
b = channel width in the z direction, m
f = friction factor
H = distance from rotational axis to inlet, m
h = convective heat transfer coefficient, W/m^2-°C
k = conductivity of fluid
Nu_x = locally averaged Nusselt number over a wall $= ha/k$
Nu = circumferentially averaged Nusselt number
$\overline{Nu}$ = the integrated mean value of the circumferentially averaged Nu over the flow channel from inlet $x=0$ to be designated distance $x=1.0$ $(X=20a)$
Pr = Prandtl number
q_w = local wall heat flux, W/m^2
Re = Reynolds number $= U_o a/\nu$
Ro = Rossby number $= \Omega a/U_o$
Ta = Taylor number $= \Omega a^2/\nu$ = ReRo
T_o = inlet mean temperature, °C
T = fluid temperature, °C
T_w = wall temperature, °C
U, V, W = velocity components in the x, y, z directions, respectively, m/s
U_o = inlet mean velocity, m/s
u, v, w = dimensionless velocity components in (x, y, z) directions, respectively
X = distance in the axial direction measured from flow inlet, m
Y = coordinate in the direction of rotation, m
Z = distance along rotational axis, m
x, y, z = dimensionless coordinate system with origin at center of channel cross section
x', y', z' = dimensionless coordinate system with origin at intersection of left side wall and trailing wall
ζ = aspect ratio $= b/a$
ν = kinematic viscosity, m^2/s
Ω = angular velocity of rotation, s^{-1}

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Contributed by the Heat Transfer Division for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received by the Heat Transfer Division September 1991; revision received August 1993. Keywords: Numerical Methods, Rotating Flows. Associate Technical Editor: R. J. Simoneau.