DOUBLE-DIFFUSIVE CONVECTION IN A TILTED ENCLOSURE FILLED WITH A NON-DARCIAN POROUS MEDIUM

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ABSTRACT

The problem of laminar double-diffusive convective flow of a viscous Newtonian fluid in a tilted square enclosure filled with a non-Darcian porous medium in the presence of heat generation or absorption effects is considered. Constant temperatures and concentrations are applied on two opposing walls of the inclined enclosure while the other two walls are kept insulated. The problem is formulated in terms of the vorticity-stream function procedure. A numerical solution based on the finite-difference method is obtained. The numerical results are validated by direct comparisons with various special cases of the problem. Representative results illustrating the effects of the heat generation or absorption coefficient, inclination angle, Darcy number, thermal Rayleigh number and the non-Darcian parameter on the contour maps of the streamlines, temperature and concentration are reported. In addition, results for the average Nusselt and Sherwood numbers are presented and discussed for various parametric conditions. It is found that tilting the enclosure produces reductions in the average Nusselt and Sherwood numbers while heat absorption increases them. Also, increases in the Darcy number increase the average Nusselt and Sherwood numbers while heat generation reduces them.

KEYWORDS: Double-Diffusive Convection, Inclined Square Enclosure, Porous Media, Heat Generation or Absorption.

1. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>enclosure aspect ratio = H/W</td>
</tr>
<tr>
<td>c</td>
<td>concentration of species</td>
</tr>
<tr>
<td>c_h</td>
<td>high species concentration (source)</td>
</tr>
<tr>
<td>c_l</td>
<td>low species concentration (sink)</td>
</tr>
<tr>
<td>c_p</td>
<td>fluid specific heat at constant pressure</td>
</tr>
<tr>
<td>c_s</td>
<td>porous medium material specific heat</td>
</tr>
<tr>
<td>C</td>
<td>dimensionless concentration = (c-c_l)/(c_h-c_l)</td>
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<tr>
<td>D</td>
<td>species diffusivity</td>
</tr>
<tr>
<td>Da</td>
<td>Darcy number = W^2/μ</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>H</td>
<td>enclosure height</td>
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<tr>
<td>K</td>
<td>effective thermal conductivity</td>
</tr>
<tr>
<td>Le</td>
<td>Lewis number = α_c/ν</td>
</tr>
<tr>
<td>N</td>
<td>buoyancy ratio = β_c(c_h-c_l)/[β_T(T_h-T_c)]</td>
</tr>
<tr>
<td>Nu</td>
<td>average Nusselt number at heated wall</td>
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<tr>
<td>P</td>
<td>fluid pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number = ν/α_c</td>
</tr>
<tr>
<td>Q_g</td>
<td>heat generation or absorption coefficient</td>
</tr>
<tr>
<td>Ra_T</td>
<td>thermal Rayleigh number = gβ_T(T_h-T_c)W^3/(α_cν)</td>
</tr>
<tr>
<td>Sh</td>
<td>average Sherwood number at heated wall</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>T_h</td>
<td>hot wall temperature (source)</td>
</tr>
<tr>
<td>T_c</td>
<td>cold wall temperature (sink)</td>
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<tr>
<td>u</td>
<td>horizontal velocity component</td>
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<td>U</td>
<td>dimensionless horizontal velocity = uW/α_c</td>
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<td>vertical velocity component</td>
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<td>dimensionless vertical velocity = vW/α_c</td>
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<td>W</td>
<td>enclosure width</td>
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<td>x</td>
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<td>y</td>
<td>vertical coordinate</td>
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<td>Y</td>
<td>dimensionless vertical coordinate = y/W</td>
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Greek Symbols

<table>
<thead>
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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>α</td>
<td>enclosure inclination angle</td>
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<tr>
<td>α_c</td>
<td>effective thermal diffusivity of the porous medium</td>
</tr>
<tr>
<td>β_T</td>
<td>thermal expansion coefficient</td>
</tr>
<tr>
<td>β_c</td>
<td>compositional expansion coefficient</td>
</tr>
<tr>
<td>ε</td>
<td>porosity of the porous medium</td>
</tr>
<tr>
<td>φ</td>
<td>dimensionless heat generation or absorption coefficient = Q_gW^2/(ρc_pα_c)</td>
</tr>
<tr>
<td>κ</td>
<td>permeability of the porous medium</td>
</tr>
<tr>
<td>μ</td>
<td>fluid dynamic viscosity</td>
</tr>
<tr>
<td>ν</td>
<td>fluid kinematic viscosity = μ/ρ</td>
</tr>
<tr>
<td>θ</td>
<td>dimensionless temperature = (T-T_c)/(T_h-T_c)</td>
</tr>
<tr>
<td>ρ</td>
<td>fluid density</td>
</tr>
<tr>
<td>ρ_p</td>
<td>porous medium material density</td>
</tr>
<tr>
<td>σ</td>
<td>specific heats ratio = [ερc_p + (1-ε)ρ_sc_s]/(ρc_p)</td>
</tr>
<tr>
<td>τ</td>
<td>dimensionless time = α_c t/W^2</td>
</tr>
<tr>
<td>ω</td>
<td>vorticity</td>
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</table>
\[ \psi = \frac{\text{dimensionless stream function}}{\psi / \alpha_e} \]

\[ \psi = \text{stream function} \]

\[ \Omega = \frac{\text{dimensionless vortic} \text{ity}}{\omega W^2 / \alpha_e} \]

\[ \Lambda = \text{a constant used to indicate presence or absence of porous medium inertia} \]

\[ \nabla^2 = \text{Laplacian operator} \]

**Subscripts**

\[ w = \text{wall condition} \]

\[ \infty = \text{ambient condition} \]

1. INTRODUCTION

Heat and mass transfer by natural convection in porous media is a very important subject in view of its occurrence in many natural and technological processes. These processes include migration of water in geothermal reservoirs, underground spreading of chemical wastes and other pollutants, grain storage, evaporative cooling, solidification (Karimi-Fard et al. [1]), migration of moisture in fibrous insulation and nutrient transfer in sea-bed, petroleum extraction, prevention of sub-soil pollution, chemical reactors, drying, and crystal growth from liquid phase and many others. Extensive reviews of fundamental studies of thermal convection in porous media are provided by Kakac et al. [2] and Nield and Bejan [3]. Many studies in the open literature employ the Darcy’s law to study natural convection in porous media-filled cavities and do not take into account the species concentration equation. In some natural convection situations, the heat transfer aspect can not completely describe the phenomenon. Both heat and mass transfer must be considered. The combination of temperature and concentration gradients in the fluid will lead to buoyancy-driven flows. This has an important influence on the solidification process in a binary system. When heat and mass transfer occur simultaneously, it leads to a complex fluid motion called double-diffusive convection. Double-diffusive convection occurs in a wide range of scientific fields such as oceanography, astrophysics, geology, biology and chemical processes (see, for instance, Beghein et al. [4]). Double-diffusive convection is often studied using the Darcy formulation and Boussinesq approximation. The majority of the many works in the literature related to double diffusive convection in a cavity, can be classified into two categories: cavity with imposed uniform heat and mass fluxes and cavity with uniform temperature and concentration. In the absence of porous media, many works have been reported. For example, Chang and Lin [5] have examined thermo-solutal opposing convection in a salt-water solution at high thermal Rayleigh numbers. Bergman and Hyun [6] have analyzed numerically double-diffusive convection in liquid metals with low Prandtl numbers. Waver and Viskanta [7] have examined experimentally natural convection in binary gases with low Lewis numbers for both aiding and opposing flows. Tsitverblit [8] has studied numerically natural convection of stably stratified salt solution in a rectangular enclosure. Nishimura et al. [9] have examined oscillatory double-diffusive convection in a rectangular enclosure with combined horizontal temperature and concentration gradients. Regarding porous media models, the phenomenological relation between the pressure drop across a saturated porous medium and the flow rate was first established by Darcy in 1856. Two notable modifications of Darcy’s law are the Brinkman’s (in 1947) and the Forchheimer’s (in 1901) extensions which account for the viscous stresses adjacent to the boundary walls and the non-linear drag effect due to the solid matrix, respectively. In the presence of a porous medium, Chan et al. [10] utilized Brinkman’s extension of the Darcy law to study natural convection in an enclosure. Other works dealing with porous cavities are those of Lauriat and Prasad [11], Vasseur et al. [12]. Many authors have used the Forchheimer’s extension for representing the non-linear solid matrix drag (Lauriat and Prasad [13], Hisato [14] and Karimi-Fard et al. [1]). Some authors have considered a modified from of the Navier-Stokes equations including all fluid forces and the solid matrix drag in the momentum equations. In this respect, Vafai and Tien [15] have established the steady-state governing equations for a porous medium using the local volume averaging technique. They represented the drag forces on the solid matrix through the Ergun’s relation [16]. The usage of Ergun’s relation for representing the non-linear solid matrix drag is of recent origin (Nithiarasu et al. [17, 18]).

Natural convection heat transfer induced by internal heat generation has received considerable attention in recent years because of its numerous applications in geophysics and energy-related engineering problems. Such applications include heat removal from nuclear fuel debris, underground disposal of radioactive waste materials, storage of foodstuff, and exothermic chemical reactions in packed-bed reactor (see, for instance, Kakac et al. [19]). Also, hot dike complexes in a volcanic region can provide the energy source for the heating of groundwater which can be tapped for power generation. This can be modeled as a porous enclosure with heat generation. Acharya and Goldstein [20] have studied numerically two-dimensional natural convection of air in an externally heated vertical or inclined square box containing uniformly distributed internal energy sources. Their numerical results showed two distinct flow pattern systems depending on the ratio of the internal to the external Rayleigh numbers. Also, it was found that the average heat flux ratio along the cold wall increased with increasing external Rayleigh numbers and decreasing internal Rayleigh numbers. Recently, Churbanov et al. [21] have studied numerically unsteady natural convection of a heat generation fluid in a vertical rectangular enclosure with isothermal or adiabatic rigid walls. Their results were obtained using a finite-difference scheme in the two-dimensional stream-function-vorticity formulation. Steady-state as well as oscillating solution were obtained and compared with other numerical and experimental published data. Other related works dealing with temperature-dependent heat generation effects can be found in the papers by Vajravelu and Nayfeh [22], Chamkha [23] and Khanafer and Chamkha [24].

The objective of the present work is to consider a generalized Darcy-Brinkman-Forchheimer model in simulation of double-diffusive convective flow of a viscous Newtonian fluid in a tilted square enclosure filled with a non-Darcian porous medium in the presence of heat generation or absorption effects that are linearly proportional to temperature difference. This has direct applications in exothermic reaction in packed-bed reactors and in the heat recovery from geothermal systems and particularly in the field of large storage systems of agricultural products.
3. GOVERNING EQUATIONS

The schematic of the problem under consideration is shown in Figure 1. The temperatures are uniformly imposed along two opposing walls while the other two walls are assumed adiabatic and impermeable to mass transfer. The inclined enclosure is completely filled by a uniform fluid-saturated porous medium in local thermal equilibrium with the fluid. The fluid is assumed to be incompressible, heat generating or absorbing and viscous. The viscous dissipation is assumed to be negligible. The Boussinesq approximation with opposite thermal and compositional buoyancy forces is used for the body force in the momentum equations.

The governing equations for the problem under consideration are based on the balance laws of mass, nonlinear momentum, thermal energy, and concentration in two dimensions. Taking into account all previous assumptions, these equations can be written in dimensional form as

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{1}{\varepsilon} \frac{\partial u}{\partial t} + \frac{1}{\varepsilon} \frac{\partial u}{\partial x} + \frac{1}{\varepsilon^2} \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\varepsilon} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
+ g \beta (T - T_e) \sin \alpha - \beta_c (c - c_e) \sin \alpha - \frac{\mu}{\rho_k} u &= 0
\end{align*}
\]

(1)

\[
\frac{1}{\varepsilon} \frac{\partial v}{\partial t} + \frac{1}{\varepsilon} \frac{\partial v}{\partial x} + \frac{1}{\varepsilon^2} \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\varepsilon} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

(2)

\[
\frac{1}{\varepsilon} \frac{\partial T}{\partial t} + \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} = \frac{K}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_e}{\rho c_p} (T - T_e)
\]

(3)

\[
\frac{\partial c}{\partial t} + \frac{\partial u}{\partial x} \frac{\partial c}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial c}{\partial y} = D \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)
\]

(4)

where \(u\) and \(v\) are the velocity components in the \(x\) and \(y\) directions, \(p\), \(T\) and \(c\) are the pressure, temperature and concentration, respectively, \(\beta\) and \(\beta_c\) are the thermal and compositional expansion coefficients, respectively. The parameter \(\tau\) is time and \(\kappa\), \(\sigma\), \(\nu\), \(\mu\), \(\mu_p\) and \(\alpha\) are the porous medium permeability, ratio of specific heats, and porosity of the porous medium, the fluid kinematic and dynamic viscosities, specific heat at constant pressure, fluid density and the enclosure inclination angle, respectively. \(D\) is the species diffusivity, \(T_e\) and \(c_e\) are the cold wall temperature and concentration, respectively. \(g\) is the gravitational acceleration. \(Q_e\) is the dimensional heat generation or absorption coefficient, respectively. The parameter \(\Lambda\) is employed to indicate the presence or absence of the porous medium inertia effects such that when \(\Lambda = 0\), these effects are absent and when \(\Lambda = 1\), these effects are present.

The initial and boundary conditions for the problem can be written as:

\[
\begin{align*}
t &= 0, \quad x = x_e, \quad y = y_e & : & \quad u = 0, \quad v = 0, \quad T = T_e, \quad c = c_e \\
x = 0, \quad y = 0 & : & \quad u = 0, \quad v = 0, \quad T = T_{in}, \quad c = c_{in} \\
x = W, \quad y = 0 & : & \quad u = 0, \quad v = 0, \quad T = T_{in}, \quad c = c_{in} \\
x = x_e, \quad y = 0 & : & \quad u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial c}{\partial y} = 0 \\
x = x_e, \quad y = H & : & \quad u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial c}{\partial y} = 0
\end{align*}
\]

(6)

where \(W\) and \(H\) are the width and height of the enclosure, respectively. It should be noted herein that for simplicity, the present work assumes that two of the four walls do not possess heat or mass. These are the two walls at which temperature and concentration gradients in the transverse direction are equal to zero (\(y=0\) and \(y=H\)). However, in some practical applications, it is not necessary that thermally insulated walls are also mass isolated ones since a given wall may allow for heat transfer but not for mass and vise versa.

The vorticity can be defined in the usual way as

\[
\omega = -\left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

(7)

where \(\psi\) is the dimensional stream function and \(\omega\) is the dimensional vorticity. Equations (1) through (7) are non-dimensionalized using the following dimensionless variables:

\[
\begin{align*}
\tau &= \frac{\alpha_e x}{W^2}, \quad X = \frac{x}{W}, \quad Y = \frac{y}{W}, \quad U = \frac{u}{W \alpha_e}, \quad V = \frac{v}{W \alpha_e}, \\
\theta &= \frac{T - T_e}{T_h - T_e}, \quad C = \frac{c - c_e}{c_h - c_e}, \quad p = \frac{p}{\rho \alpha_e}, \quad \Omega = \frac{\alpha_e W^2}{\rho}, \quad \Psi = \frac{\psi}{\alpha_e}, \\
\alpha_e &= \frac{K}{\rho c_p}, \quad \nu = \frac{\mu}{\rho}, \quad Pr = \frac{\nu}{\alpha_e}, \quad Le = \frac{\alpha_e}{D}, \quad Da = \frac{K}{W^2}, \\
N &= \frac{P_e (c_h - c_e)}{\beta (T_h - T_e)}, \quad \phi = \frac{Q_e W^2}{\rho c_p \alpha_e}, \quad Ra_T = \frac{\beta (T_h - T_e)}{c_e \alpha_e}, \quad \sigma = \frac{\varepsilon \rho c_p}{\rho c_p} \frac{(1 - e) \rho_c}{\rho c_p}
\end{align*}
\]

(8)
The Prandtl number, Le is Lewis number, RaT is the thermal Rayleigh number, Da is the Darcy number, \( \phi \) is the dimensionless heat generation or absorption coefficient and \( N \) is the buoyancy ratio.

Differentiating Equation (3) with respect to \( x \) and Equation (2) with respect to \( y \) and then subtracting the resulting equations eliminates the pressure gradient terms and combines the original two equations into a single equation in terms of the vorticity. By employing Equations (8), the resulting dimensionless equations can be written as:

\[
\Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} = -\nabla^2 \Psi
\]  
(9)

\[
\frac{1}{\varepsilon} \left( \frac{\partial \Omega}{\partial \tau} + \frac{1}{\varepsilon} U \frac{\partial \Omega}{\partial X} + \frac{1}{\varepsilon^2} V \frac{\partial \Omega}{\partial Y} \right) = \frac{1}{\varepsilon} \nabla^2 \Omega - \frac{Pr}{Da} \frac{\partial \Omega}{\partial \tau}
\]

\[
-\frac{1.75}{\sqrt{150}} \left( \sqrt{\frac{U^2 + V^2}{\varepsilon^{3/2}}} \right)^{n-1} \Omega + Ra_T Pr \frac{\partial \Omega}{\partial X} \cos \alpha - \frac{\partial \Omega}{\partial Y} \sin \alpha
\]

\[
-\frac{Pe}{Ra_T} Pr \frac{\partial \Omega}{\partial X} \cos \alpha - \frac{Pe}{Ra_T} Pr \frac{\partial \Omega}{\partial Y} \sin \alpha
\]  
(10)

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = 1 \nabla^2 U + \phi U
\]  
(11)

\[
\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Le} \nabla^2 C
\]  
(12)

where the superscript \((n-1)\) is only employed to indicate that the velocity components \((U \text{ and } V)\) in the porous medium inertia term of Equation (10) are taken constant as they are evaluated at the previous time step.

The dimensionless initial and boundary conditions become

\(\tau = 0:\)

\[
U = V = \Psi = 0, \quad \theta = 0, \quad C = 0
\]  
(13a)

\(Y=0:\)

\[
U = V = \Psi = 0, \quad \Omega = \left( \frac{\partial \Omega}{\partial Y^2} \right), \quad \frac{\partial \Omega}{\partial Y} = 0, \quad \frac{\partial C}{\partial Y} = 0
\]  
(13b)

\(Y=H/W:\)

\[
U = V = \Psi = 0, \quad \Omega = \left( \frac{\partial \Omega}{\partial Y^2} \right), \quad \frac{\partial \Omega}{\partial Y} = 0, \quad \frac{\partial C}{\partial Y} = 0
\]  
(13c)

\(X=0:\)

\[
U = V = \Psi = 0, \quad \Omega = \left( \frac{\partial \Omega}{\partial X^2} \right), \quad \theta = 1, \quad C = 1
\]  
(13d)

\(X=1:\)

\[
U = V = \Psi = 0, \quad \Omega = \left( \frac{\partial \Omega}{\partial X^2} \right), \quad \theta = 0, \quad C = 0
\]  
(13e)

It should be noted that for a non-inclined enclosure \((\alpha = 0)\) and in the absence of the heat generation or absorption effects \((\phi = 0)\), Equations (9) through (13) reduce to the equations reported by Nathiarasu et al. [17].

The Nusselt and Sherwood numbers are averaged and evaluated along the left vertical boundary of the enclosure which may be expressed in dimensionless form as

\[
\overline{Nu} = -\int_0^1 \left( \frac{\partial \theta}{\partial X} \right) dY
\]  
(14a)

\[
\overline{Sh} = -\int_0^1 \left( \frac{\partial C}{\partial X} \right) dY
\]  
(14b)

where the aspect ratio \((A=H/W)\) for a square enclosure is unity.

4. NUMERICAL METHOD

The numerical algorithm used to solve Equations (9) through (13) is based on the finite-difference methodology. First, central difference quotients are used to approximate the second-order derivatives while the first-order derivatives are approximated by backward difference in time and central difference in space. The equations are transformed into a tri-diagonal set of algebraic equations. This tri-diagonal system of algebraic equations can be easily solved by the well-known Thomas algorithm at each specific dimensionless time. These equations are solved in the \(x\)-direction for the concentration, temperature, vorticity and the stream function and then the solution is swept in the \(y\)-direction.

The following respective form of a tri-diagonal equation will be obtained:

\[
Z_{i+1,j} [E] + Z_{i,j} [B] + Z_{i+1,j} [A] = [D]
\]  
(15)

where \(Z\) is a typical unknown dependent variable at time level \(n+1\) that can be \(\Psi\), \(\Omega\), \(\theta\) or \(C\) and \([E]\), \([B]\) and \([A]\) are coefficients of the dependent variable \(Z\) and subscripts \(i\) and \(j\) denote the \(X\) and \(Y\) locations, respectively. These coefficients may be constant or functions of other dependent or independent variables. The \([D]\) term represents all of the variables which include variable \(Z\) at time level \(n\) or constants appearing alone in each equation which are not multiplied by the variable \(Z\) at time level \(n+1\).

The numerical computation is carried out for 4x41 grid nodal points for a time step of \(10^{-6}\), \(\Delta X = 1/40\) and \(\Delta Y = 1/40\). The convergence criterion required that the difference between the current and previous iterations for all of the dependent variables be \(10^{-4}\).

5. SOLUTION PROCEDURE

1. All dependent variables are initialized to zero.
2. The new boundary condition values at time level \((n+1)\) are calculated for all walls from the previous values at time level \((n)\).
3. The new concentration values at time level \((n+1)\) are calculated from the previous time level \((n)\) values, and then a subroutine is called to solve the obtained tri-diagonal equations for all the concentration values at all the internal grid points.
4. The temperature, vorticity, and the stream function are calculated in the same way as in step (3), respectively.
5. The velocity components $U$ and $V$ are calculated at time level $(n+1)$ from the values at time level $(n)$ explicitly for all the internal grid points.

6. The error is calculated for the concentration, temperature, and the vorticity at the last time step (only for a steady solution).

7. To obtain the solution at the next time step $(n+2)$, the same procedure is followed by starting with step (2).

This procedure is for unsteady solution. If a steady-state solution is required, then the concentration and the temperature are only needed to be updated for a number of internal loops for each single time step. Then, at the end of this single time step, the vorticity, stream function, and the velocity components $(U$ and $V$) need to be updated.

8. The average Nusselt and Sherwood numbers are then calculated at the hot wall.

6. VALIDATION

In order to check on the accuracy of the numerical technique employed for the solution of the problem considered in the present study, it was validated by performing simulation for steady-state double-diffusive convection flow in a vertical rectangular enclosure with combined horizontal temperature and concentration gradients and in the absence of the porous medium and the heat generation or absorption effects which was reported earlier by Nishimura et al. [9]. Figures 2 and 3 present comparisons for the streamlines, isotherms, concentration contours and density contours of the present work at $N=0.8$ (thermal-dominated flow) and $N=1.3$ (compositional-dominated flow) with those of Nishimura et al. [9]. Also, Table 1 presents some comparisons for the average Nusselt number with the previous works of Lauriat and Prasad [13] and Nithiarasu et al. [17]. These various comparisons show good agreement between the results and lend confidence in the numerical results to be reported in the next section.

![Figure 2. A comparison for thermal-dominated flow $N=0.8$](image)

![Figure 3. A comparison for compositional-dominated flow $N=1.3$](image)

<table>
<thead>
<tr>
<th>$Ra^*$</th>
<th>$Da$</th>
<th>[13]</th>
<th>[17]</th>
<th>Present work</th>
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<tr>
<td>100</td>
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<td>100</td>
<td>$10^2$</td>
<td>1.70</td>
<td>1.71</td>
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5. RESULTS AND DISCUSSION

In this section, representative numerical results for the steady-state streamline, temperature, and concentration contours within the enclosure will be reported. In addition, representative steady-state results for the average Nusselt number $Nu$ and the average Sherwood number $Sh$ for various physical conditions will be presented and discussed. In all the graphical results to be presented subsequently, $Da = 5 \times 10^2$, $Le = 2.0$, $Pr = 1.0$, $Ra = 10^3$, $\alpha = 0$, $\Lambda = 1.0$, $\phi = 0$, $\epsilon = 0.6$, and $\sigma = 1.0$ unless otherwise stated.

Figures 4 and 5 depict the effects of the heat generation or absorption coefficient $\phi$ on the streamline, temperature and concentration contours for heat absorption $(\phi < 0)$ and heat generation $(\phi > 0)$ for $N = 0.8$, respectively. It is observed that the presence of a heat sink (heat absorption, $\phi < 0$) within the enclosure causes the thermal recirculation within the enclosure to move slightly upward and a smaller thermal recirculation is formed in the opposite corner near the right wall of the enclosure. In addition, the waves in the core
region for temperature and concentration contours tend to move upwards. This is obvious from Figure 4. This indicates that heat absorption causes higher heat transfer rates. However, the presence of a heat source (heat generation, $\phi > 0$) produces less heat transfer rates and the thermal recirculation tends to move slightly downward. A smaller thermal recirculation is formed in the upper opposite corner near the left wall of the enclosure. In addition, the waves in the core region of the enclosure for temperature and concentration tend to move downward as illustrated in Figure 5.

Figures 6 and 7 present similar results as those presented in Figures 4 and 5 but for $N=1.5$. Similar to the case of $N=0.8$, the presence of a strong heat sink tends to cause the formation of a smaller recirculation but in the left lower corner of the enclosure with the primary recirculation in the core region is slowed down as depicted by the reduction in the value of the maximum stream function. The exact opposite effect is predicted for case of heat generation ($\phi > 0$) where a smaller recirculation in the right upper corner is formed and the primary recirculating eddy is enhanced as the strength of the heat source is increased. Also, in the presence of a heat sink, the temperature and concentration contours are stretched at the bottom part of the enclosure and are crowded at the right top part of the enclosure. On the other hand, in the presence of a heat source, the temperature and concentration contours are stretched at the top part of the enclosure and are crowded at the right bottom part of the enclosure.
The influence of the heat generation or absorption coefficient $\phi$ and the Darcy number $Da$ on the values of $Nu$, $Sh$ and $\Psi_{max}$ for $N=1.5$ is depicted in Figures 8 through 10, respectively. It is observed that heat generation ($\phi > 0$) decreases the average Nusselt number, the average Sherwood number and the maximum stream function while heat absorption ($\phi < 0$) increases all of them. In addition, all of $Nu$, $Sh$ and $\Psi_{max}$ increase as $Da$ increases. Physically, the behavior of heat and mass transfer through the porous medium can be classified into three regimes depending on the value of $Da$. The first regime for $Da < 10^{-4}$ ($Ra_t*Da < 10$) is classified as the conduction regime where very slow flow occurs due high stresses caused by low permeability of the porous medium. The second regime for $10^{-4} < Da < 5x10^{-4}$ ($10 < Ra_t*Da < 50$) is classified as the mixed conduction-convection regime where the flow speed is moderate. Finally, the third regime for $Da > 5x10^{-4}$ ($Ra_t*Da > 50$) is classified as the convection regime where high permeability of the porous medium allows higher flow rates.

![Figure 8. Effects of $\phi$ on the average Nusselt number for $N=1.5$](image1.png)

![Figure 9. Effects of $\phi$ on the average Sherwood number for $N=1.5$](image2.png)

![Figure 10. Effects of $\phi$ on the maximum stream function for $N=1.5$](image3.png)

Figure 7. Effects of $\phi$ on the stream function, temperature, and concentration contours for $N=1.5$ and $Da=5x10^{-4}$ where (a) $\phi=1.0$ and (b) $\phi=2.0$
Figure 11 presents the effect of buoyancy ratio $N$ on the values of $\overline{\text{Nu}}$ for $\text{Da} = 10^{-3}$ and $\text{Ra}_{\gamma} = 2 \times 10^6$ at different values of $\phi$. The $\overline{\text{Nu}}$ profiles have a V-shape with $N$ where $\overline{\text{Nu}}$ is minimum at $N = 1.0$ and it increases by increasing or decreasing the value of $N$ from $N = 1.0$ in a symmetrical fashion. This suggests that the buoyancy-driven flows that come from the combination of temperature and concentration gradients in the fluid increase as the difference between the value of $N$ and unity increases.

![Figure 11. Effects of $\phi$ on the average Nusselt number for $\text{Da}=10^{-3}$ and $\text{Ra}_{\gamma}=2 \times 10^6$.](image)

Figures 12 through 14 present steady-state contours for the streamline, temperature, and concentration at various values of the enclosure inclination angle ($\alpha = 0^\circ$, $30^\circ$, $60^\circ$) for $N = 0.8$ and both $\text{Da} = 6 \times 10^{-4}$ and $\text{Da} = 10^{-3}$, respectively. For $N = 0.8$, the thermal buoyancy dominates and a large clockwise thermal recirculation is predicted with the isotherms being almost horizontally uniform in the core region within the enclosure for large Darcy numbers. Furthermore, the concentration and temperature contours are crowded in opposite corners and are parallel lines in the core of the enclosure but for high Darcy numbers the concentration contours are turning in the core of the enclosure like a wave. For small Darcy numbers ($\text{Da} = 6 \times 10^{-4}$), as the enclosure is tilted, the streamline contours are distorted with the maximum values of the stream function occurring for $\alpha = 0^\circ$ which indicates a slower clockwise thermal recirculation. The temperature and concentration contours become more parallel to sidewalls for $\alpha = 60^\circ$. But for large Darcy numbers ($\text{Da} = 10^{-3}$), as the enclosure is tilted, the clockwise thermal recirculation is increased and the maximum values of the stream function increases in the core region of the enclosure compared with the case of $\alpha = 0^\circ$. Also, the concentration and temperature contours are turning more and more in the core of the enclosure and the isotherms are becoming less horizontally uniform as $\alpha$ increases.

![Figure 12. Effects of $\text{Da}$ on stream function, temperature, and concentration contours for $N=0.8$ and $\alpha=0^\circ$ where (a) $\text{Da}=6 \times 10^{-4}$ and (b) $\text{Da}=10^{-2}$.](image)

![Figure 13. Effects of $\text{Da}$ on stream function, temperature, and concentration contours for $N=0.8$ and $\alpha=30^\circ$ where (a) $\text{Da}=6 \times 10^{-4}$ and (b) $\text{Da}=10^{-2}$.](image)
Figures 14 and 15 display the effects of \( \alpha \) on the stream function, temperature, and concentration contours at steady state for \( N = 1.5 \), \( \text{Ra}_T = 10^4 \), and \( \text{Da} = 5 \times 10^{-2} \). As the inclination angle \( \alpha \) increases further and further, the compositional recirculation enhances in the core region of the enclosure for \( \alpha = 15^\circ \) and then starts to distort for larger tilt angles and the streamlines become more crowded near the vertical walls, and also, the eye in the core of the enclosure splits into two recirculations situated in opposite corners of enclosure. Also, the temperature and concentration contours become more parallel to the vertical walls. An inspection of the maximum value of the stream function for angles \( \alpha = 0^\circ, 15^\circ, 30^\circ \) and \( 45^\circ \) reveals that it is highest for \( \alpha = 0^\circ \) and decreases as \( \alpha \) increases. This indicates that the flow behavior is fastest at \( \alpha = 0^\circ \) and becomes slower as \( \alpha \) increases.

Figures 16 through 19, display the effects of the enclosure inclination angle \( \alpha \) on the profiles \( \text{Nu} \), \( \text{Sh} \) and \( \Psi_{\text{max}} \) with \( \text{Da} \) for \( N = 1.5 \), respectively. Again, it is observed that as the Darcy number increases, the values of all of \( \text{Nu} \), \( \text{Sh} \) and \( \Psi_{\text{max}} \) increase. However, as the enclosure is tilted, the flow within the enclosure reduces resulting in lower heat and mass transfer coefficients at the hot wall of the enclosure. This is depicted in the reductions in all of \( \text{Nu} \), \( \text{Sh} \) and \( \Psi_{\text{max}} \) as \( \alpha \) increases shown in Figures 17 through 19.
Figures 20 and 21 display the effects of $Ra_T$ on the profiles $Nu$ and $Sh$ with $Da$, respectively. It is observed that as the thermal Rayleigh number increases, the maximum values of $Nu$ and $Sh$ increase and it can be reached at lower values of the Darcy number compared with maximum values of $Nu$ and $Sh$ at lower values of the thermal Rayleigh number. In addition, the maximum value of $Da$ that can be reached before the oscillatory behavior occurs in the enclosure is decreased as the thermal Rayleigh number increases. Physically, the behavior of heat and mass transfer through the porous medium can be classified into three regimes depending on the values of $Da$. The first regime ($Ra_T^*Da < 10$) is classified as the conduction regime where very slow flow occurs due high stresses caused by low permeability of the porous medium. The second regime ($10 < Ra_T^*Da < 100$) is classified as the mixed conduction-convection regime where the flow speed is moderate. Finally, the third regime ($Ra_T^*Da > 100$) is classified as the convection regime where high permeability of the porous medium allows higher flow rates.

Figures 22 and 23 present the effects of the non-Darcian porous medium parameter on the values of $Nu$, $Sh$ for $N = 0.8$. It is observed that $Nu$ and $Sh$ are predicted to decrease when the non-Darcian effect is present especially at high values of $Da$. This represents the porous medium inertia effect on the heat and mass transfer in an enclosure.

6. CONCLUSIONS

The characteristics of heat and mass transfer by natural convection flow of a heat generating or absorption fluid inside an inclined square non-Darcian porous medium enclosure was studied numerically. The governing equations for a generalized model based on the Darcy-Brinkman-Forchheimer model were developed, non-dimensionalized and transformed into the vorticity-stream function formulation using appropriate dimensionless parameters. The resulting equations were then solved numerically by an implicit, iterative, finite-difference scheme. Comparisons with previously published work on special cases of the problem were performed and found to be in good agreement. Graphical results for various parametric conditions were presented and discussed. It was found that the heat and mass transfer mechanisms and the flow characteristics inside the enclosure depended strongly on the porous medium Darcian and non-Darcian effects, heat generation or absorption effects, buoyancy ratio, inclination angle and thermal Rayleigh number. The effect of heat absorption increased the heat transfer within enclosure and decreased the mass transfer. Due to the inclusion of the porous medium inertial force, the predicted heat and mass transfer rates were predicted to be lower for the generalized model as compared to the Darcy-Brinkman model. Also, the effect of increasing the enclosure inclination angle was found to decrease the average Nusselt and Sherwood numbers along the hot wall of the enclosure as well as the maximum stream function within the enclosure. In addition, the present study showed that the Darcy and thermal Rayleigh numbers effects need to be considered together for a proper understanding of heat and mass transfer in the Darcian or non-Darcian flow regimes. As the value of the product of multiplying the Darcy number by the Rayleigh number increased, the heat and mass transfer
approach changed from a conduction approach to a convection approach after which an oscillatory flow behavior was predicted.

REFERENCES

11. G. Lauriat and V. Prasad, Natural convection in a vertical porous cavity: A numerical study for Brinkman-extended