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# Two-Phase Thermal Asymptotic Suction Profile

**A. J. Chamkha**[1]

## Nomenclature

$c$ = fluid-phase specific heat at constant pressure
Ec = fluid-phase Eckert number
$\mathbf{e}_x$, $\mathbf{e}_y$ = unit vectors in $x$ and $y$ directions, respectively
$F$ = nondimensionalized fluid-phase tangential velocity
$\mathbf{f}$ = interphase force per unit volume acting on the particle phase
$H$ = nondimensionalized fluid-phase temperature
$H_0$ = nondimensionalized fluid-phase wall temperature
$\underline{\underline{I}}$ = unit tensor
$k$ = fluid-phase thermal conductivity
$P$ = fluid-phase pressure
Pr = fluid-phase Prandtl number
$Q_p$ = interphase heat transfer rate per unit volume to the particle phase
$\mathring{q}_w$ = wall heat transfer
$T$ = fluid-phase temperature
$\mathbf{V}$ = fluid-phase velocity vector
$x$, $y$ = Cartesian coordinate variables
$\alpha$ = velocity inverse Stokes number
$\beta$ = viscosity ratio
$\gamma$ = specific heat ratio
$\delta_t$ = thermal boundary layer
$\epsilon$ = temperature inverse Stokes number
$\eta$ = transformed normal coordinate
$\kappa$ = particle loading
$\mu$ = fluid-phase viscosity coefficient
$\rho$ = fluid-phase density
$\underline{\underline{\sigma}}$ = fluid-phase stress tensor
$\tau_T$ = temperature relaxation time
$\tau_v$ = momentum relaxation time
$\phi$ = volume fraction of particles
$\omega$ = slip coefficient
$\nabla$ = gradient operator

### Subscripts

$\infty$ = free stream
$p$ = particle phase
$s$ = surface

### Superscripts

$T$ = transpose of a second-order tensor

## Introduction

Many industrial processes employ particle–fluid suspensions. Understanding such processes through analysis is essential for their optimization. The problem considered in this paper is steady two-phase flow past an infinite porous flat plate. This is one of the few two-phase flow problems for which the governing equations can be exactly reduced to ordinary differential equations for a variety of physical models and solved in closed form. Soo (1967) and Marble (1970), among others, have reported continuum equations governing two-phase particulate suspensions. Chamkha and Peddieson (1989) employed these equations with some modifications and reported exact solutions for the flow fields of the problem described above without considering the thermal aspects of it.

The purpose of this paper is to report exact solutions for the convective heat flux and the temperature distributions for both phases. The fluid phase is assumed incompressible. The particle phase is assumed viscous, incompressible, and pressureless. The particle volume fraction is assumed finite and uniform.

## Governing Equations

Consider the two-dimensional steady laminar flow in a half-space bounded by an infinite porous flat plate. The half-space occupies the region $y>0$ with the plate being fixed and coincident with the plane $y=0$. The flow is a parallel stream with velocity $V_\infty$ and at a temperature $T_\infty$ in the $x$ direction. The fluid phase exhibits a uniform suction with velocity $V_s$ at the plate surface and the plate is maintained at a constant temperature $T_s$ (see Fig. 1). The particles are assumed spherical in shape and uniformly distributed in the carrier fluid. Radiative heat transfer from one particle to another is neglected.

The governing equations, which are based on the balance laws of mass, linear momentum, and energy, generalize the dusty-gas equations given by Marble (1970) through the inclusion of particle-phase viscous stresses, and are applicable to any size particles. These are given by

$$\begin{gathered}\nabla\cdot((1-\phi)\mathbf{V})=0,\quad \nabla\cdot(\phi\mathbf{V}_p)=0\\ \rho(1-\phi)(\mathbf{V}\cdot\nabla V)=\nabla\cdot\underline{\underline{\sigma}}-f,\quad \rho_p\phi\mathbf{V}_p\cdot\nabla\mathbf{V}_p=\nabla\cdot\underline{\underline{\sigma}}_p+\mathbf{f}\\ \rho c(1-\phi)\mathbf{V}\cdot\nabla T=(1-\phi)k\nabla^2T+\underline{\underline{\sigma}}:\nabla\mathbf{V}+Q_p+(\mathbf{V}-\mathbf{V}_p)\cdot\mathbf{f}\\ \rho_p c_p\phi\mathbf{V}_p\cdot\nabla T_p=\underline{\underline{\sigma}}_p:\nabla\mathbf{V}_p-Q_p\end{gathered}\tag{1}$$

where

$$\begin{gathered}\underline{\underline{\sigma}}=(1-\phi)(-P\underline{\underline{I}}+\mu(\nabla\mathbf{V}+\nabla\mathbf{V}^T)),\quad \underline{\underline{\sigma}}_p=\mu_p\phi(\nabla\mathbf{V}_p+\nabla\mathbf{V}_p^T)\\ \mathbf{f}=\rho_p\phi(\mathbf{V}-\mathbf{V}_p)/\tau_v,\quad Q_p=\rho_pc_p\phi(T_p-T)/\tau_T\end{gathered}\tag{2}$$

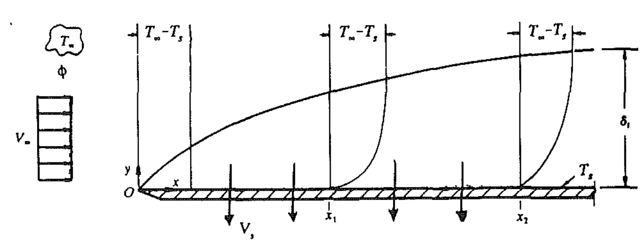


Fig. 1 Flat plate schematic

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Contributed by the Heat Transfer Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Heat Transfer Division August 1992; revision received February 1993. Keywords: Forced Convection, Multiphase Flows. Associate Technical Editor: J. H. Kim.

It is evident from Eqs. (1) and (2) that both the fluid phase and the particulate phase are coupled through drag and heat transfer between them.

Equations (1) and (2) can be nondimensionalized by using

$$y=\mu\eta/(\rho V_\infty),\quad \mathbf{V}=\mathbf{V}_\infty F(\eta)\mathbf{e}_x-V_s\mathbf{e}_y$$

$$\mathbf{V}_p=V_\infty F_p(\eta)\mathbf{e}_x-V_s\mathbf{e}_y,\quad T=T_\infty H(\eta),\ T_p=T_\infty H_p(\eta) \tag{3}$$

and rearranging to give

$$F''+r_vF'+\kappa\alpha(F_p-F)=0,\quad \beta F_p''+r_vF_p'+\alpha(F-F_p)=0$$

$$H''+r_v\,\mathrm{Pr}H'+\kappa\mathrm{Pr}\gamma\epsilon(H_p-H)+\mathrm{Ec}\ \mathrm{Pr}(F')^2+\mathrm{Ec}\ \mathrm{Pr}\kappa\alpha(F_p-F)^2=0$$

$$r_vH_p'+\epsilon(H-H_p)+\mathrm{Ec}\beta/\gamma(F_p')^2=0 \tag{4}$$

where a prime denotes ordinary differentiation with respect to $\eta$ and

$$r_v=V_s/V_\infty,\ \kappa=\rho_p\phi/(\rho(1-\phi)),\ \alpha=\mu/(\rho\tau_vV_\infty^2),\ \mathrm{Pr}=\mu c/k,$$

$$\gamma=c_p/c,\ \epsilon=\mu/(\rho\tau_TV_\infty^2),\ \mathrm{Ec}=V_\infty^2/(cT_\infty),\ \beta=\mu_p/\mu \tag{5}$$

are the suction parameter, the particle loading, the velocity inverse Stokes number, the fluid-phase Prandtl number, the specific heat ratio, the temperature inverse Stokes number, the Eckert number, and the viscosity ratio, respectively.

Equations (4) are solved subject to the following boundary conditions:

$$F(0)=0,\ F(\infty)=1,\quad F_p'(0)=\omega F_p(0),\quad F_p(\infty)=1,$$

$$H(0)=H_0,\ H(\infty)=1,\ H_p(\infty)=1 \tag{6}$$

Equation (6$c$) is borrowed from rarefied gas dynamics and used herein because the exact forms of boundary conditions for a particulate phase at a surface are not understood at present.

The wall heat flux coefficient is an important physical property of the thermal characteristics of this type of flow. It can be defined as

$$\mathring{q}_w=-H'(0)/(\mathrm{PrEc}) \tag{7}$$

where the minus sign indicates the direction of the heat transfer.

## Results and Discussion

Chamkha and Peddieson (1989) reported among other things the solutions for the velocity profiles for both the fluid and particle phases for this problem. Attention will be focused herein on the solutions for the temperature fields for both phases and the wall heat flux.

The solutions for the fluid-phase velocity in the $x$ direction $F$ and the particle-phase velocity in the $x$ direction $F_p$ can, respectively, be shown to be

$$F=1-AC_3\exp(-\lambda_1\eta)-BC_4\exp(-\lambda_2\eta)$$

$$F_p=1-C_3\exp(-\lambda_1\eta)-C_4\exp(-\lambda_2\eta)$$

$$A=1+(r_v\lambda_1-\beta\lambda_1^2)/\alpha,\ B=1+(r_v\lambda_2-\beta\lambda_2^2)/\alpha$$

$$C_4=\lambda_1(1+\omega(\beta\lambda_1-r_v)/\alpha)/((\omega+\lambda_1)B-(\omega+\lambda_2)A)$$

$$C_3=(\omega+C_4(\omega+\lambda_2))/(\omega+\lambda_1) \tag{8}$$

$\lambda_1$ and $\lambda_2$ are the absolute values of the two negatives roots of the quartic equation

$$\beta\lambda^4+r_v(1+\beta)\lambda^3+(r_v^2-\alpha(1+\kappa\beta))\lambda^2-r_v\alpha(1+\kappa)\lambda=0 \tag{9}$$

Equations (4$c$) and (4$d$) governing the fluid-phase temperature and the particle-phase temperature fields, respectively, can be combined into a third-order differential equation in $H$. This can be shown to be (using the solutions for $F$ and $F_p$ in Eq. (8))

$$H'''+CH''+DH'=X\exp(-2\lambda_1\eta)+Y\exp(-2\lambda_2\eta)+Z\exp(-(\lambda_1+\lambda_2)\eta) \tag{10}$$

where

$$C=(r_v^2\mathrm{Pr}-\epsilon)/r_v,\quad D=-\mathrm{Pr}\epsilon(1+\kappa\gamma)$$

$$X=\mathrm{EcPr}C_3^2(2\lambda_1^3A^2+2\kappa\alpha\lambda_1(A-1)^2+\epsilon\lambda_1^2A^2/r_v+\kappa\alpha\epsilon(A-1)^2/r_v+\beta\kappa\epsilon\lambda_1^2/r_v)$$

$$Y=\mathrm{EcPr}C_4^2(2\lambda_2^3B^2+2\kappa\alpha\lambda_2(B-1)^2+\epsilon\lambda_2^2B^2/r_v+\kappa\alpha\epsilon(B-1)^2/r_v+\beta\kappa\epsilon\lambda_2^2/r_v)$$

$$Z=2\mathrm{EcPr}C_3C_4(\lambda_1\lambda_2AB(\lambda_1+\lambda_2)+\kappa\alpha(A-1)(B-1)(\lambda_1+\lambda_2)+\lambda_1\lambda_2\epsilon AB/r_v+\kappa\alpha\epsilon(A-1)(B-1)/r_v+\beta\kappa\epsilon\lambda_1\lambda_2/r_v) \tag{11}$$

Without going into the details (for brevity), Eq. (10) can be solved subject to the appropriate boundary conditions given in Eqs. (6) to yield

$$H=1+C_2\exp(-m_2\eta)+N\exp(-2\lambda_1\eta)+O\exp(-2\lambda_2\eta)+P_1\exp(-(\lambda_1+\lambda_2)\eta)$$

$$m_2=(C+(C^2-4D)^{1/2})/2,\ C_2=H_0-(1+N+O+P_1)$$

$$N=-X/(2\lambda_1(4\lambda_1^2-2\lambda_1C+D)),$$

$$O=-Y(2\lambda_2(4\lambda_2^2-2\lambda_2C+D))$$

$$P_1=-Z((\lambda_1+\lambda_2)((\lambda_1+\lambda_2)^2-(\lambda_1-\lambda_2)C+D)) \tag{12}$$

Knowing $H$, the solution for $H_p$ from Eq. (4$d$) can then be determined and shown to be

$$H_p=1+Q\exp(-m_2\eta)+R\exp(-2\lambda_1\eta)+S\exp(-2\lambda_2\eta)+U\exp(-(\lambda_1+\lambda_2)\eta)$$

$$Q=\epsilon C_2/(r_v m_2+\epsilon),\quad R=\epsilon N/(2\lambda_1r_v+\epsilon)$$

$$S=\epsilon O/(2\lambda_2r_v+\epsilon),\quad U=\epsilon P_1/((\lambda_1+\lambda_2)r_v+\epsilon) \tag{13}$$

The wall heat transfer coefficient $\mathring{q}_w$ can be determined by differentiating Eq. (12$a$) once, evaluating it at $\eta=0$, and then substituting the result into Eq. (7). This can be shown to give

$$\mathring{q}_w=(m_2C_2+2\lambda_1N+2\lambda_2O+(\lambda_1+\lambda_2)P_1)/(\mathrm{Pr\ Ec}) \tag{14}$$

It should be mentioned that in the absence of particle-phase viscous effects ($\beta=0$), the solutions for $H$, $H_p$, and $\mathring{q}_w$ reported earlier reduce to those given by Chamkha (1992). It is difficult to gain insight into the thermal behavior of this problem from the form of the results given herein. For this reason, numerical evaluations of the results are performed and graphic solutions are presented and discussed below.

Figures 2 and 3 present the temperature profiles for the fluid and particle phases, respectively, for various values of the temperature inverse Stokes number $\epsilon$. Increases in the values of $\epsilon$ cause the thermal interaction between the two phases to increase. This, in turn, increases the interphase energy transfer between the two phases, and, therefore increases the fluid-phase temperature at any position above the plate. This is evident from Fig. 2. For large values of $\epsilon$, thermal equilibrium between the two phases occurs. This is apparent from both Figs. 2 and 3 since the temperature profiles associated with $\epsilon=100$ for both phases are essentially the same.

Figure 4 illustrates the behavior of the wall heat flux for different values of the particle loading $\kappa$ and the temperature inverse Stokes number $\epsilon$. Increases in the values of $\kappa$ have a tendency to increase the fluid-phase temperature $H$ and cause it to approach the free-stream temperature faster. This decreases the thermal boundary layer and increases the slope of the temperature profile at the wall, which, in turn, increases the wall heat transfer. This is reflected in the increases in $\mathring{q}_w$

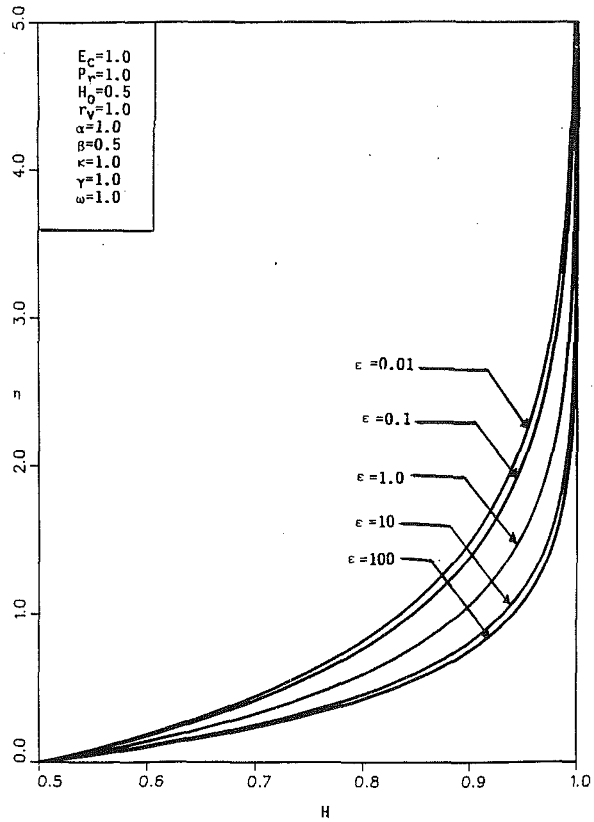


Fig. 2 Fluid-phase temperature profiles

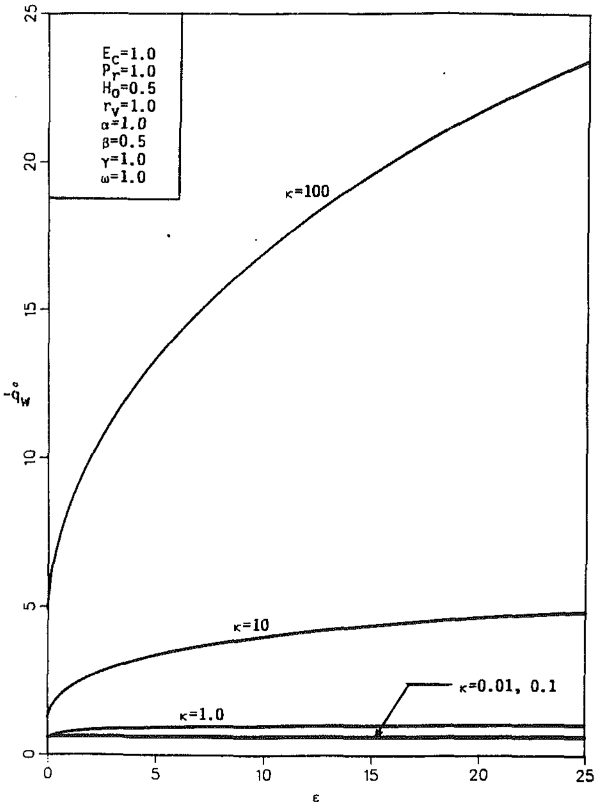


Fig. 4 Wall heat flux coefficient versus $\epsilon$

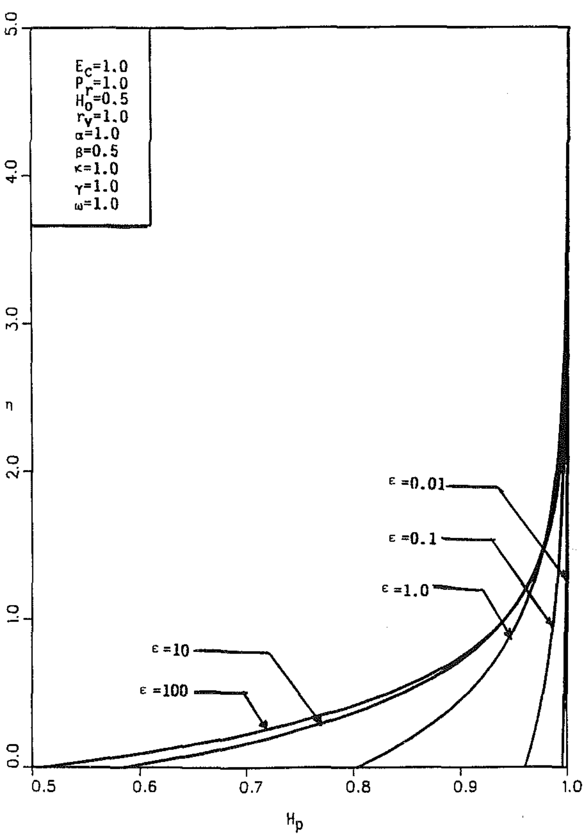


Fig. 3 Particle-phase temperature profiles

as $\kappa$ increases, shown in Fig. 4. As mentioned before, the fluid-phase temperature above the plate increases by increasing the value of $\epsilon$. This causes the same effect mentioned above, namely an increase in the wall heat flux.

In comparison with the inviscid results ($\beta = 0$) reported by Chamkha (1992), it is observed that the addition of particle-phase viscous effects causes the wall heat transfer to decrease.

## Conclusion

The thermal aspects of flow of a dusty gas past an infinite porous flat plate are considered and the solutions are reported in closed form. Numerical evaluations of the exact solutions are performed and presented graphically to illustrate the influence of the various physical parameters on the solutions. It is concluded that, owing the presence of particles, the wall heat transfer coefficient is enhanced. In addition, the inclusion of particle-phase viscosity to the dusty gas model causes a decrease in the heat transfer toward the wall. It is not possible to evaluate these results in the absence of experimental data at present. It is hoped that these results can be utilized as a vehicle for understanding more complex problems.

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