Unsteady Laminar Free Convection from a Vertical Cone with Uniform Surface Heat Flux

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Abstract. Numerical solutions of, unsteady laminar free convection from an incompressible viscous fluid past a vertical cone with uniform surface heat flux is presented in this paper. The dimensionless governing equations of the flow that are unsteady, coupled and non-linear partial differential equations are solved by an efficient, accurate and unconditionally stable finite difference scheme of Crank-Nicolson type. The velocity and temperature fields have been studied for various parameters Prandtl number and semi vertical angle. The local as well as average skin-friction and Nusselt number are also presented and analyzed graphically. The present results are compared with available results in literature and are found to be in good agreement.

Keywords: cone, finite-difference method, heat transfer, natural convection, unsteady.

Nomenclature

$f''(0)$	local skin-friction in [9]
$f'(\eta)$	dimensionless velocity in X-direction in [9]
Gr_L	Grashof number
Gr_L^*	modified Grashof number
g	acceleration due to gravity
k	thermal conductivity
L	reference length
Nu'_x	local Nusselt number
$\overline{Nu'}_L$	average Nusselt number
Nu_X	non-dimensional local Nusselt number
$\overline{Nu}$	non-dimensional average Nusselt number
Pr	Prandtl number
q	uniform wall heat flux per unit area
R	dimensionless local radius of the cone
r	local radius of the cone
T'	temperature
T	dimensionless temperature
t'	time
t	dimensionless time
U	dimensionless velocity in X-direction
u	velocity component in x-direction
V	dimensionless velocity in Y-direction
v	velocity component in y-direction
X	dimensionless spatial co-ordinate
x	spatial co-ordinate along cone generator

Y	dimensionless spatial co-ordinate along the normal to the cone generator	y	spatial co-ordinate along the normal to the cone generator

Greek symbols

α	thermal diffusivity	ϕ	semi vertical angle of the cone
β	volumetric thermal expansion	μ	dynamic viscosity
η	dimensionless independent variable in [9]	ν	kinematic viscosity
Δt	dimensionless time-step	τ'_x	local skin-friction
ΔX	dimensionless finite difference grid size in X-direction	τ_X	dimensionless local skin-friction
		$\overline{\tau}'_L$	average skin-friction
ΔY	dimensionless finite difference grid size in Y-direction	$\overline{\tau}$	dimensionless average skin-friction
		θ	temperature in [9]

Subscripts

w	condition on the wall	∞	free stream condition

1 Introduction

Natural convection flows under the influence of gravitational force have been investigated most extensively because they occur frequently in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces the natural convection. Recently heat flux applications are widely using in industries, engineering and science fields. Heat flux sensors can be used in industrial measurement and control systems. Examples of few applications are detection fouling (Boiler Fouling Sensor), monitoring of furnaces (Blast Furnace Monitoring/General Furnace Monitoring) and flare monitoring. Use of heat flux sensors can lead to improvements in efficiency, system safety and modeling.

Several authors have developed similarity solutions for the axi-symmetrical problems of natural convection laminar flow over vertical cone in steady state. Merk and Prins [1, 2] developed the general relation for similar solutions on iso-thermal axi-symmetric forms and they showed that the vertical cone has such a solution in steady state. Further, Hossain et al. [3] have discussed the effects of transpiration velocity on laminar free convection boundary layer flow from a vertical non-isothermal cone and concluded that due to increase in temperature gradient, the velocity as well as the surface temperature decreases. Ramanaiah et al. [4] discussed free convection about a permeable cone and a cylinder subjected to radiation boundary condition. Alamgir [5] has investigated the overall heat transfer in laminar natural convection from vertical cones using the integral method. Pop et al. [6] have studied the compressibility effects in laminar free convection from a vertical cone. Recently, Pop et al. [7] analyzed the steady laminar mixed convection boundary-layer flow over a vertical isothermal cone for fluids of any Pr for the both cases of buoyancy assisting and buoyancy opposing flow conditions. The resulting non-similarity boundary layer equations are solved numerically using the Keller-box scheme for fluids of any Pr from very small to extremely large values ($0.001 \leq Pr \leq 10000$). Takhar et al. [8] discussed the effect of thermo physical quantities on the free convection flow of gases over iso-thermal vertical cone in steady state, in which thermal conductivity, dynamic viscosity and specific heat at constant pressure were to be assumed a power law variation with absolute temperature. They concluded that the heat transfer increases with suction and decreases with injection.

Recently theoretical studies on laminar free convection flow of axi-symmetric bodies have received wide attention especially in case of uniform and non-uniform surface heat flux. Similarity solutions for the laminar free convection from a right circular cone with prescribed uniform heat flux conditions for various values of Prandtl number (i.e. $Pr = 0.72, 1, 2, 4, 6, 8, 10, 100$) and expressions for both wall skin friction and wall temperature distributions at $Pr \to \infty$ were presented by Lin [9]. Na et al. [10, 11] studied the non-similar solutions for transverse curvature effects of the natural convection flow over a slender frustum of a cone. Later, Na et al. [12] studied without transverse curvature effects on the laminar natural convection flow over a frustum of a cone. In above investigations the constant wall temperature as well as the constant wall heat flux was considered. The effects of amplitude of the wavy surfaces associated with natural convection over a vertical frustum of a cone with constant wall temperature or constant wall heat flux was studied by Pop et al. [13]. Rama Subba Reddy Gorla et al. [14] presented numerical solution for laminar free convection from a vertical frustum of a cone without transverse curvature effect (i.e. large cone angles when the boundary layer thickness is small compared with the local radius of the cone) to power-law fluids.

Further, Pop et al. [15] focused the theoretical study on the effects of suction or injection on steady free convection from a vertical cone with uniform surface heat flux condition. Kumari et al. [16] studied free convection from vertical rotating cone with uniform wall heat flux. Hasan et al. [17] analyzed double diffusion effects in free convection under flux condition along a vertical cone. Hossain et al. [18, 19] studied non-similarity solutions for the free convection from a vertical permeable cone with non-uniform surface heat flux and the problem of laminar natural convective flow and heat transfer from a vertical circular cone immersed in a thermally stratified medium with either a uniform surface temperature or a uniform surface heat flux. Using a finite difference method, a series solution method and asymptotic solution method, the solutions have been obtained for the non-similarity boundary layer equations.

Many investigations have been done free convection past a vertical cone/frustum of cone in porous media. Yih [20, 21] studied in saturated porous media combined heat and mass transfer effects over a full cone with uniform wall temperature/concentration or heat/mass flux and for truncated cone with non-uniform wall temperature/variable wall concentration or variable heat/variable mass flux. Recently Chamkha et al. [22] studied the problem of combined heat and mass transfer by natural convection over a permeable cone embedded in a uniform porous medium in the presence of an external magnetic field and internal heat generation or absorption effects with the cone surface is maintained at either constant temperature, concentration or uniform heat and mass fluxes. Grosan et al. [23] considering the boundary conditions for either a variable wall temperature or variable heat flux studied the similarity solutions for the problem of steady free convection over a heated vertical cone embedded in a porous medium saturated with a non-Newtonian power law fluid driven by internal heat generation. Wang et al. [24] studied the steady laminar forced convection of micro polar fluids past two-dimensional or axi-symmetric bodies with porous walls and different thermal boundary conditions (i.e. constant wall temperature/constant wall heat flux). Further, solutions of the transient free convection flow problems over a vertical/impulsively started vertical plate, cylinder/moving cylinder and inclined plate have been obtained by following investigators Soundalgekar et al. [25], Muthucumaraswamy et al. [26, 27] and Ganesan et al. [28–30] using implicit finite difference method. Recently, Bapuji et al. [31] discussed numerical solutions of flow past plane/axi-symmetrical shape bodies. Also, Bapuji et al. [32, 33] solved numerical solutions of problem namely laminar natural convection from an isothermal and non-isothermal vertical cone in transient state using implicit finite difference method.

The present investigation, namely unsteady laminar free convection from a vertical cone with uniform surface heat flux has not received any attention. Hence, the present work is considered to deal with transient free convection vertical cone with uniform surface heat flux. The governing boundary layer equations are solved by an implicit finite difference scheme of Crank-Nicolson type with various parameters Pr and ϕ. In order to check the accuracy of the numerical results, the present results are compared with the available results of Lin [9], Pop et al. [15], Na et al. [12] and are found to be in excellent agreement.

2 Mathematical analysis

An axi-symmetric transient laminar free convection of a viscous incompressible flow past vertical cone with uniform surface heat flux is considered. It is assumed that the viscous dissipation effects and pressure gradient along the boundary layer are negligible. Also, assumed that the cone surface and the surrounding fluid that is at rest are with the same temperature T'_∞. Then at time $t' > 0$, it is assumed that heat is supplied from cone surface to the fluid at uniform rate q and it is maintained. The co-ordinate system chosen (as shown in Fig. 1) such that x measures the distance along surface of the cone from the

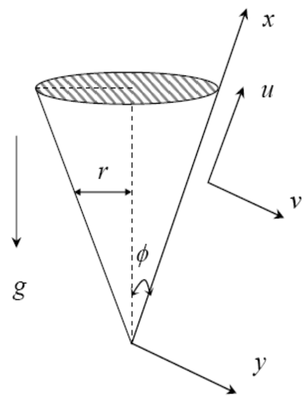

Fig. 1. Physical model and co-ordinate system.

apex ($x = 0$) and y measures the distance normally outward. Here, ϕ is the semi vertical angle of the cone and r is the local radius of the cone. The fluid properties assumed constant except for density variations, which induce buoyancy force and it plays main role in free convection. The governing boundary layer equations of continuity, momentum and energy under Boussinesq approximation are as follows:

equation of continuity

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{1}$$

equation of momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T' - T'_\infty)\cos\phi + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

equation of energy

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2}. \tag{3}$$

The initial and boundary conditions are

$$\begin{aligned}
&t' \le 0: \quad u = 0, \quad v = 0, \quad T' = T'_\infty \quad \text{for all } x,\ y,\\
&t' > 0: \quad u = 0, \quad v = 0, \quad \frac{\partial T'}{\partial y} = \frac{-q}{k} \quad \text{at } y = 0,\\
&\qquad\qquad u = 0, \qquad\qquad T' = T'_\infty \quad \text{at } x = 0,\\
&\qquad\qquad u \to 0, \qquad\qquad T' \to T'_\infty \quad \text{as } y \to \infty.
\end{aligned} \tag{4}$$

The physical quantities of interest are the local skin friction τ'_x and the local Nusselt number Nu'_x are given respectively by,

$$\tau'_x = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu'_x = \frac{x}{T'_w - T'_\infty}\left(-\frac{\partial T'}{\partial y}\right)_{y=0}. \tag{5}$$

Also, the average skin friction $\overline{\tau}'_L$ and the average heat transfer coefficient $\overline{h}$ over the cone surface are given by

$$\overline{\tau}'_L = \frac{2\mu}{L^2}\int_0^L x\left(\frac{\partial u}{\partial y}\right)_{y=0} dx, \quad \overline{h} = \frac{2k}{L^2}\int_0^L \frac{x}{T'_w - T'_\infty}\left(-\frac{\partial T'}{\partial y}\right)_{y=0} dx. \tag{6}$$

The average Nusselt number given by

$$\overline{Nu'}_L = \frac{L\overline{h}}{k} = \frac{2}{L}\int_0^L \frac{x}{T'_w - T'_\infty}\left(-\frac{\partial T'}{\partial y}\right)_{y=0} dx. \tag{7}$$

Further, we introduce the following non-dimensional variables:

$$\begin{aligned} X &= \frac{x}{L}, \quad Y = \frac{y}{L}\,Gr_L^{1/5}, \quad R = \frac{r}{L}, \\ U &= \left(\frac{L}{\nu}Gr_L^{-2/5}\right)u, \quad V = \left(\frac{L}{\nu}Gr_L^{-1/5}\right)v, \\ t\ &= \left(\frac{\nu}{L^2}\,Gr_L^{2/5}\right)t', \quad T = \frac{T' - T'_\infty}{qL/k}\,Gr_L^{1/5}, \end{aligned} \tag{8}$$

where $Gr_L = g\beta qL^4/\nu^2 k$ is the Grashof number based on L, $Pr = \nu/\alpha$ is the Prandtl number and $r = x\sin\phi$. Equations (1), (2) and (3) are reduced to the following non-dimensional form:

$$\frac{\partial}{\partial X}(RU) + \frac{\partial}{\partial Y}(RV) = 0, \tag{9}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = T\cos\phi + \frac{\partial^2 U}{\partial Y^2}, \tag{10}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\,\frac{\partial^2 T}{\partial Y^2}. \tag{11}$$

The corresponding non-dimensional initial and boundary conditions are

$$\begin{array}{lllll} t \le 0: & U = 0, & V = 0, & T = 0 & \text{for all } \ X,\ Y, \\ t > 0: & U = 0, & V = 0, & \dfrac{\partial T}{\partial y} = -1 & \text{at } \ Y = 0, \\ & U = 0, & & T = 0 & \text{at } \ X = 0, \\ & U \to 0, & & T \to 0 & \text{as } \ Y \to \infty. \end{array} \tag{12}$$

The local non-dimensional skin-friction τ_X and local Nusselt number Nu_X given by (5) become

$$\tau_X = Gr_L^{3/5}\left(\frac{\partial U}{\partial Y}\right)_{Y=0}, \quad Nu_X = \frac{X\,Gr_L^{1/5}}{T_{Y=0}}\left(-\,\frac{\partial T}{\partial Y}\right)_{Y=0}. \tag{13}$$

Also, the non-dimensional average skin-friction $\overline{\tau}$ and the average Nusselt number $\overline{Nu}$ are reduced to

$$\begin{aligned} \overline{\tau} &= 2Gr_L^{3/5}\int_0^1 X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX, \\ \overline{Nu} &= 2Gr_L^{1/5}\int_0^1 \frac{X}{T_{Y=0}}\left(-\,\frac{\partial T}{\partial Y}\right)_{Y=0} dX. \end{aligned} \tag{14}$$

3 Solution procedure

The unsteady, non-linear, coupled and partial differential equations (9), (10) and (11) with the initial and boundary conditions (12) are solved by employing a finite difference scheme of Crank-Nicolson type which is discussed by many authors Soundalgekar and Ganesan [25], Ganesan and Rani [28], Muthucumaraswamy and Ganesan [26, 27], Ganesan and Palani [30]. Recently, the heat transfer problem deals with, unsteady free convection flow past a vertical cone are solved numerically by an implicit finite-difference method of Crank-Nicolson type as described in detail by Bapuji et al. [32, 33]. The finite difference scheme of dimensionless governing equations is reduced to tri-diagonal system of equations and is solved by Thomas algorithm as discussed in Carnahan et al. [34]. The region of integration is considered as a rectangle with X_{max} ($X_{max} = 1$) and Y_{max} ($Y_{max} = 26$) where corresponds to $Y = \infty$ which lies very well out side both the momentum and thermal boundary layers. The maximum of Y was chosen as 26, after some preliminary investigation so that the last two boundary conditions of (12) are satisfied within the tolerance limit 10^{-5}. The mesh sizes have been fixed as $\Delta X = 0.05$, $\Delta Y = 0.05$ with time step $\Delta t = 0.01$. The computations are carried out first by reducing the spatial mesh sizes by 50 % in one direction, and later in both directions by 50 %. The results are compared. It is observed in all cases, that the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate. The scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it tends to zero as Δt, ΔY and ΔX tend to zero. Hence, the scheme is compatible. Stability and compatibility ensure the convergence.

4 Results and discussion

In order to prove the accuracy of our numerical results, the present results in steady state at $X = 1.0$ obtained and considering the modified Grashof number $Gr_L^* = Gr_L \cos\phi$, (i.e. the numerical solutions obtained from the equations (9)–(11) are independent of semi vertical angle of the cone ϕ) are compared with available similarity solutions in literature. The velocity and temperature profiles of the cone for $Pr = 0.72$ are displayed in Fig. 2 and the numerical values of local skin-friction τ_X, temperature T, for different values of Prandtl number are shown in Table 1 are compared with similarity solutions of Lin [9] in steady state using suitable transformation (i.e. $Y = (20/9)^{1/5}\eta$, $T = (20/9)^{1/5}(-\theta(0))$, $U = (20/9)^{3/5} f'(\eta)$, $\tau_X = (20/9)^{2/5} f''(0)$). It is observed that the results are in good agreement with each other. It is also noticed that the present results agree well with those of Pop and Watanabe [15], Na and Chiou [12] (as pointed out in Table 1).

In Figs. 3–6, transient velocity and temperature profiles are shown at $X = 1.0$, with various parameters Pr and ϕ. The value of t with star ($*$) symbol denotes the time taken to reach steady state. In Fig. 3, transient velocity profiles are plotted for various values of ϕ and $Pr = 0.71$. When ϕ increases near the cone apex, it leads to decrease in the impulsive force along the cone surface. Hence, the difference between temporal maximum velocity values and steady state values decreases with increasing the values of semi vertical angle

Table 1. Comparison of steady state local skin-friction and temperature values at $X = 1.0$ with those of Lin [9]

	Temperature			Local skin friction		
	Lin results [9]		Present results	Lin results [9]		Present results
Pr	$-\theta(0)$	$-(\frac{20}{9})^{1/5}\theta(0)$	T	$f''(0)$	$(\frac{20}{9})^{2/5}f''(0)$	τ_X
0.72	1.52278 1.52278*	1.7864	1.7796	0.88930 0.88930*	1.2240	1.2154
1	1.39174	1.6327 1.6329**	1.6263	0.78446	1.0797	1.0721
2	1.16209	1.3633	1.3578	0.60252	0.8293	0.8235
4	0.98095	1.1508	1.1463	0.46307	0.6373	0.6328
6	0.89195	1.0464	1.0421	0.39688	0.5462	0.5423
8	0.83497	0.9796	0.9754	0.35563	0.4895	0.4859
10	0.79388	0.9314	0.9272	0.32655	0.4494	0.4460
100	0.48372	0.5675	0.5604	0.13371	0.1840	0.1813

*Values taken from Pop and Watanabe [15] when suction/injection is zero.
**Values taken from Na and Chiou [12] when solutions for flow over a full cone.

of the cone ϕ. The tangential component of buoyancy force reduces as the semi vertical angle increases. This causes the velocity to reduce as angle ϕ increases. The momentum boundary layer becomes thick, and the time taken to reach steady state increases for increasing ϕ. In Fig. 4, transient temperature profiles are shown for different values of ϕ with $Pr = 0.71$. It is observed the temperature and boundary layer thickness increase with increasing ϕ. The difference between temporal maximum temperature values and steady state values decrease with increasing ϕ. In Figs. 5 and 6, transient velocity and temperature profiles are plotted for various values of Pr with $\phi = 15°$. Viscous force increases and thermal diffusivity reduces with increasing Pr, causing a reduction in the velocity and temperature as expected. It is observed from the figures that the difference between temporal maximum values and steady state values are reduced when Pr increases. It is also noticed that the time taken to reach steady state increases and thermal boundary layer thickness reduces with increasing Pr.

The study of the effects of the parameters on local as well as the average skin-friction, and the rate of heat transfer is more important in heat transfer problems. The derivatives involved in equations (13) and (14) are obtained using five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula. The variations of local skin-friction τ_X and local Nusselt number Nu_X for different values of ϕ, at various positions on the surface of the cone ($X = 0.25$ and 1.0) in the transient period are shown in Figs. 7 and 8 respectively. It is observed from the Fig. 7 that local skin-friction τ_X decreases with increasing ϕ, due to the fact that velocity decreases with increasing angle ϕ as shown in Fig. 3 and the influence of ϕ on skin friction τ_X increases as ϕ increases in the transient period along the surface of the cone moving away from apex. Fig. 8 reveals that local Nusselt number Nu_X values decrease with increasing

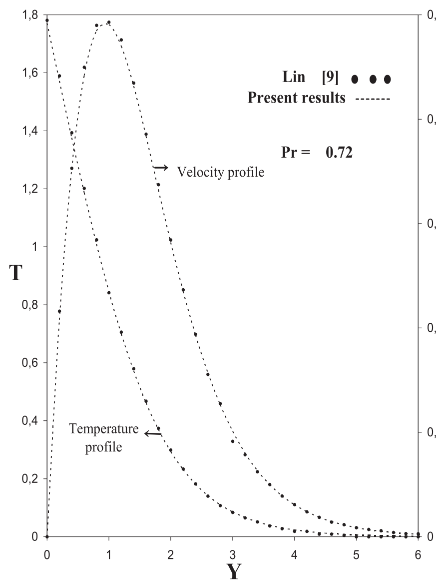

Fig. 2. Comparison of steady state temperature and velocity profiles at $X = 1.0$.

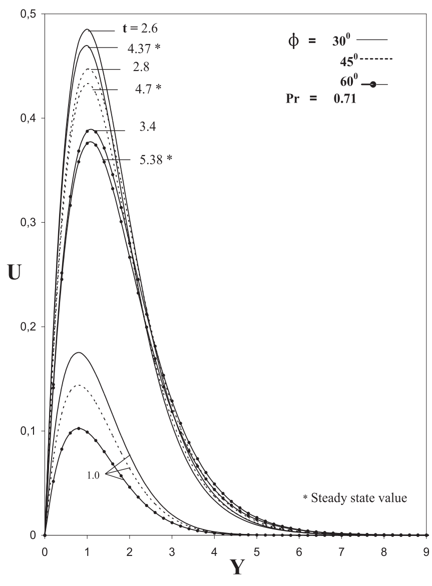

Fig. 3. Transient velocity profiles at X=1.0 for different values of ϕ.

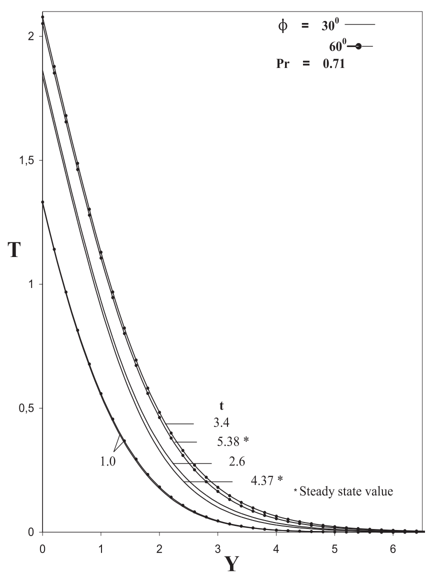

Fig. 4. Transient temperature profiles at $X = 1.0$ for different values of ϕ.

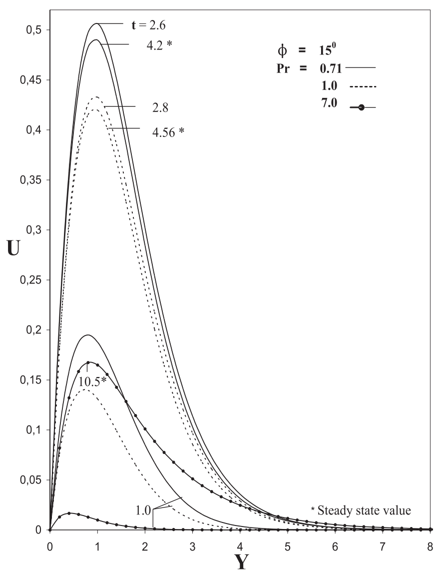

Fig. 5. Transient velocity profiles at X=1.0 for different values of Pr.

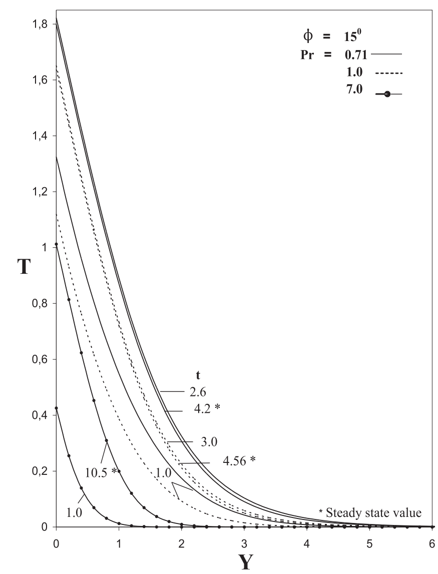

Fig. 6. Transient temperature profiles at $X = 1.0$ for different values of Pr.

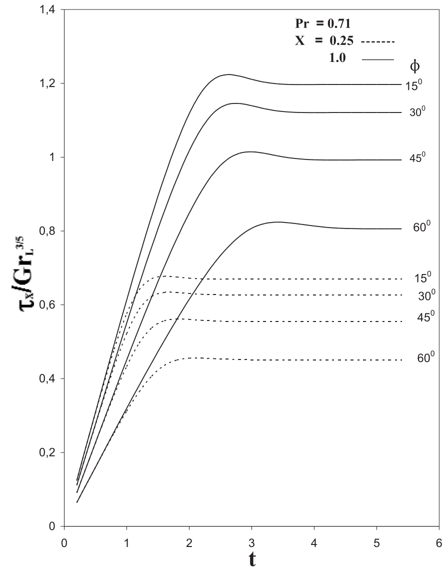

Fig. 7. Local skin friction at $X = 0.25$ and 1.0 for different values of ϕ in transient state.

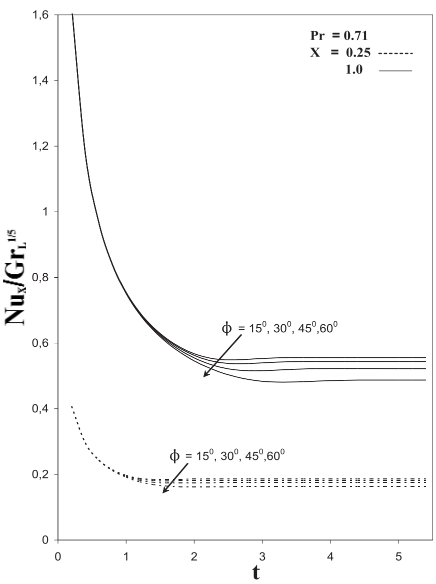

Fig. 8. Local Nusselt number at $X = 0.25$ and 1.0 for different values of ϕ in transient state.

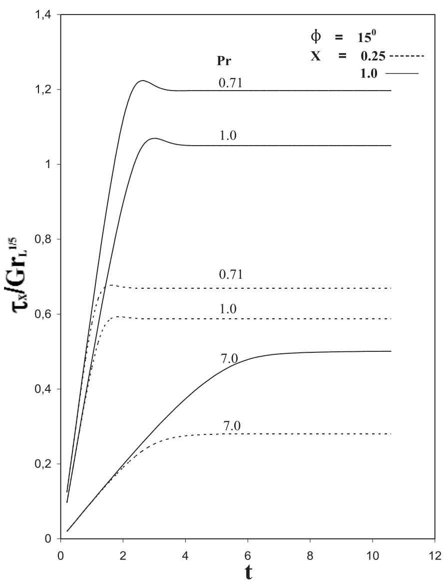

Fig. 9. Local skin friction at $X = 0.25$ and 1.0 for different values of Pr in transient state.

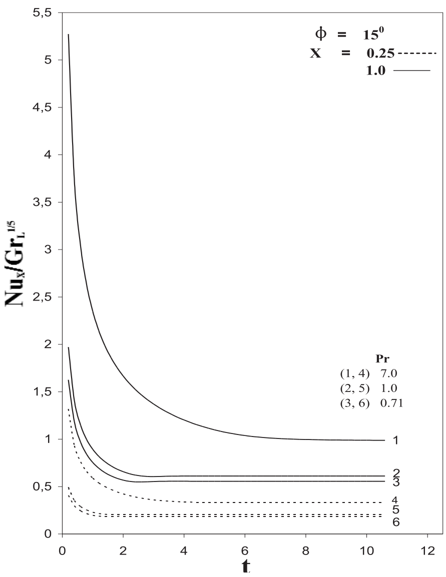

Fig. 10. Local Nusselt number at $X = 0.25$ and 1.0 for different values of Pr in transient state.

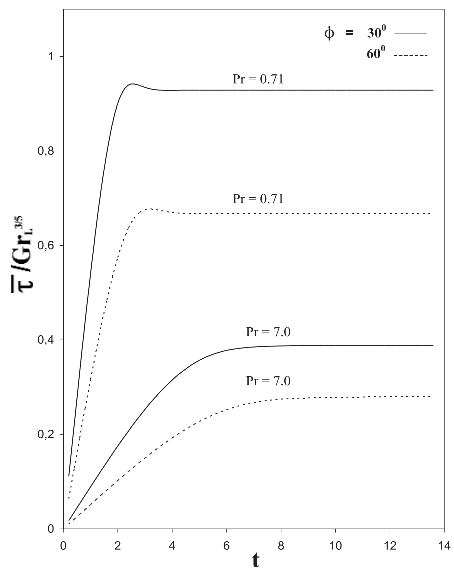

Fig. 11. Average skin friction for different values of ϕ and Pr in transient state.

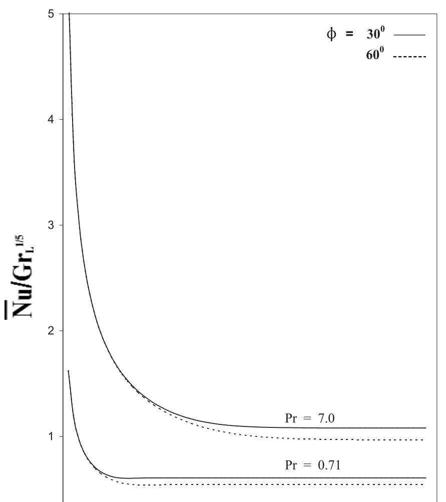

Fig. 12. Average Nusselt number for different values of ϕ and Pr in transient state.

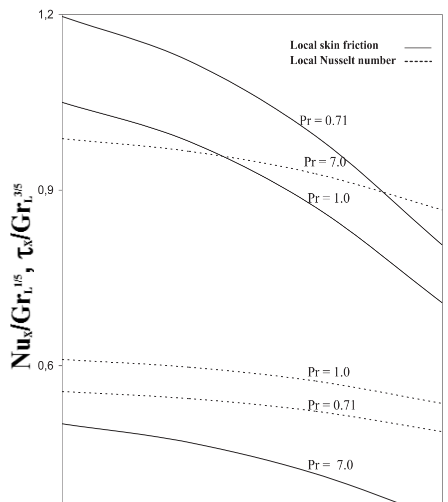

Fig. 13. Local Nusselt number, local skin friction at $X = 1.0$ for different values of Pr in steady state.

angle ϕ as temperature distribution increases with ϕ which is shown in Fig. 4. It is observed that this effect is less near the cone apex. The variation of the local skin-friction τ_X and the local Nusselt number Nu_X in the transient regime is displayed in Figs. 9 and 10 for different values of Pr and at various positions on the surface of the cone ($X = 0.25$ and 1.0). The local wall shear stress decreases as Pr increases because velocity decreases with an increasing value of Pr as shown in Fig. 5. In transient period initially local skin friction almost constant through out the surface and gradually increases with time along the surface until it reaches steady state. Local Nusselt number Nu_X increases with increasing Pr and it is clear from the Fig. 10, that decreasing rate of Nu_X increases when the distance increases from the cone vertex along the surface of the cone.

The influence of ϕ and Pr on average skin-friction $\overline{\tau}$ transient period are shown in Fig. 11 and it is more for smaller values of angles ϕ and lower values of Pr. Fig. 12, displays the influence of average Nusselt number $\overline{Nu}$ in transient period for various values of Pr and ϕ. It is clear that $\overline{Nu}$ is more for smaller values of ϕ and larger values of Pr. Finally, steady state local skin-friction τ_X and local Nusselt number Nu_X profiles are at $X = 1.0$ plotted in Fig. 13, against semi vertical angle of the cone ϕ for various values of Pr. It is observed that the local shear stress τ_X increases as Pr or ϕ decreases, local Nusselt number Nu_X reduces as ϕ increases or Pr decreases.

5 Conclusions

A numerical study has been carried out for the unsteady laminar free convection from a vertical cone with uniform surface heat flux. The dimensionless governing boundary layer equations are solved by an implicit finite-difference method of Crank-Nicolson type. Present results are compared with available results in literature and are found to be in good agreement. The following conclusions are made:

1. The time taken to reach steady state increases with increasing Pr or ϕ.
2. The velocity reduces when the parameters ϕ, Pr are increased.
3. Temperature increases with increasing ϕ and decreasing Pr values.
4. Momentum boundary layers become thick when ϕ is increased.
5. Thermal boundary layer becomes thin when ϕ is reduced or Pr is increased.
6. The difference between temporal maximum values and steady state values (for both velocity and temperature) become less when Pr or ϕ increases.
7. The influence of ϕ over the local skin friction τ_X and local Nusselt number Nu_X are less near the vertex of the cone and then increases slowly with increasing distance from the vertex.
8. Local and average skin-frictions increases when the value of ϕ or Pr is reduced.
9. Local and average Nusselt numbers reduce with increasing ϕ or decreasing Pr.

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