MODELLING OF DEEP BED FILTRATION WITH SCALE-DEPENDENT DIFFUSION AND SUPERFICIAL VELOCITY

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A one-dimensional continuum deep bed filtration model with scale-dependent diffusion or dispersion coefficient and filter superficial velocity is considered. The diffusion coefficient and superficial velocity are assumed to increase according to a polynomial function with the filter longitudinal distance. A general mass transfer that characterizes particle attachment and detachment effects is considered. Since the general problem is nonlinear and possesses no analytical solutions, a numerical solution based on an efficient implicit finite-difference method is obtained. Comparisons with exact and approximate analytical solutions for special cases of the filtration model are performed and found to be in excellent agreement.

A parametric study of some physical parameters is conducted and the results are presented graphically to illustrate interesting features of the solutions. It is found that the presence of a particle detachment mechanism has significant effects on the solute concentration and specific deposit profiles at all filtration times. In addition, the scale-dependent polynomial-type diffusion coefficient and superficial velocity are predicted to yield significant changes in the solute concentration, specific deposit and filter efficiency at all filtration time stages compared with their corresponding constant cases.

INTRODUCTION

Most deep bed filtration simulations are carried out by assuming that the diffusivity is independent of the filter position or absent and that the filter superficial velocity is constant. It is of interest to know the degree to which deviations from these assumptions affect predictions of filter performance. In the present work, several aspects of this question are investigated. Exact and approximate solutions are obtained for special cases of dynamic filtration with constant diffusivity and superficial velocity. These solutions hold for constant entrance concentration and a general attachment/detachment mass transfer function.

Previously published analytical and numerical solutions to deep bed filtration problems are obtained based on the assumption of a homogeneous porous medium. In reality, the filter porous medium through which the contaminants move is seldom homogeneous and significant spatial variability of transport properties are expected especially for long filters. As a result of the heterogeneity of the porous medium in a filter, the dispersion coefficients in all directions vary with the space coordinate and the resulting contaminant or solute transport equation (balance of filtrate mass) contains spatially-dependent diffusion or dispersion coefficients. To the authors' knowledge, no deep bed filtration models with scale-dependent diffusion coefficients have been reported in the literature.

The popular class of attachment functions discussed by Tien is shown to lead to a formulation allowing a single solution to be representative of all possible variations of inlet concentrations, superficial velocities and initial porosities. For the sake of illustration of the effects of variable diffusivity and superficial velocity, the linear chromatography-like attachment/detachment function (see, for instance, Tien or Adin and Rajagopalan) is considered. Numerical solutions are obtained by an implicit finite-difference method and it is shown how one can adjust the computational domain from finite to semi-infinite to ensure proper appli-
cation of the exit concentration boundary condition. It should be pointed out that deep bed filtration models differ mainly in the forms of the attachment/ detachment functions employed therein. The complexity of these functions is determined by the number of phenomena they attempt to describe. The simplest models are those involving pure attachment such as that proposed by Iwasaki\(^5\) which can be modified to account for filter clogging in a number of ways. The papers listed in Tieri (i.e. Table 2.1) are representative of this work. Soo and Radke\(^6\) and Soo et al.\(^3\) showed that it was possible to use such models to describe attachment by simultaneous straining and interception.

Models allowing for both attachment and detachment (particle unloading or re-entrainment of previously captured particles) have been developed. Representative of this work are the papers by Adin\(^9\), Adin and Rebhurn\(^1\), and Adin and Rajagopalan\(^2\). As discussed in the latter paper and by Rajagopalan and Chu\(^8\), it is possible to reinterpret such models to describe the reduction in attachment rate associated with the covering of filter grain surfaces. Privman et al.\(^9\) dealt with the phenomenon explicitly. Song and Elimelech\(^10\) extended this work by introducing an internal variable which describes the time evolution of the surface covering. It is interesting to note that all these models except the one associated with pure attachment predict s-shaped breakthrough curves that are qualitatively similar. The objective of the present work is to consider a deep bed filtration theory in which the diffusion coefficient and superficial velocity vary with the longitudinal distance of the filter and to illustrate how these variations affect the filter performance.

**PROBLEM FORMULATION**

Consider a one-dimensional continuum deep bed filtration model with an arbitrary general mass transfer function \(M'\). The solute transport in the porous medium of the filter takes place from a continuous source of constant inlet solute concentration (c) and the superficial velocity and dispersion or diffusion coefficient are assumed to be spatially-dependent. The governing equations are based on the filtrate and filter mass balances for monodispersed particles (see for instance, Tieri\(^1\)) described respectively by:

\[
\begin{align*}
\frac{\partial c(z,t)}{\partial t} + \frac{\partial}{\partial z} \left[ d(z) \frac{\partial c(z,t)}{\partial z} \right] &= - \lambda M'(\varepsilon,c) \quad \text{(1)} \\
\frac{\partial c(z,t)}{\partial t} &= - \lambda M'(\varepsilon,c) \quad \text{(2)}
\end{align*}
\]

where \(t\) is the time, \(z\) the one-dimensional spatial coordinate (0 \(\leq\) \(z\) \(\leq\) \(L\), where \(L\) is the filter length), \(c(z,t)\) the solute concentration, \(u(z)\) the spatially-dependent superficial velocity, \(d(z)\) the spatially-variable hydrodynamical dispersion coefficient representing the sum of both the effects of molecular diffusion \(D_m\) and mechanical dispersion \(a u v\), where \(a\) is usually called the dispersivity and \(u v\) is the average superficial velocity), \(M'(\varepsilon,c)\) is a general mass transfer function, \(\lambda\) the inverse filtration length, \(\varepsilon(z,t)\) the filter porosity, and \(\varepsilon_i\) the initial filter porosity. It should be noted that the first term on the left hand side of equation (1) represents the transient or accumulation effect. The second term on the left hand side of the equation represents the advection or convection effect which is defined as the transport of solute by the mean velocity in the flow stream. The first term on the right side of equation (1) accounts for the dispersion or diffusion effect which is responsible for spreading of the solute in the medium. The last term in equation (1) represents the filtration attachment and/or detachment mechanisms that may take place (depending on the nature and properties of the solute and the porous medium of the filter).

The initial and boundary conditions for this problem are:

\[
\begin{align*}
c(z,0) &= 0, \quad \varepsilon(z,0) = \varepsilon_i \quad \text{(3)} \\
c(0,t) &= c_i, \quad \frac{\partial c(L,t)}{\partial z} = 0
\end{align*}
\]

It is convenient to work with dimensionless equations which can be accomplished by using

\[
\begin{align*}
\eta &= \frac{z}{L}, \quad \tau = \frac{t}{T}, \quad C(\eta,\tau) = \frac{c(z,t)}{c_i}, \quad U(\eta) = \frac{u(z)}{u} \\
D(\eta) &= \frac{d(z)}{D}, \quad S = \varepsilon - \varepsilon_i, \quad M = \frac{M'}{u c_i}, \quad S = \frac{s}{u c_i T/L}
\end{align*}
\]

where \(T, u'\) and \(D'\) are respective characteristic quantities of process time, superficial velocity and diffusion. The parameter \(s\) denotes the filter specific deposit.

Substituting equation (4) into equations (1)-(3) gives:

\[
N_1 \frac{\partial C(\eta,\tau)}{\partial \tau} + \frac{\partial}{\partial \eta} \left[ \frac{\partial C(\eta,\tau)}{\partial \eta} \right] = N_2 \frac{\partial}{\partial \eta} \left[ D(\eta) \frac{\partial C(\eta,\tau)}{\partial \eta} \right] - N M(S,C)
\]

\[
\frac{\partial S(\eta,\tau)}{\partial \tau} = N M(S,C)
\]

\[
C(\eta,0) = 0, \quad S(\eta,0) = 0
\]

\[
C(0,\tau) = 1.0, \quad \frac{\partial C(1,\tau)}{\partial \eta} = 0
\]

where

\[
N_1 = \lambda L, \quad N_2 = \frac{\varepsilon L}{u' T}, \quad N_3 = \frac{D'}{u'' L}
\]

are dimensionless numbers which characterize the
filtration process. In particular, \( N_1 \) measures the influence of attachment rate, \( N_2 \) the influence of disturbance front propagation and \( N_3 \) the influence of diffusion.

The linear 'chromatography-like' attachment/detachment model for the mass transfer function \( M(C,S) \) (see, for instance, Tien\(^3\) or Adin and Rajagopalan\(^4\)) is often used in deep bed filtration modeling since it allows for a variety of possible physical situations. For this reason, it is adopted in the current work and given by

\[
M(S,C) = C \cdot \frac{N_6 S}{N_5} \tag{9}
\]

where \( N_4 \) is a dimensionless constant characterizing the attachment rate and \( N_5 \) is a dimensionless parameter characterizing the detachment rate. Substituting equation (9) into equations (5) and (6) and simplifying yields

\[
N_2 \frac{\partial C(\eta, \tau)}{\partial \tau} + U(\eta) \frac{\partial C(\eta, \tau)}{\partial \eta} + C(\eta, \tau) \frac{\partial U(\eta)}{\partial \eta} = \frac{\partial C(\eta, \tau)}{\partial \eta} \frac{dD(\eta)}{d\eta} = N_6 C(\eta, \tau) \cdot N_5 S(\eta, \tau) \tag{10}
\]

\[
\frac{\partial S(\eta, \tau)}{\partial \tau} = N_6 C(\eta, \tau) - N_5 S(\eta, \tau) \tag{11}
\]

In deep bed filtration modelling, in addition to the solute concentration and the filter specific deposit (or porosity), information about the time evolution of the filter efficiency is of great importance. The filter efficiency is defined as

\[
E(\tau) = 1 - \frac{c(L, t)}{c_0} = 1 - C(L, \tau) \tag{12}
\]

Results from field studies on geological media suggest that the dispersivity is scale-dependent, i.e. the dispersivity increases with distance (\( x \)) from the pollution source (see, for instance, Huang et al.\(^1\)). The growth with distance of the dispersion process is a consequence of the heterogeneous nature of the subsurface environment of the media. Most geological materials are extremely non-homogeneous due to the presence of irregular stratifications, fissures and fractures, and lenses of high or low permeability. These non-homogeneities cause the hydraulic properties to vary spatially, leading to spatial fluctuations in the fluid velocity, and eventually to a dispersivity which increases with distance or time (see Pickens and Grisak\(^2\) and Pachepsky et al.\(^3\) and the references therein). This conclusion is especially true for deep sand filters.

Numerical solutions for the general deep bed filtration model given by equations (10) and (11), and subject to the initial and boundary conditions in equations (7), are obtained for the following scale-dependent diffusion coefficient and superficial velocity:

\[
D(\eta) = 1 + N_2 \eta^n \tag{13}
\]

\[
U(\eta) = 1 + N_5 \eta^m \tag{14}
\]

where \( N_0, \eta_1, N_2 \) and \( n_2 \) are dimensionless constants. It can be seen that when \( N_0 = 0 \) in equation (13), the constant diffusion case is recovered. Also, when \( N_5 = 0 \) in equation (14), the case of constant superficial velocity is obtained. It should be noted that Zoppou and Knight\(^4\) considered a solute transport problem with variable velocity and diffusivity. In their notation, Zoppou and Knight assumed that \( D(\eta) = \eta^n \) but neglected the molecular contribution to the diffusion coefficient. In the apparent absence of experimental data, equations (13) and (14) were used in order to illustrate the influence of heterogeneity of the medium on the filter performance. Of course, other types of variation in diffusion coefficient with filter distance such as exponential and periodic functions can be employed. Such instances can be explored once experimental verification can be performed.

**ANALYTICAL SOLUTIONS**

It is helpful to present some analytical solutions for special cases of equations (10) and (11), subject to equations (7), to use as comparison standards for the numerical procedure used in the present work.

For some cases of constant diffusion \( D = 1 \) and superficial velocity \( U = 1 \), linear partial differential equations result. These can be solved by Laplace transformation methods assuming a semi-infinite region in order to facilitate the process of obtaining exact and approximate closed-form solutions.

A relatively simple exact solution is possible for the special case of \( N_5 = 0 \) (no detachment). Equation (10) becomes like an advection-convection-reactive equation usually seen in solute transport in soil. In this case, equation (10) is uncoupled from equation (11) and its solution has the form:

\[
C(\eta, \tau) = \exp \left[ - \left( 1 + 4N_6/N_5 \right) \frac{1}{2} \left( \frac{2N_5}{N_6} \right) \right] \cdot \frac{\text{erf} \left( \frac{\left( 1 + 4N_6/N_5 \right) \left( \frac{2N_5}{N_6} \right) \eta - \tau \right)}{2} + \exp \left( \frac{\left( 1 + 4N_6/N_5 \right) \left( \frac{2N_5}{N_6} \right) \eta + \tau}{2} \right) \right] \tag{15}
\]

where

\[
\text{erf}(\xi) = 1 - \text{erf}(\xi) = \frac{2}{\sqrt{\pi}} \int_{\xi}^{\infty} \exp(-\theta^2) d\theta \tag{16}
\]

where \( \text{erf} \) and \( \text{erfc} \) are the error function and compli-
Filtration Solutions

In the absence of diffusion \( (N_s = 0) \), equation (15) becomes

\[
C(\eta, \tau) = \exp(-N_s \eta) H(\tau - N_s \eta)
\]  

(17)

where \( H \) is the unit step function. It is obvious that equation (17) exhibits a discontinuity at \( \eta = \tau/N_s \). For a small amount of diffusion \( (N_s \ll 1) \), this discontinuity is replaced by a narrow continuous transition layer.

For the above cases, the corresponding solution for \( S \) can be found as follows:

\[
s(\eta, \tau) = N_s \int_0^\tau C(\eta, \tau') d\tau'
\]  

(18)

An exact solution exists for the complete form of equations (10) and (11) subject to equations (7) (for a semi-infinite region) but it involves integrals that require numerical evaluation (see Adin and Rajagopal\(^{7}\) for the special case of \( N_s = 0 \)). A simpler approximate solution valid for moderate and large times is obtained by Charrka et al.\(^{15}\) based on the method of Rasmussen\(^{16}\). This solution can be written as

\[
C(\eta, \tau) = \frac{1}{2} (A_1 + A_2 A_3)
\]  

(19)

where

\[
A_1 = \text{erfc} \left[ \left( \frac{A_1 N_s}{N_s + N_s} \right)^{1/2} \frac{A_2 \eta / N_s - \tau}{2} \right]
\]

\[
A_2 = \exp \left( \frac{(A_1)^2 \eta}{N_s + N_s (A_1)^2} \right)
\]

\[
A_3 = \text{erfc} \left[ \left( \frac{A_1 N_s}{N_s + N_s} \right)^{1/2} \frac{A_2 \eta / N_s + \tau}{2} \right]
\]

\[
A_4 = N_s + N_s N_s
\]

which reduces to

\[
C(\eta, \tau) = \frac{1}{2} (A_1 + A_2 A_3) H(\tau - N_s \eta)
\]  

(20)

where

\[
A_5 = \text{erfc} \left[ \left( \frac{A_1 N_s}{N_s} \right)^{1/2} \frac{A_2 \eta / N_s - \tau}{2} \right]
\]

\[
A_6 = \exp \left( \frac{(A_1)^2 \eta}{N_s} \right)
\]

In their most general form, equations (10) and (11) are non-linear. Therefore, an analytical solution to these equations is unlikely and a numerical procedure is required. Many existing computer codes employ a finite-difference approach for the solution of filtration equations. It is logical to investigate the applicability of this methodology to equations (10) and (11).

In the present work, an implicit iterative tri-diagonal finite difference method similar to that discussed by Blottner\(^{7}\) is employed. A two-point backward difference quotient is used to represent the dimensionless time \( \tau \) derivative and three-point central difference quotients are used to represent the dimensionless space \( \eta \) derivatives. The computation starts at \( \tau = 0 \) and marches forward in time. At each time, a system of non-linear algebraic equations must be solved to determine the \( \eta \) distributions of \( C \) and \( S \) and an iterative procedure is employed. At each iteration, an equivalent linear system of algebraic equations (the linearization being effected by representing some quantities by their values from the previous iteration) must be solved. These equations have a tri-diagonal form and can be solved by Potter’s method variables which can be determined by a forward sweep in the \( \eta \) direction. The physical variables can then be found from a corresponding backward sweep. This process avoids the need for matrix inversion. Iteration is continued until convergence is obtained at a given time. The procedure moves forward to the next time.

In the present work a finite computational domain is required. For this, one can either choose a computational length such that \( \eta_c \gg 1 \) to approximate a semi-infinite region and use the computed solution only in the region \( 0 \leq \eta \leq 1 \) or directly impose a non-reflecting boundary condition at the filter exit (\( \eta = 1 \)). In all the numerical results presented, the computational domain was divided up into 196 points in the \( \eta \) direction and 101 points in the \( \tau \) direction. Variable step sizes in the \( \eta \) direction and constant step sizes of 0.01 in the \( \tau \) direction were employed. The initial step size and growth factor used in the \( \eta \) direction were respectively 0.001 and 1.035. These values gave \( n = 23 \) and were arrived at by performing various numerical experiments to assess grid independence. Only the computed solution in the region \( 0 \leq \eta \leq 1 \) was used. The

\[
A_s = \text{erfc} \left[ \left( \frac{A_1 N_s}{N_s} \right)^{1/2} \frac{A_2 \eta / N_s - \tau}{2} \right]
\]

for \( N_s = 0 \). In this case, the corresponding expression for \( S \) can be computed from

\[
S(\eta, \tau) = \frac{N_s}{N_s} \frac{\partial C(\eta, \tau)}{\partial \tau} - N_s \frac{\partial^2 C(\eta, \tau)}{\partial \eta^2} + N_s C
\]  

(21)

NUMERICAL METHOD

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convergence criterion required that the difference between the current and previous iterations was $10^{-7}$. The numerical method was validated by favourable comparisons with the approximate and exact results (equations (16) and (19)) obtained previously. These comparisons are shown in the graphical results (i.e., Figures 1, 5 and 13) reported below and lend confidence to the accuracy of the numerical method.

RESULTS AND DISCUSSION
Various numerical results were obtained and a representative set of results is presented in Figures 1-16. These results were chosen to illustrate the influence of the scale-dependent dispersion constants $n_1$ and $N_D$ and the spatially-dependent superficial velocity constants $n_2$ and $N_2$ on the temporal development of the solute concentration and specific deposit profiles ($C(n, \tau)$ and $S(n, \tau)$) as well as the time development of the filter efficiency and the specific deposit at the filter exit ($E(n)$ and $S(1, \tau)$). In Figures 1-12 both attachment and detachment mechanisms are present ($N_1 \neq 0$ and $N_2 \neq 0$), while in Figures 13-16 the detachment mechanism is absent ($N_1 \neq 0$ and $N_2 = 0$).

Figures 1-4 respectively present the temporal development of the filter solute concentration and specific deposit profiles as well as the filter efficiency and exit specific deposit for various values of the dispersion or diffusion index $n_1$ for the case of constant superficial velocity ($N_D = 0$). Also presented in these figures is the case of constant diffusion for which $N_D = 0$ and the approximate analytical solution for $C(n, \tau)$ given by equation (16). It is clear from Figure 1 that the approximate solution obtained from equation (16) is in excellent agreement with the constant diffusion solution for $r \geq 0.5$. All of the results shown in Figures 1-4 capture the transition from small values of the dimensionless time ($\tau$) to moderate values ($\tau = 1.0$). In general, it is predicted that increases in the dispersion index increase the solute concentration and specific deposit close to the filter entrance while they reduce far downstream for all shown time values; as a consequence there is a decrease in the concentration boundary-layer thickness. Such behaviour as $n_1$ increases appears to be more significant at larger time values. It should be noted, however, that the concentration level and specific deposit values for the constant diffusion case ($N_D = 0$) are higher close to the filter entrance and lower at its exit than for the cases of

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Figure 1: Effects of dispersion index on the temporal development of concentration profiles.

Figure 2: Effects of dispersion index on the temporal development of specific deposit.

Figure 3: Effects of dispersion index on the temporal development of efficiency.

Figure 4: Effects of dispersion index on the temporal development of exit specific deposit.

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variable diffusion \((N_0 \neq 0)\) for all times (see Figures 1 and 2). Physically, the decrease in the concentration boundary-layer thickness as \(n_2\) increases means that solute concentration at the exit of the filter reduces for each time. This has a direct increasing effect on the filter efficiency as expected from its definition in equation (12) and is shown in Figure 3. Also, it is seen from Figure 3 that filter efficiency deteriorates as filtration time passes by. While the filter efficiency increases as \(n_1\) increases, it remains lower than what is predicted for the constant diffusion case represented by \(N_0 = 0\).

In addition, further inspection of Figures 2 and 4 shows that the specific deposit at the filter entrance increases as time progresses and that the same trend is predicted for the specific deposit at the filter exit. The result is expected since an increase in the specific deposit of the filter implies a decrease in the filter porosity. In this way it is understood that as the filtration time increases so the porosity of the filter decreases due to the capture of particles.

Figures 5-8 respectively illustrate the effects of the dispersion or diffusion coefficient \(N_0\) on the temporal development of the solute concentration and specific deposit profiles and the filter efficiency and exit specific deposit for the case of constant superficial velocity \((N_0 = 0)\). Physically speaking, increasing the dispersive or diffusive effects has the general tendency to increase the ability of the solute to transport more easily through the porous medium of the filter which leads to increases in the solute concentration. It is clearly seen from Figures 5-8 that as \(N_0\) decreases so the concentration level and the specific deposit decrease in the filter zone close to the entrance while they increase close to the filter exit. The filter zone close to the filter entrance increases as filtration time increases. As a result the filter exit concentration and specific deposit increase which yields decreases in the filter efficiency and porosity as \(N_0\) increases. Also, the changes in \(C(t, r)\), \(S(t, r)\), \(E(t)\) and \(S(1, t)\) are more pronounced for larger values of \(N_0\). These behaviours are obvious from Figures 5-8.

Figures 9-12 respectively depict the influence of the superficial velocity index \(n_2\) on the solute concentration and specific deposit profiles at five different dimensionless times \((r = 0.1, 0.2, 0.5, 0.75\) and \(1.0))\) and the temporal development of the filter efficiency and exit specific deposit. The corresponding cases of constant superficial velocity are also shown. For a fixed value of \(N_0 = 1.0\), increasing the index from \(n_2 = 0\) (for which the superficial velocity is uniform in the filter with a value of 2) to \(n_2 = 1\) (for which the superficial velocity increases gradually with the filter length from a value of unity to reach the value of 2 at the filter exit) causes the solute concentration and the specific deposit to decrease everywhere in the filter (except the entrance) at all times. The decrease in the exit concentration causes the filter efficiency to increase. Hence, the case of uniform superficial velocity with a value of unity has the lowest velocity and, therefore, produces the highest efficiency. These trends are clear from Figures 9-12.

Finally, the effects of the dispersion or diffusion index \(n_1\) on all of \(C(t, r)\), \(S(t, r)\), \(E(t)\) and \(S(1, t)\) are respectively illustrated in Figures 5-8.
tively presented in Figures 13-16 for constant superficial velocity and in the absence of filter detachment mechanism \((N_b = 0)\). Also shown is the case of constant diffusion for which \(N_b = 0\) and the analytical solution for \(C(n, t)\) given by equation (19). It is clear from Figure 13 that the analytical solution obtained from equation (19) is in excellent agreement with the constant diffusion solution for all considered values of \(r\).

Again, increases in the values of \(n\), tend to increase the solute concentration and specific deposit close to the filter entrance while they reduce it far downstream for all filtration times. As a result, the concentration and the specific deposit at the filter exit decrease as \(n\) increases which causes enhancements in the filter efficiency. Furthermore, as in Figures 1-4, the concentration level and the specific deposit values for the constant diffusion case \((N_b = 0)\) are higher close to the filter entrance and lower at its exit than those for the cases of variable diffusion \((N_b \neq 0)\) for all times. Therefore, the filter efficiency is higher for the constant diffusion case than for the variable diffusion cases. By comparison of Figures 13-16 with Figures 1-4, it can be observed that in the absence of the particle detachment mechanism \((N_b = 0)\) the solute concentration reduces at a much faster rate as it enters the filter than when the detachment mechanism is present \((N_b \neq 0)\).

CONCLUSIONS

A one-dimensional continuum deep bed filtration model with scale-dependent dispersion or diffusion coefficient and superficial velocity was considered. The diffusion coefficient and the superficial velocity were assumed to increase as polynomial functions with the

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**Figure 8:** Effects of dispersion coefficient on the temporal development of exit specific deposit.

**Figure 9:** Effects of velocity index on the temporal development of concentration profiles.

**Figure 10:** Effects of velocity index on the temporal development of specific deposit.

**Figure 11:** Effects of velocity index on the temporal development of efficiency.

**Figure 12:** Effects of velocity index on the temporal development of exit specific deposit.
filter length. The general governing equations were non-linear and cannot be solved in closed form. However, for some special cases linear equations were obtained and exact and approximate solutions were reported. The general equations were solved numerically by an efficient, explicit, iterative tri-diagonal finite difference method. The accuracy of the numerical method was verified by favourable comparisons with the analytical solutions. Several numerical solutions based on the general model were reported. A parametric study was conducted and the results were presented graphically to illustrate interesting features of the solutions. In general, the scale-dependent polynomial-type dispersion coefficient and superficial velocity were predicted to obtain significant changes in the solute concentration, specific deposit and filter efficiency at most time stages compared with the constant diffusion and constant superficial velocity cases. It was found that allowing the diffusion or dispersion coefficient to vary with the filter length caused the efficiency of the filter to decrease. However, in reference to the polynomial form of diffusion used, as the diffusion index was increased the filtration efficiency was increased but it remained below that of the constant diffusion case. On the other hand, as the diffusion coefficient multiplying the scale function increased so the filter efficiency decreased. Similarly, varying the superficial velocity with the filter length caused the filter efficiency to decrease compared to the constant superficial velocity case.

**NOMENCLATURE**

- $c$: concentration of solute (kg m$^{-3}$)
- $c_o$: solute concentration at filter entrance (kg m$^{-3}$)
- $C$: dimensionless concentration of solute
- $d$: hydrodynamic dispersion or diffusion coefficient (m$^2$ s$^{-1}$)
- $D$: dimensionless hydrodynamic dispersion or diffusion coefficient
- $D_o$: molecular diffusion coefficient (m$^2$ s$^{-1}$)
- $E$: filter efficiency
- $erf$: error function
- $erfc$: complementary error function
- $H$: unit step function
- $L$: filter length (m)
- $M$: dimensionless general mass transfer function
- $M^*$: general mass transfer function
- $n_1, n_2$: constants used in polynomial-type diffusion

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**Figure 13:** Effects of dispersion index on the temporal development of concentration profiles.

**Figure 15:** Effects of dispersion index on the temporal development of efficiency.

**Figure 14:** Effects of dispersion index on the temporal development of specific deposit.

**Figure 18:** Effects of dispersion index on the temporal development of exit specific deposit.
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