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UNSTEADY MAGNETOHYDRODYNAMIC TWO FLUID FLOW AND HEAT TRANSFER IN A HORIZONTAL CHANNEL

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ABSTRACT

Unsteady magnetohydrodynamic flow of two immiscible fluids with time-dependent oscillatory wall transpiration velocity is investigated through a horizontal channel. The flow above the interface is assumed to be electrically conducting while the other fluid and the channel walls are assumed to be electrically insulating. Separate solutions for each fluid are obtained and these solutions are matched at the interface using suitable matching conditions. The partial differential equations governing the flow and heat transfer are transformed to ordinary differential equations and closed-form solutions are obtained. Results are presented graphically for various values of Hartman number, frequency parameter, viscosity ratio, conductivity ratio and Prandtl number on the flow and heat transfer characteristics.

KEYWORDS: Unsteady flow; MHD; immiscible fluids and perturbation method.

NOMENCLATURE

A	real positive constant
C_p	specific heat at constant pressure
$\bar{g}$	gravitational acceleration
K	thermal conductivity
M	Hartmann number
P	pressure gradient
Pr	Prandtl number
T	temperature
T_w	wall temperature
t	time
u, v	velocity components of velocity along and perpendicular to the plates, respectively
$\bar{u}_1$	average velocity
v_0	scale of suction

Greek Symbols

α	ratio of viscosity
β	ratio of thermal conductivity
δ	Kronecker delta
ρ_0	fluid density
ω	frequency parameter
ωt	periodic frequency parameter
ν	kinematic viscosity
μ	viscosity of fluid
ε	coefficient of periodic parameter
θ	non-dimensional temperature

Subscripts

1, 2 quantities for Region-I and Region-II respectively.

1. INTRODUCTION

The study of magnetohydrodynamics (MHD) flow for an electrically conducting fluid flow and heat transfer through a horizontal channel has attracted the interest of many authors in view of its important applications in many engineering problems involving petroleum industries, plasma physics, geophysics, MHD pump, MHD flowmeter and cooling of nuclear reactors. MHD is used in the development of an MHD generator as a new source of energy, another technology which has assumed importance is nuclear fusion. Recently this study has been largely concerned with the flow and heat transfer characteristics in various physical situations, for example Hartmann [1] carried out the pioneer work in the study of steady magnetohydrodynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. Alireza and Shai [2] studied the effect of temperature-dependent transport properties on the developing MHD flow and heat transfer in parallel plate channel whose walls are held at constant and equal temperature.

Problem involving a multiphase flow, heat transport arises in many scientific and engineering disciplines. Important applications include MHD power generation, petroleum industries and magneto-fluid dynamics. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe (Packham and Shail [3], Malashetty and Leela [4]). Loharsbi and Sahai [5] studied two-phase MHD flow and heat transfer in a parallel plate channel with one of the fluids being electrically conducting. Shail [6] analyzed the Hartmann flow of a conducting fluid in a channel between two horizontal insulating plates of infinite extent, there being a layer of non-conducting fluid between the conducting liquid and the upper channel wall. Recently Malashetty and Umavathi [7] investigated two-phase MHD flow and heat transfer in an inclined channel. Chamkha [8] considered the steady laminar flow of two immiscible electrically conducting and heat absorbing immiscible fluids in an infinitely long impermeable parallel-plate channel filled with a uniform porous medium. Fully developed flow and heat transfer in horizontal channel consisting of an electrically conducting fluid layer sandwiched between two fluids layers is studied analytically by Umavathi et al. [9].

All the above authors pertain to steady flow. Most of the practical problems deal with unsteady flow. Soundalgekar and Bhat [10] analyzed oscillatory MHD channel flow and heat transfer under a transverse magnetic field. Chamkha [11] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Israle-Coobey et al. [12] investigated the influence of viscous dissipation and radiation on unsteady MHD free convective flow past a heated vertical plate with time dependent suction in an optically thin environment. Recently, Umavathi et al. [13, 14] studied oscillatory flow and heat transfer in two immiscible fluids.

The objective of the present work is to consider unsteady magnetohydrodynamic flow of two immiscible fluids in a horizontal channel. The flow above the interface is electrically conducting while the fluid below the interface is electrically non-conducting.

2. PROBLEM FORMULATION

The geometry under consideration consists of two infinite parallel plates extending in the X and Y directions (Fig. 1). The regions $0 \le y \le h$ and $-h \le y \le 0$ are denoted as Region-I and Region-II respectively. The fluid in Region-I is a conducting fluid having density ρ_1, viscosity μ_1 and thermal conductivity K_1. A constant magnetic field of strength B_0 is applied transverse to the flow field. The Region-II is filled with a non-conducting fluid having density ρ_2, viscosity μ_2 and thermal conductivity K_2.

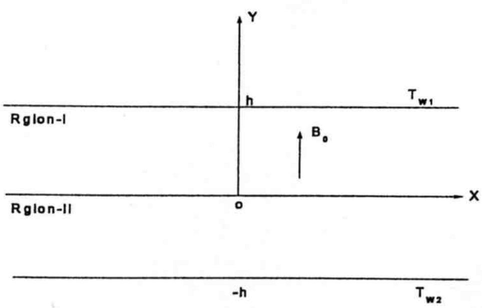

Fig. 1 Physical configuration

It is assumed that the flow is unsteady, fully developed and that fluid properties are constants. The flow in both regions is assumed to be driven by a common pressure gradients $-\frac{\partial p}{\partial x}$ and temperature gradients $\Delta T = T_{w1} - T_{w2}$ where T_{w1} is the temperature of the boundary at y = h and T_{w2} is the temperature of the boundary at y = -h. Under these assumptions the governing equations of motion and energy (Loharsbi and Sahai 1988) are:

Region-I

$$\frac{\partial v_1}{\partial y} = 0 \tag{1}$$

$$\rho_0\left(\frac{\partial u_1}{\partial t} + v_1\frac{\partial u_1}{\partial y}\right) = \mu_1\frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} - \sigma_1 u_1 B_0^2 \tag{2}$$

$$\rho_0 Cp\left(\frac{\partial T_1}{\partial t} + v_1\frac{\partial T_1}{\partial y}\right) = K_1\frac{\partial^2 T_1}{\partial y^2} + \mu_1\left(\frac{\partial u_1}{\partial y}\right)^2 + \sigma_1 B_0^2 u_1^2 \tag{3}$$

Region-II

$$\frac{\partial v_2}{\partial y} = 0 \tag{4}$$

$$\rho_0\left(\frac{\partial u_2}{\partial t} + v_2\frac{\partial u_2}{\partial y}\right) = \mu_2\frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p}{\partial x} \tag{5}$$

$$\rho_0 C_p\left(\frac{\partial T_2}{\partial t} + v_2\frac{\partial T_2}{\partial y}\right) = K_2\frac{\partial^2 T_2}{\partial y^2} + \mu_2\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{6}$$

where u is the x-component of fluid velocity, v is the y-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions, which required that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y=0.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$\begin{aligned} &u_1(h) = 0 \\ &u_2(-h) = 0 \\ &u_1(0) = u_2(0) \\ &\mu_1\frac{\partial u_1}{\partial y} = \mu_2\frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \end{aligned} \tag{7}$$

The thermal boundary and interface conditions on temperature for both fluids are given by