Unsteady buoyancy driven saline water over a vertical flat plate

Rebhi. A. Damseh*
Mechanical Engineering Department,
Al-Huson University College,
Al-Balqa Applied University, P.O. Box 50, Irbid, Jordan
E-mail: rdamseh@yahoo.com
*Corresponding author

A.J. Chamkha
Manufacturing Engineering Department,
The Public Authority for Applied Education and Training,
P.O. Box 42325, Shuweikh, 70654, Kuwait
E-mail: achamkha@yahoo.com

Abstract: The effect of heat and mass transfer on transient laminar free convection flows for saline water over a vertical flat surface is studied. The non-Boussinesq equation to approximate temperature variations with density in buoyancy term is applied. The governing equations are written in their dimensionless form and then solved numerically using an implicit finite difference technique. Two parameters are found to describe the problem; the first one is the density parameter, the second one is the dimensionless Grashof number. Velocity profiles, temperature profiles and Nusselt numbers are discussed.

Keywords: natural convection; mass transfer; saline water; unsteady.


Biographical notes: Rebhi A. Damseh is an Associate Professor in Mechanical Engineering/Refrigeration (August 2007-present) at Al-Balqa Applied University-Huson University College, Jordan. He is the Head of Mechanical Engineering Department. His research interests: refrigeration, new refrigerants, numerical methods in fluid flow, solid particle deposition on mechanical surfaces, fluid flow in porous media and radiation conduction interaction.

Ali J. Chamkha is a Full Professor in the Manufacturing Engineering Department at the Public Authority for Applied Education and Training, Kuwait. Throughout his industry and university careers after his PhD graduation in December 1989, he was and still is involved in many broad research areas. His research interest areas include multiphase fluid-particle dynamics, fluid flow in porous media, heat and mass transfer and magnetohydrodynamics.

1 Introduction
The natural convection heat transfer mode in various geometries has received a great deal of attention (Kao, 1976; Na, 1978; Aziz and Na, 1982). There are several transport processes in industry and in nature where buoyancy forces arise from both thermal and mass diffusion caused by temperature gradients and the concentration differences of dissimilar chemical species. The combined effect of heat and mass transfer on different problems can be seen in El-Hakiem and El Amin (2001), Magyari and Keller (1999), Takhar et al. (2000) and Chamkha (2001). Most of the past studies have used the Boussinesq approximation i.e., taking into account that the fluid density varies linearly with temperature. However, this is not applicable for water at low temperatures, where in the local thermodynamic equilibrium a density extreme in pure or saline water at atmospheric pressure may occur. When flows are driven by temperature and salinity gradients around the level of a density extreme, maximum density conditions may strongly influence the resulting motion (Ingham et al., 2001; Duwairi et al., 2006).

The density-temperature relationship is linear for air whereas in water this relationship is linear at high temperatures and non-linear at low temperatures. The density of pure water is maximum at 3.98°C. The density increases as the temperatures decreases approaching 3.98°C, while the density decreases as the temperature decreases
from 3.98°C to 0°C. The convective flows generate in pure cold and saline water have been analysed by Gebhart et al. (1979), Carey et al. (1980) and El-Henawy et al. (1986) for different thermally driven flows adjacent to vertical and horizontal surfaces, in these studies the steady state conditions are assumed and a set of similarity solutions have been obtained.

The influence of mass transfer on the natural convection problem in cold and saline water is not addressed yet. In this paper the transient free convection boundary layer over an isothermal vertical flat plate which is generated in both cold and saline water and in the presence of mass transfer is studied. Numerical solutions using MacCormack’s technique for different dimensionless velocity, temperature and concentration profiles and different local coefficient of friction, Nusselt numbers and Sherwood number are presented.

2 Basic equations

Consider unsteady laminar free convection boundary layer flow over an isothermal vertical flat plate in both pure and saline water. With respect to an arbitrary origin on this planar wall, the x-axis is taken along plate and the y-axis is taken perpendicular to it into the fluid. Initially, the plates and the fluid are at ambient temperature $T_\infty$ subsequently, the temperature of the plate is impulsively increased to the constant value $T_w$. We also assume that the density of the pure or saline water is given by the following equation (Gebhart and Mollendroff, 1977):

$$\rho(s, p, T) = \rho_m(s, p)[1 - \beta_s(s, p)[T - T_m(s, p)]^{g(s, p)}]$$  \hspace{1cm} (1)

where $\rho_m$ and $T_m$ are the maximum density and temperature for a given pressure $p$ and salinity $s$ and $q$ is the exponent. This equation has very high accuracy to 20°C, salinity of 40% and to a pressure of 100 bars.

If we suppose that another substance is present in water (milk, sugar, dust) it will be diluted in water in a simple, linear way. The variation of concentration with temperature and salinity is unique in nature and it is caused by water special chemical structure. There is no scientific evidence that any other substance will influence water special chemical structure. There is no scientific evidence that any other substance will influence water salinity. In this study, evidence that any other substance will influence water salinity is unique in nature and it is caused by water special chemical structure. There is no scientific evidence that any other substance will influence water salinity. In this study, evidence that any other substance will influence water special chemical structure. There is no scientific evidence that any other substance will influence water salinity.

where $C(s, p, T) = C_m(s, p)[1 - \beta_s(s, p)[T - T_m(s, p)]^{g(s, p)}]$  \hspace{1cm} (2)

where $C_m$ is the maximum concentration.

The governing equations of this flow can be written in non-dimensional form as:

$$\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2}$$  \hspace{1cm} (4)

$$\frac{\partial \phi}{\partial \tau} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial y^2}$$  \hspace{1cm} (5)

where $R = \frac{T_m(s, p) - T_\infty}{T_w - T_\infty}, \text{ and } R_i = \frac{C_m(s, p) - C_i}{C_m - C_i}$  \hspace{1cm} (6)

is the extreme temperature parameter. The initial and boundary conditions of Equations (3)–(6) are:

$$t \leq 0, \quad u = 0, \quad v = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for all } \quad x \geq 0, \quad y \geq 0$$

$$t > 0, \quad u = 0, \quad v = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{for } \quad y = 0, \quad x \geq 0$$

$$u = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for } \quad y \rightarrow \infty.$$  \hspace{1cm} (7)

The non-dimensional variables are defined by:

$$t = Gr \frac{l^2}{(u/l)^2}, \quad x = \tilde{x}/l, \quad y = Gr \frac{l}{(u/l)}$$

$$u = Gr \frac{l}{(u/l)} \tilde{u}, \quad v = Gr \frac{l}{(u/l)} \tilde{v}, \quad \theta = T - T_\infty \frac{T_\infty - T_w}{T_w - T_\infty}, \quad \phi = C - C_m \frac{C_m - C_i}{C_m - C_i}, \quad Gr_i = g \beta_i l \frac{T_\infty - T_w}{y^2},$$

$$Gr \frac{l^2}{(u/l)^2} \frac{T_\infty - T_w}{y^2} \beta_i \frac{T_\infty - T_w}{y^2}$$  \hspace{1cm} (8)

where $l$ is the characteristic length of the plate, $\tilde{x}$, $\tilde{y}$, $\tilde{u}$, $\tilde{v}$, $\tilde{T}$ represent the dimensional variables, and $Gr_i$, $Gr*$ is the thermal and modified Grashof number respectively. The values of local coefficient of friction, local Nusselt number and Sherwood number are given by:

$$C_i = \frac{\partial u(x, 0, t)}{\partial y}$$  \hspace{1cm} (9)

$$NuGr_i^{1/4} = -\frac{\partial \theta(x, 0, t)}{\partial y}$$  \hspace{1cm} (10)

$$ShGr_i^{1/4} = -\frac{\partial \phi(x, 0, t)}{\partial y}.$$  \hspace{1cm} (11)

The boundary layer equations describe the conservation of mass, momentum, energy, and concentration with a non-Boussinesq equation are formulated and solved in their time dependent formulation using the MacCormack’s technique, which is an explicit finite difference technique and a second order accuracy in space and time, the details of this solution is clearly explained by Anderson (1995). In order to verify the accuracy of the present method comparison of results with similarity solutions obtained by Pantokratoras (1999) are shown in Table 1 for the steady laminar free convection over a vertical isothermal impermeable plate (zero blowing and suction) with $Gr*$ = 0 and a linear density-temperature relationship. The comparison is found to be in excellent agreement. As mentioned before the numerical solution used is a time marching technique giving the downstream velocity and temperature profiles using the known upstream profiles. In the present work the above quantities have been calculated by obtaining explicitly the flow field variables at grid point $(i, j)$ at time $\tau + \Delta \tau$ from the known flow field
variables at grid points \((i, j), (I + 1, j), (i - 1, j), (i + 1, j)\) and \((i, j + 1)\) at time \(\tau\). The flow field variables at all other grid points at time \(\tau + \Delta \tau\) are obtained in like fashion. Then the local coefficient of friction, local Nusselt numbers and Sherwood numbers are calculated from Equations (9)–(11).

Table 1 \(\theta'\) at different time and at \(x = 0.5, R = 0, Pr = 11.4, q = 1, Gr^* = 0\). Result in parenthesis is that of similarity solution obtained by Pantokratoras (1999)

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>(\theta')</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>1.0324</td>
</tr>
<tr>
<td>4</td>
<td>1.01444</td>
</tr>
<tr>
<td>(\infty)</td>
<td>1.06028 [1.06]</td>
</tr>
</tbody>
</table>

3 Results and discussion

In this paper the transient free convection heat and mass transfer effects in saline water from vertical surfaces are investigated. The governing equations are written in their dimensionless form using a set of dimensionless variables then solved using a finite-difference implicit method.

The value of \(q\) in these results is taken to be 1.8; this exponent was found to be in the range of \(1 \leq q \leq 2\) (Gebhart and Mollendroff, 1977). The temperature ratio parameter \(R\) is considered to vary between 0.1 and –1.0 while the concentration parameter ratio \(R_1\) is given the value –1 in all of the predicted results, the concentration buoyancy effect is studied throughout the given values of the modified Grashof number \(Gr^*\).

Figures 1–3 depict the dimensionless velocity, temperature and concentration profiles inside the boundary layer at the midpoint of the plate \((x = 0.5)\). The steady state conditions are considered for different values of \(R\), the other dimensionless parameters are taken to be \(Pr = 11, Sc = 800, q = 1.8\) and \(Gr^* = 1\). Obviously, increasing the dimensionless temperature ratio \(R\) aids to decrease the velocity due to the important rule that the buoyancy forces played in transferring momentum between fluid layers and increase temperature and concentration of the fluid inside the boundary layer. It is clear from Figures 4 and 5 that Nusselt numbers and Sherwood numbers decreases as \(R\) increases, the temperature rise inside the boundary layer at increasing values of time leads to decrease the heat transfer and mass transfer phenomenon. The effect of \(R\) on local coefficient of friction is drawn in Figure 6. The coefficient of friction increases as the dimensionless temperature ratio \(R\) decreases at all point of the time field. Moreover, it’s interesting to note that the steady state time increase as \(\sqrt{R}\) becomes increasingly more positive.

Figures 7–9 show the effect of modified Grashof number on local coefficient of friction, local Nusselt numbers and local Sherwood numbers. Increasing \(Gr^*\) had increased coefficient of friction and consequently this will increase velocities inside boundary layer. Increasing \(Gr^*\) had also increased Nusselt numbers and Sherwood numbers because of higher buoyancy forces. Different values of local Nusselt numbers against transient time for different dimensionless groups are listed in Table 1.
Figure 4  Nusselt number variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $Gr^* = 1$)

Figure 5  Sherwood number variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $Gr^* = 1$)

Figure 6  Skin-friction coefficient variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $Gr^* = 1$)

Figure 7  Skin-friction coefficient variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $R = 0.1$)

Figure 8  Nusselt number variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $R = 0.1$)

Figure 9  Sherwood number variations at the midpoint of the plate as a function of the dimensionless time $t$ ($Pr = 11$, $Sc = 800$, $q = 1.8$, $R = 0.1$)
4 Concluding remarks

Numerical solutions for unsteady heat and mass transfer, laminar flow of saline water on vertical flat plate were reported. Based on the obtained graphical results, the following conclusions were deduced:

- The fluid velocity decreased as either of the temperature ratio parameter $R$ increased or the modified Grashof number $Gr^*$ was decreased. The coefficient of friction increased as $R$ decreased and or $Gr^*$ is increased.
- The temperature and concentration profiles inside the boundary layer are increased as $R$ increased; this triggers the Nusselt number and Sherwood number to decrease.
- The effect of $Gr^*$ is to increase the Nusselt number and Sherwood number, this can be noticed for dimensionless time $t > 4$.

References


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Concentration</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Concentration where density is maximum</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Wall concentration</td>
</tr>
<tr>
<td>$C_\infty$</td>
<td>Ambient fluid concentration</td>
</tr>
<tr>
<td>$g$</td>
<td>Magnitude of acceleration due to gravity</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Thermal Grashof number</td>
</tr>
<tr>
<td>$Gr^*$</td>
<td>Modified Grashof number</td>
</tr>
<tr>
<td>$K$</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>$L$</td>
<td>Characteristic length of plate</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $\nu/\alpha$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Exponent in density and concentration, Equations (1) and (2)</td>
</tr>
<tr>
<td>$R$</td>
<td>Density parameter defined in Equation (6)</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Density parameter defined in Equation (6)</td>
</tr>
<tr>
<td>$S$</td>
<td>Salinity</td>
</tr>
<tr>
<td>$Sc$</td>
<td>Schmidt number, $\nu/D$</td>
</tr>
<tr>
<td>$t$</td>
<td>Non-dimensional time</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Temperature where the density is maximum</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Wall temperature</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Ambient fluid temperature</td>
</tr>
<tr>
<td>$u$, $v$</td>
<td>Non-dimensional velocity components along $x$- and $y$-axes</td>
</tr>
<tr>
<td>$x$, $y$</td>
<td>Non-dimensional coordinates along and normal to the plate</td>
</tr>
</tbody>
</table>
Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Coefficient in the concentration, Equation (2)</td>
</tr>
<tr>
<td>$\beta_T$</td>
<td>Coefficient in the density, Equation (1)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Non-dimensional temperature</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density, Equation (1)</td>
</tr>
</tbody>
</table>

$\rho_m$  Maximum density  
$\rho_\infty$  Density of ambient fluid  
$\nu$  Kinematic viscosity  
$\phi$  Non-dimensional temperature

Superscript

- Dimensional quantities