Mixed Convection Flow of Non-Newtonian Fluid From a Slotted Vertical Surface With Uniform Surface Heat Flux

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In the present paper, the combined convection flow of an Ostwald–de Waele type power-law non-Newtonian fluid past a vertical slotted surface has been investigated numerically. The boundary condition of uniform surface heat flux is considered. The equations governing the flow and the heat transfer are reduced to local non-similarity form. The transformed boundary layer equations are solved numerically using implicit finite difference method. Solutions for the heat transfer rate obtained for the rigid surface compare well with those documented in the published literature. From the present analysis, it is observed that, an increase in χ leads to increase in skin friction as well as reduction in heat transfer at the surface. As the power-law index n increases, the friction factor as well as heat transfer increase.

Dans cet article, on a étudié numériquement l'écoulement de convection combinée d'un fluide non-newtonien de loi de puissance de type Ostwald-de Waele en aval d'une surface perforée verticale. La condition limite d'un flux de chaleur de surface uniforme est considérée. Les équations gouvernant l'écoulement et le transfert de chaleur sont réduites à la forme de non-similarité locale. Les équations de couche limite transformées sont résolues numériquement par la méthode des différences finies implicites. Les solutions pour la vitesse de transfert de chaleur obtenues pour la surface rigide se comparent bien à celles qui sont décrites dans la littérature scientifique publiée. À partir de la présente analyse, on observe qu'une augmentation de χ mène à une augmentation du frottement superficiel ainsi qu'à une réduction du transfert de chaleur à la surface. Lorsque l'indice de loi de puissance n augmente, le facteur de friction et le transfert de chaleur augmentent également.

Keywords: mixed convection, non-Newtonian fluids, slotted surface

INTRODUCTION

Several industrial fluids are non-Newtonian in their flow characteristics. In a Newtonian fluid, the shear stress is directly proportional to the rate of shear strain, whereas in a non-Newtonian fluid, the relationship between the shear stress and the rate of shear strain is nonlinear. Most of the particulate slurries such as china clay and coal in water, multiphase mixtures such as oil–water emulsions, paints, synthetic lubricants, biological fluids such as blood, synovial fluid, and saliva and foodstuffs such as jams, jellies, soups, and marmalades are examples of non-Newtonian fluids. Because of the large apparent viscosity of the non-Newtonian fluids, they are associated with low Reynolds and Grashof number. Therefore, laminar flow situations of non-Newtonian fluids frequently exist when compared to Newtonian fluids. A review of non-Newtonian fluid flow problems may be found in Astrita and Marrucci (1974), Darby (1976), Schowalter (1978), Tanner (1985), Shenoy and Mashelkar (1982), Anderson and Irgens (1990), and Ghosh et al. (1994).

A study of heat and mass transfer in non-Newtonian fluids is of practical importance. Visco-elastic fluids, couple stress fluids, micropolar fluids, and power-law fluids are a few different types of non-Newtonian fluids. The simple and most important type

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is the power-law fluid (Ostwald–de Waele model) for which the rheological equation of the state between the stress components τ_{ij} and strain components e_{ij} is defined by Skelland (1967)

$$\tau_{ij} = -p\delta_{ij} + K\left|\sum_{m=1}^{3}\sum_{l=1}^{3} e_{lm}e_{lm}\right|^{(n-1)/2} e_{ij}$$

where p is the pressure, δ_{ij} is the Kronecker delta, and K and n are the consistency coefficient and power-law index of the fluid, respectively. For $n > 1$, the fluid is said to be dilatant or shear thickening; for $n < 1$, the fluid is called shear thinning or pseudo-plastic; and for $n = 1$, the fluid is simply a Newtonian fluid. Several fluids studied in the literature suggest that the range $0 < n \leq 2$ is valid for the power-law index n. Considerable amount of research work has been done in this field by taking into account the heat and mass transfer. Schowalter (1960) and Acrivos et al. (1960) preformed a theoretical analysis of steady boundary layer flow of power-law fluids. Lee and Ames (1966) extended this work to find the similarity solutions for non-Newtonian power-law fluids. Kim et al. (1993) presented similarity solutions of the steady boundary layer equations of a non-Newtonian fluid. The effects of surface mass transfer in non-Newtonian power-law boundary layer flows were considered by Thompson and Snyder (1968) and Kim and Eraslan (1969). The steady incompressible boundary layer flow of flow of a non-Newtonian power-law fluid on a two-dimensional body in the presence of a magnetic field was studied by Sarpkaya (1961) and Djukic (1973). Pakdemirli (1993) derived boundary layer equations for power-law fluids using a special coordinate system that made the equations independent of the body shape. Kleinstreuer and Wang (1988) developed transformation parameters for analysing the forced convection of non-Newtonian fluids past vertical cylinders, rotating spheres, and axis symmetric bodies. Haung and Lin (1993) solved the problem of mixed convection from a vertical plate to power-law fluids. Wang (1993, 1995) presented a boundary layer analysis for a laminar mixed convection from a vertical or horizontal plate to non-Newtonian fluids. Pop and Gorla (1991) presented similarity solutions for the mixed convection in non-Newtonian fluids from stationary or moving horizontal surfaces.

The boundary layer flow over a slotted plate has drawn the attention of Laplace and Arquis (1998). The media are often characterized by their pressure drop coefficient, which is mostly determined experimentally. When faced with a tangential flow, non-slip hypothesis is generally taken and this assumption is no longer valid when the perforation density is very large. In a boundary layer flow, this will have consequence for the displacement thickness. Working on ideal periodic perforated media placed in a purely tangential flow, it has been proved by Laplace and Arquis (1998) that the shear stress at the wall is equal to the slip velocity times a coefficient λ applies to slotted plate. By means of an experimental study, the slip condition (called the Navier condition hereafter) was determined by Beavers and Joseph (1967) for a fluid-porous medium interface. A theoretical justification for it was given by Saffman (1971).

The objective of the present paper is to investigate the combined convection flow of a power-law non-Newtonian fluid past a slotted vertical surface. The equations that govern the flow and heat transfer are reduced to local non-similarity form. The transformed boundary layer equations are solved numerically using implicit finite difference method together with Keller box elimination technique (1978) for all the values of χ. The solutions of a rigid surface are compared with literature values. The results are expressed in terms of the local skin friction and local rate of heat transfer coefficients against χ for different Prandtl numbers, Pr, and the slip velocity coefficient C.

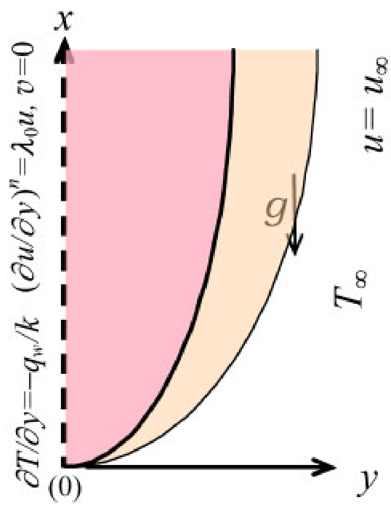

Figure 1. The flow configuration and coordinate. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

ANALYSIS

Consider a steady laminar mixed convection flow of non-Newtonian power-law fluid along a slotted vertical surface subject to uniform surface heat flux, q_w, as shown in Figure 1.

The streamwise coordinate χ is measured in the direction of forced flow and the transverse coordinate y is measured normal to the surface of the plate. The fluid properties are considered to be constant except that the density variations within the fluid are allowed to contribute to the buoyancy force. When the wall temperature is higher than the ambient temperature, that is, $T_w > T_\infty$ or $Z = 1$, the buoyancy force will aid the upwardly directed uniform stream, and when $T_w < T_\infty$, or $Z = -1$, the resulting buoyancy force will retard the forced flow.

By employing the Boussinesq approximations and making use of the power-law viscosity model, the governing equations for the problem under consideration are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = Zg\beta\,(T - T_\infty) + \frac{K}{\rho}\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

The associated boundary conditions are as follows:At $y = 0$,

$$\left(\frac{\partial u}{\partial y}\right)^n = \lambda_0 u, \quad v = 0$$

and

$$\frac{\partial T}{\partial y} = -\frac{q_w}{k} = \text{constant} \quad (4)$$

As $y \to \infty$

$$u = u_\infty \quad \text{and} \quad T = T_\infty$$

In Equation (2), n is the fluid flow index that designates the nature of the fluid. When $n=1$, the equation governs the flow of a Newtonian fluid. K is the power-law fluid consistency index, ρ is the density, β is the volume expansion coefficient, and α is the thermal diffusivity. Due to the nature of the non-Newtonian power-law fluid, similarity solutions only exist at infinite Prandtl number, that is, when inertia terms in Equation (2) are neglected. But when the inertia force becomes the same order of magnitude as of the viscous force, similarity solutions do not exist. Therefore, using the dimensional analysis, one can obtain the following similarity transformations in order to facilitate the solutions of Equations (1)–(4) reducing them to non-similar equations. We introduce the following group of transformations:

$$\chi = \left[\lambda_0 \alpha^{n-1} \Lambda^{2-3n} L^{2-n}\right]^{1/n} \left(\frac{x}{L}\right)^{1/(n+1)}$$

$$\eta = \Lambda \left(\frac{x}{L}\right)^{1/(n+1)} \frac{y}{L}$$

$$\psi = \alpha \Lambda \left(\frac{x}{L}\right)^{1/(n+1)} F(\chi, \eta)$$

$$\theta = \frac{T - T_\infty}{q_w L/k \Lambda (x/L)^{1/(n+1)}}$$

where $\psi(x, y)$ is the stream function that satisfies the continuity equation, with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

The dimensionless buoyancy parameter Λ is defined as

$$\Lambda = \mathrm{Re}^{1/(n+1)} + Ra^{1/(3n+2)} = \frac{\mathrm{Re}^{1/(n+1)}}{(1-\zeta)} = \frac{Ra^{1/(3n+2)}}{\zeta}$$

where

$$\mathrm{Re} = \frac{\rho u_\infty{}^{2-n} L^n}{K}$$

and

$$Ra = \frac{\rho g \beta \left|q_w\right| L^{2(n+1)}}{K \alpha^n k}$$

$$\zeta = \frac{Ra^{1/(3n+2)}}{[\mathrm{Re}^{1/(3n+1)} + Ra^{1/(3n+2)}]} \tag{5}$$

In the above equations, *Re* and *Ra* are the generalized Reynolds number and generalized Rayleigh number, respectively. The mixed convection parameter ζ covers the entire domain of mixed convection from pure forced convection ($\zeta = 0$) to the pure free convection ($\zeta = 1$).

Substituting the transformation (5) into Equations (1)–(3), we have the following transformed non-similar equations:

$$\mathrm{Pr}^{2-n} \left\{ \left|F''\right|^{n-1} F'' \right\}' + \left(\frac{1}{n+1}\right) FF'' + \mathrm{Pr}^{2-n} \left(\frac{\chi}{C}\right)^{n+1} \zeta^{3n+1} \theta = \left(\frac{1}{n+1}\right) \chi \left[F' \frac{\partial F'}{\partial \chi} - F'' \frac{\partial F}{\partial \chi}\right] \tag{6}$$

$$\left(\frac{\chi}{C}\right)^{(n-1)/(n+1)} \theta'' + \frac{1}{(n+1)} \left[F\theta' - F'\theta\right] = \left(\frac{1}{n+1}\right) \chi \left[F' \frac{\partial \theta}{\partial \chi} - \theta' \frac{\partial F}{\partial \chi}\right] \tag{7}$$

The transformed boundary conditions are given by

$$F(\chi, 0) = 0, \quad F''(\chi, 0) = \chi \left[F'(\chi, 0)\right]^{1/n}, \quad \theta'(\chi, 0) = -1 \tag{8}$$

$$F'(\chi, \infty) = \mathrm{Pr}[1-\zeta]^{(n+1)/(2-n)}, \quad \theta(\chi, \infty) = 0$$

In Equations (6) and (7), *Pr*, the generalized Prandtl number, and *C* are defined as

$$\mathrm{Pr} = \frac{(K/\rho)^{1/(2-n)} L^{(2-2n)/(2-n)} \Lambda^{3(n-1)/(2-n)}}{\alpha} \tag{9}$$

$$C = [\lambda_0 \alpha^{n-1} \Lambda^{2-3n} L^{2-n}]^{1/n}$$

It is usually the friction factor C_f and Nusselt number *Nu* that are of interest in practical applications. The local skin friction coefficient C_{fx} is given by

$$C_{fx} = \frac{\tau_w}{(\rho u_\infty^2/2)} \tag{10}$$

where τ_w is the wall shear stress.

The local friction factor group is given by

$$\frac{1}{2} C_{fx} \mathrm{Re}_x^{1(n+1)} = \mathrm{Pr}^{-n} (1-\zeta)^{3n/(n-2)} [F''(\chi, 0)]^n \tag{11}$$

The wall heat transfer rate may be written by Fourier's law by

$$q_w = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

The heat transfer coefficient h may be written as

$$h = \frac{q_w}{T_w - T_\infty} \tag{12}$$

The local Nusselt number may be written as

$$Nu_x = \frac{hx}{k}$$

$$= \frac{\Lambda(\chi/C)}{\theta(\chi, 0)} \quad \text{where} \, \chi = C\left(\frac{x}{L}\right)^{1/(n+1)} \tag{13}$$

Numerical Solution

The resulting system of coupled nonlinear equations and associated boundary conditions is solved by an implicit finite difference method by Keller (1978). Equations (6) and (7) with appropriate boundary conditions in (8) are written as a system of nonlinear algebraic equations (finite difference equations) using a "central" finite difference scheme with uniform mesh. The appropriate boundary conditions in (8) form a part of this system, which is solved by using Newton–Raphson method. The resulting system of linear algebraic equations, which turns out to be a tridiagonal

Table 1. Comparison of $(C_{fx} \mathrm{Re}_x^{1/(n+1)})/2$ for forced convection between a flat plate and non-Newtonian fluids

n	Acrivos et al. (1960)	Kim et al. (1993)	Haung and Lin (1993)	Wang (1993, 1995)	Present method
0.5	0.5755		0.5808	0.5761	0.5765
0.8			0.4054	0.4052	0.4053
1	0.3320		0.3320	0.3320	0.3320
1.2	0.2750	0.2780	0.2780	0.2778	0.2779
1.5	0.2189		0.2203	0.2201	0.2201

Table 2. Comparison of $Nu_x \mathrm{Re}_x^{-1/(n+1)}$ for forced convection between a flat plate and non-Newtonian fluids

n	χ	Haung and Lin (1993)	Wang (1993, 1995)	Present method
0.5	0.01	0.6020	0.59901	0.60012
	0.1	0.7811	0.77695	0.77697
	1	1.0111	1.00615	1.00721
1.5	0.01	1.3886	1.39334	1.39312
	0.1	1.1916	1.19482	1.19215
	1	1.0223	1.00615	1.00721

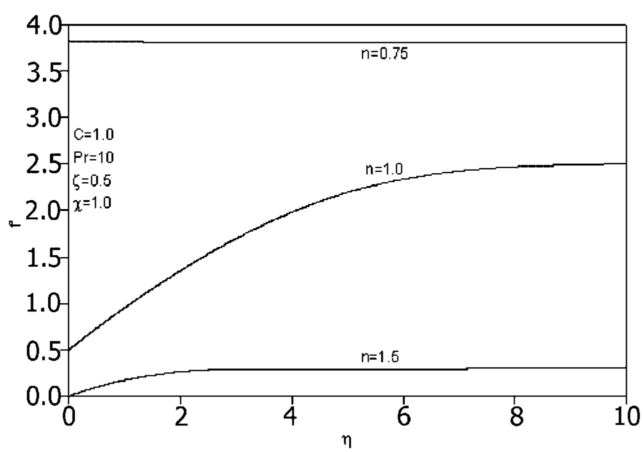

Figure 2. Velocity profiles for different values of n.

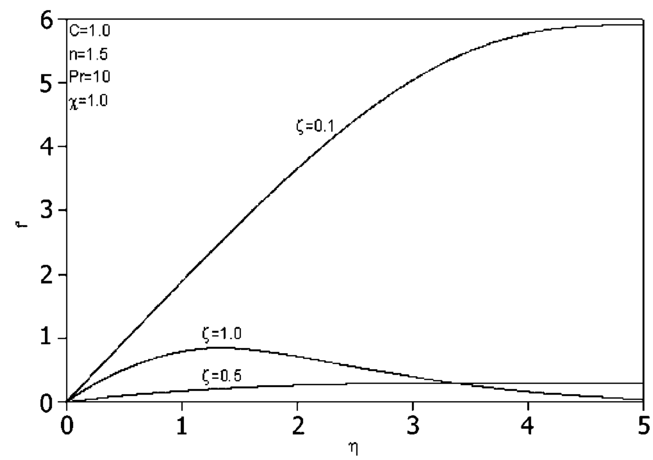

Figure 3. Velocity profiles for different values of ζ.

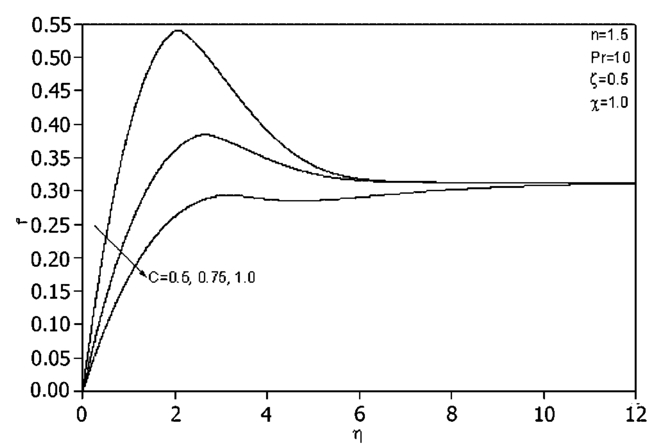

Figure 4. Velocity profiles for different values of C.

system, has been solved by a method suggested in Cebeci and Bradshaw (1984). The choice of initial guess in Newton–Raphson method and the numerical η_∞, which depends on the physical parameter n, and Pr is very crucial in the numerical procedure. The initial guess was made with the aid of the known exact solution for $n = 1$, and several trial-and-error runs were made to obtain accurate values of F, F', etc. up to a significant number of decimal places and satisfy the boundary condition at η_∞ by employing a shooting technique (Conte and de Boor, 1980) for Equations (6) and (7) and its appropriate boundary conditions in Equation (8). These numerical computations are then used to draw several graphs and tables.

The two-dimensional grid is non-uniform to accommodate the steep velocity and temperature gradients at the wall particularly in the vicinity of the singular point at $\chi = 0$; that is, the leading edge of the vertical plate. The location of the boundary layer edge η_∞ depends strongly on the power-law viscosity index n, the mixed convection parameter ζ, and the fluid Prandtl number Pr. For example, η_∞ ($n = 1.0$, $\zeta = 0$, $Pr = 10$) ≈ 8 and η_∞ ($n = 0.5$, $\zeta = 1.0$, $Pr = 10$) ≈ 40. Numerical error testing has been accomplished by straightforward repeat calculations with finer mesh to check grid independence of the results and by local mesh refinement in the η-direction with the smooth translation to the coarser region.

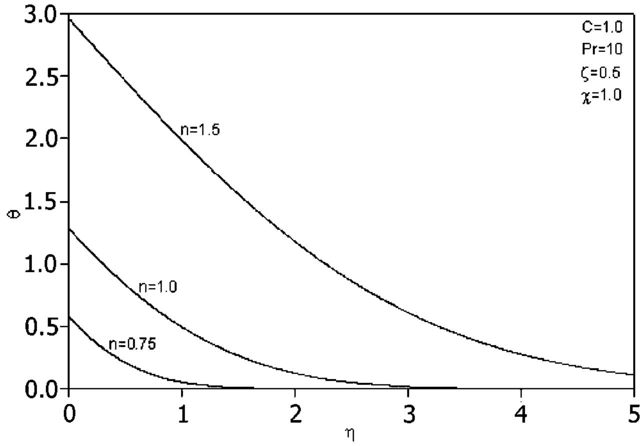

Figure 5. Temperature profiles for different values of n.

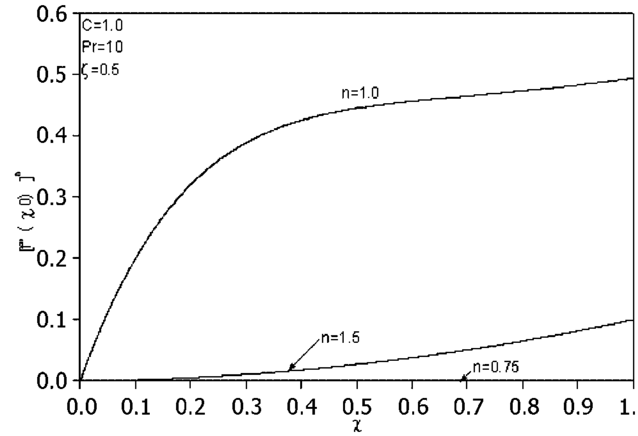

Figure 8. Effects of n on $[f^{II}\ (\chi^0)]^n$ for different values of χ.

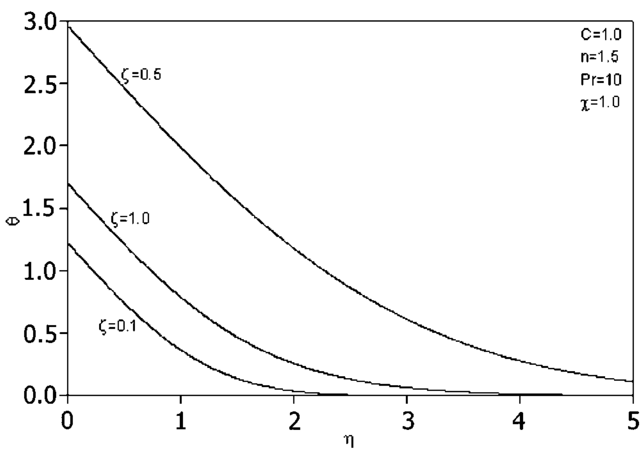

Figure 6. Temperature profiles for different values of ζ.

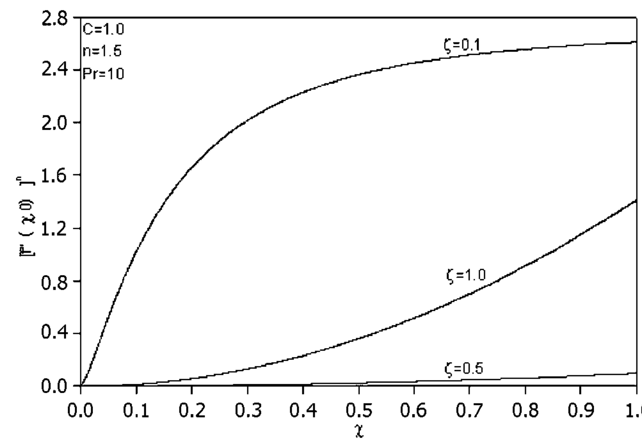

Figure 9. Effects of ζ on $[f^{II}\ (\chi^0)]^n$ for different values of χ.

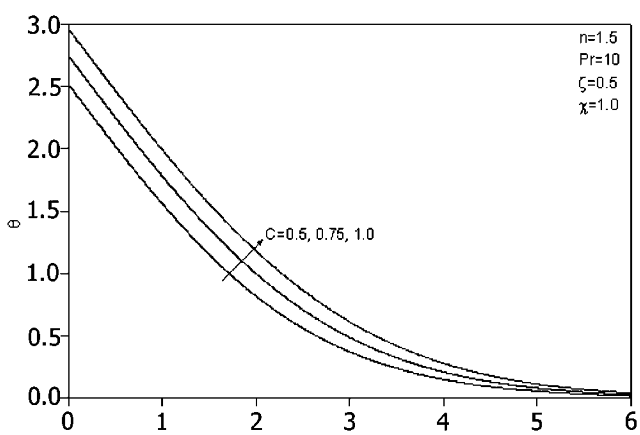

Figure 7. Temperature profiles for different values of C.

RESULTS AND DISCUSSION

To confirm the accuracy of the present results, we have compared our results with those reported in the published literature for the case of rigid vertical surface. These results are shown in Tables 1 and 2. The agreement between our results and the literature values is within 1% error.

Figures 2–4 display results for the velocity distribution within the boundary layer. $\zeta = 1$ corresponds to the pure free convection, whereas $\zeta = 0$ corresponds to pure forced convention. The evolution of the velocity profiles from forced convection ($\zeta = 0.1$) to free convection ($\zeta = 1.0$) for buoyancy assisted case is shown in Figure 3 for dilatant fluids. As the power-law viscosity index n increases, the velocity decreases. As the slip parameter C increases, the velocity decreases. For small values of C (0.5), the velocity within the

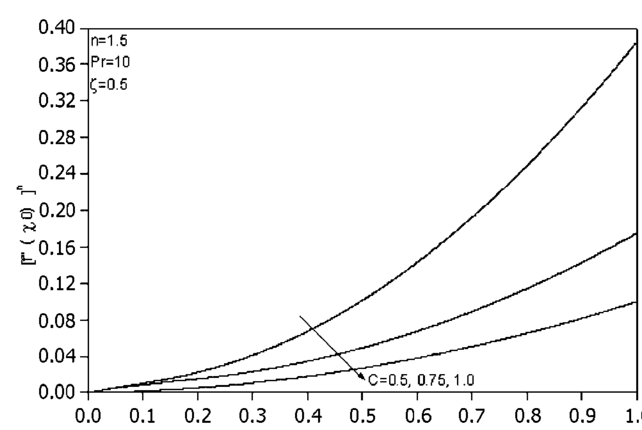

Figure 10. Effects of C on $[f^{II}\ (\chi^0)]^n$ for different values of χ.

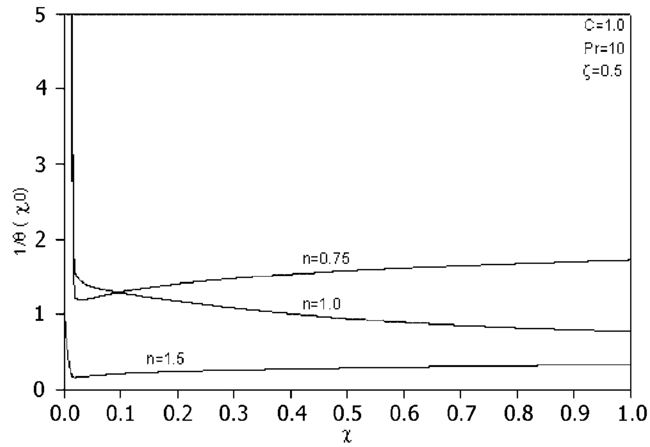

Figure 11. Effects of n on $1/\theta\ (\chi^0)$ for different values of χ.

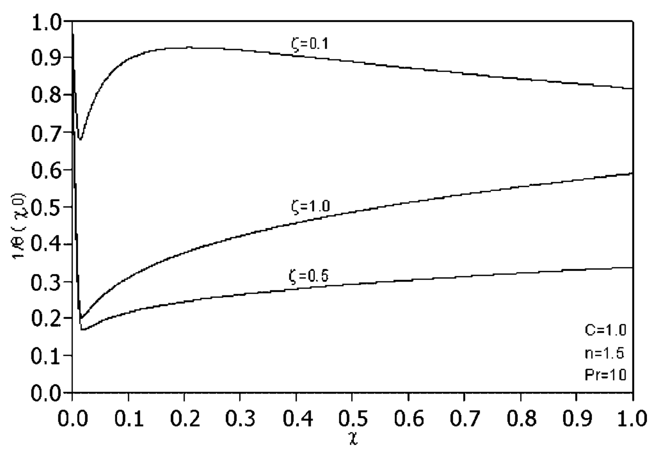

Figure 12. Effects of ζ on $1/\theta\ (\chi^0)$ for different values of χ.

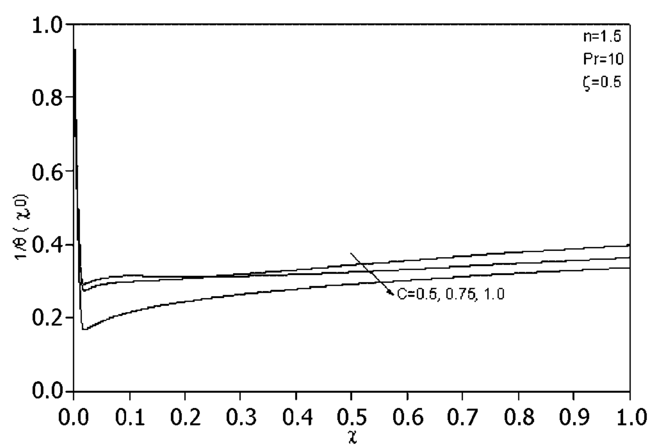

Figure 13. Effects of C on $1/\theta\ (\chi^0)$ for different values of χ.

boundary layer is higher than free stream value and exhibits an overshoot.

Figures 5–7 show the temperature distribution within the boundary layer. As the value of n increases, the temperature profile becomes broader and the thermal boundary layer thickness increases.

Figures 8–13 display results for friction factor and heat transfer rate, respectively. The friction factor and heat transfer rate increases with χ. As the slip parameter C increases, friction factor and heat transfer rate decrease.

CONCLUDING REMARKS

In the present work, we have investigated the combined convection flow of a power-law type non-Newtonian fluid past a slotted vertical surface. The governing equations for the flow and heat transfer are reduced to local non-similarity equations, treating χ as a local similarity variable. The transformed boundary layer equations have been integrated numerically applying Keller-box method for all values of χ. The results are expressed in terms of the reduced local skin friction factor, and local heat transfer coefficients against χ for varying values of n, C, ζ, and Prandtl number Pr.

Following conclusions can be made from present investigation:

1. Increase in χ leads to increase in the values of the skin friction coefficient, as well as the heat transfer rate.
2. As the viscosity index n increases, both the friction factor and heat transfer rate increase.
3. As the slip parameter C increases, the friction factor and the heat transfer rate decrease.

NOMENCLATURE

C	slip parameter
C_f	local skin friction coefficient
e_{ij}	strain components
F	dimensionless stream function
g	gravitational acceleration
h	local heat transfer coefficient
K	fluid consistency index for power-law fluids
k	thermal conductivity
L	length of the plate
Nu_x	local Nusselt number
n	power-law viscosity index
Pr	generalized Prandtl number
p	static pressure
q	local heat transfer rate
Ra	generalized Rayleigh number
Re	generalized Reynolds number based on L
Re_x	generalized Reynolds number based on x
T	temperature
u	velocity component in x direction
v	velocity component in y direction
u_∞	free stream velocity
x	streamwise coordinate measured along the surface
y	coordinate normal to the surface
Z	dimensionless constant (1 if T_w is greater than T_∞; -1 if T_w is less than T_∞)

Greek Symbols

α	thermal diffusivity
β	coefficient of thermal expansion
θ	dimensionless temperature
ρ	fluid density
ψ	non-dimensional stream function
ζ	mixed convection parameter
χ	local similarity variable
τ	shear stress

Λ	buoyancy parameter
λ_0	slip constant
η	dimensionless coordinate

Subscripts

w	wall conditions
∞	ambient conditions

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