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Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface: The full analytical solution

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ABSTRACT

In a recent paper by Damseh et al. (Int. J. Thermal Sci. 48, 1658–1663, 2009), the title problem for an infinite vertical plate has been investigated numerically using the fourth-order Runge–Kutta method. In the present paper the full analytical solution is given. Several new features emerging from this approach are discussed in detail.

1. Introduction

Modeling and analysis of the dynamics of micropolar fluids has been the subject of many research papers in recent years. This stems from the fact that these types of fluids may have many engineering and industrial applications. Micropolar fluids are defined as fluids consisting of randomly oriented molecules whose fluid elements undergo translational as well as rotational motions. Analysis of physical problems using these types of fluids has revealed several interesting phenomena and microscopic effects arising from local structure and micro-rotation of fluid elements not found in Newtonian fluids. The theory of micropolar fluids and thermo-micropolar fluids was developed by Eringen [1,2] in an attempt to explain the behavior of certain fluids containing polymeric additives and naturally occurring fluids such as the phenomenon of the flow of colloidal fluids, real fluid with suspensions, exotic lubricants, liquid crystals and animal blood. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Ariman et al. [3,4], Łukaszewicz [5] and Eringen [6].

Boundary-layer flow on a continuous surface occurs in a number of industrial and metallurgical applications. Examples may be found in continuous casting, glass fiber production, metal extrusion, hot rolling, textiles, crystal growing and wire drawing ([7,8]). Sakiadis [9] initiated the theoretical study of boundary-layer flow on a continuous semi-infinite sheet moving steadily through an otherwise quiescent fluid environment, whereas its heat transfer aspect was studied by Tsou et al. [10]. This problem [9,10] was extended by Erickson et al. [11] and Fox et al. [12] to include fluid wall suction or blowing and investigated its effects on the heat and mass transfer in the boundary layer. Crane [13] was the first to study the flow caused by an elastic sheet whose velocity varies linearly with the distance from a fixed point on the sheet. Gupta and Gupta [14] reported a similarity solution for heat and mass transfer of the boundary-layer flow over a stretching sheet subject to suction or blowing at the surface. Chen and Char [15] investigated the effects of variable surface temperature and variable surface heat flux on the heat transfer characteristics of a linearly stretching sheet with suction and blowing effects. Magyari and Keller [16] investigated heat and mass transfer in the boundary layers on an exponentially stretching continuous surface and later reported exact solutions for self-similar boundary-layer flows induced by permeable stretching surfaces [17]. All of the above studies [9–17] were restricted to Newtonian fluids.

A significant amount of research concerning micropolar fluid flow and heat transfer caused by continuously stretched or moving

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surfaces under different boundary conditions and in the presence of various physical effects has been reported. Soundalgekar and Takhar [18] investigated steady boundary-layer flow and heat transfer of a micropolar fluid due to a continuously-moving surface for constant micro-inertia density. Hady [19] considered heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection. Raptis [20] studied flow of a micropolar fluid past a continuously-moving plate in the presence of radiation. El-Arabawy [21] analyzed the effect of suction/injection on the flow of a micropolar fluid past a continuously-moving plate in the presence of radiation. Kelson and Desseaux [22] reported self-similar solutions for the boundary-layer flow of micropolar fluids driven by a stretching sheet with uniform suction or blowing through the surface. Kelson and Farrell [23] considered micropolar flow over a porous stretching sheet with strong suction or injection. Bhargava et al. [24] reported a finite element solution of mixed convection micropolar flow driven by a porous stretching sheet. Abo-Eldahab and El Aziz [25] analyzed flow and heat transfer in a micropolar fluid past a stretching surface embedded in a non-Darcian porous medium with uniform free stream. Eldabe and Ouaf [26] solved the problem of heat and mass transfer in a MHD flow of a micropolar fluid past a stretching surface with Ohmic heating and viscous dissipation effects using the Chebyshev finite difference method. Odda and Farhan [27] studied the effects of variable viscosity and variable thermal conductivity on heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing. Bhargava et al. [28] reported numerical solutions for micropolar transport phenomena over a nonlinear power-law stretching sheet. Mahmoud [29] considered thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity. Aouadi [30] reported a numerical study for micropolar flow over a stretching sheet. Ishak et al. [31] studied heat transfer over a uniformly stretching surface with variable heat flux in micropolar fluids. Demseh et al. [32] investigated combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flow over a uniformly stretched permeable surface.

In certain applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation or absorption effects are important. In addition, in many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretching of a surface. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first-order reaction in which the rate of reaction is directly proportional to the species concentration. Chamkha [33] studied the problem of heat and mass transfer by steady flow of electrically conducting fluid on a uniformly moving vertical surface in the presence of first-order chemical reaction. Kandasamy et al. [34] studied the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. Patil and Kulkarni [35] studied the effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. Khedr et al. [36] studied MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption and reported similarity solutions. Mohamed and Abo-Dahab [37] analyzed the influence of chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium with heat generation.

The objective of the present work is to report a full analytical solution for the problem considered earlier by Damseh et al. [32] on micropolar fluid flow over a uniformly stretched permeable surface with the combined effects of heat generation or absorption and first-order chemical reaction.

2. Basic equations

The basic boundary value problem of the approach of Damseh et al. [32] for the velocity $U = U(Y)$, micro-rotation $N = N(Y)$, temperature $\theta = \theta(Y)$ and concentration $\varphi = \varphi(Y)$ fields is specified by the following linear differential equations and boundary conditions

$$(1+R)U'' + U' + R\,N' + Gr_T\theta + Gr_C\varphi = 0 \tag{1}$$

$$\lambda N'' + N' - M\,R\big(U' + 2N\big) = 0 \tag{2}$$

$$\theta'' + Pr\,\theta' - Pr\,Q\,\theta = 0 \tag{3}$$

$$\varphi'' + Sc\,\varphi' - Sc\,\gamma\,\varphi = 0 \tag{4}$$

$$U(0) = 1,\; N(0) = -nU'(0),\; \theta(0) = \varphi(0) = 1, \tag{5}$$

$$U(\infty) = N(\infty) = \theta(\infty) = \varphi(\infty) = 0 \tag{6}$$

The primes denote differentiation with respect to the transverse coordinate Y. All the quantities involved in Eqs. (1)–(6) are dimensionless and everywhere the notations of [32] are used except for m of [32] which in fact coincides with the negative of the micro-gyration parameter n. The ten physical parameters of the problem $R,\ M,\ \lambda,\ Gr_T,\ Gr_C,\ Pr,\ Q,\ Sc,\ \gamma$ and n (see Nomenclature), are considered as being given quantities. The skin-friction coefficient C_f, the Nusselt number Nu and the Sherwood number Sh are obtained as [32]

$$\begin{aligned} C_f &= (1+R)U'(0) + R\,N(0),\\ Nu &= -\theta'(0),\; Sh = -\varphi'(0) \end{aligned} \tag{7}$$

3. Exact analytical solution and discussion

3.1. The general analytical solution

The temperature and concentration Eqs. (3) and (4) are fully decoupled from Eqs. (1) and (2). The solutions of the corresponding θ and φ sub-problems have been given in [32] in the form.

$$\begin{aligned} \theta(Y) &= e^{-m_1 Y}, \quad m_1 = \frac{Pr}{2} + \sqrt{\left(\frac{Pr}{2}\right)^2 + QPr},\\ \varphi(Y) &= e^{-m_2 Y}, \quad m_2 = \frac{Sc}{2} + \sqrt{\left(\frac{Sc}{2}\right)^2 + \gamma Sc} \end{aligned} \tag{8}$$

First of all, we emphasize that the solutions given by Eqs. (8) possess a limited range of validity with respect to Q and γ (recall that $Q > 0$ corresponds to heat absorption and $Q < 0$ to heat generation, [32]. Similarly, $\gamma > 0$ corresponds to a reactant consuming and $\gamma < 0$ to reactant producing chemical reaction, [32]). The θ-solution (8), e.g., becomes even unphysical when $Q < Q_{\text{crit}}$ where

$$Q_{\text{crit}} = -\frac{Pr}{4} \tag{9}$$

Indeed, in this range of Q, m_1 and thus also the Nusselt number $Nu = m_1$ resulting from (8), become complex quantities. For the Prandtl number $Pr = 0.71$ considered in most of the examples given in [32], the critical value of Q is $Q_{crit} = -0.1175$. In case of the φ solution (8) and the corresponding Sherwood number $Sh = m_2$, a similar situation would occur for $\gamma < -Sc/4 \equiv \gamma_{crit}$.

The general solution of the θ-problem is given by the linear combination

$$\theta(Y) = C_1 \exp\left[\left(-\frac{Pr}{2} - \sqrt{\left(\frac{Pr}{2}\right)^2 + QPr}\right) Y\right] + C_2 \exp\left[\left(-\frac{Pr}{2} + \sqrt{\left(\frac{Pr}{2}\right)^2 + QPr}\right) Y\right] \quad (10)$$

when $Q \neq Q_{crit}$ and by

$$\theta(Y) = (C_1 + C_2 Y)\exp\left(-\frac{Pr}{2}Y\right) \quad (11)$$

when $Q = Q_{crit}$.

It is immediately seen that in case of Eq. (10), the asymptotic condition $\theta(\infty) = 0$ requires $C_2 = 0$ only when $Q \geq 0$. In this case the wall condition $\theta(0) = 1$ implies $C_1 = 1$ and the solution (10) is unique and coincides with (8). Therefore, the range of validity of the θ-solution (8) is $Q \geq 0$. When, however $Q < 0$, the asymptotic condition $\theta(\infty) = 0$ becomes ineffective in determination of C_2, and we have a one-parameter family of non-unique solutions in both cases (10) and (11). It is worth mentioning here the θ-solution (8) can formally be applied even in the extended range $Q > Q_{crit}$ when the additional assumption $C_2 = 0$ is adopted. This, however, is an arbitrary, ad-hoc assumption which is not required by the asymptotic condition $\theta(\infty) = 0$. In other words, the θ-solution (8) is a proper unique solution only for $Q \geq 0$. In the range $Q_{crit} < Q < 0$, however, it is a particular ad-hoc solution, and in the range $Q < Q_{crit}$ a complex, non-physical solution. From the above considerations we may conclude that the full solution of temperature boundary value problem can be written in the form.

$$\theta(Y) = \begin{cases} e^{-m_1 Y}, & \text{when } Q \geq 0 \\ e^{-\frac{Pr}{2}Y} \dfrac{\cos h\left[\left(\sqrt{PrQ + \left(\frac{Pr}{2}\right)^2}\right)Y + \alpha\right]}{\cos h\,\alpha}, & \text{when } Q_{crit} < Q < 0 \\ 1 + \alpha Y e^{-\frac{Pr}{2}Y}, & \text{when } Q = Q_{crit} \\ e^{-\frac{Pr}{2}Y} \dfrac{\cos\left[\left(\sqrt{Pr|Q| - \left(\frac{Pr}{2}\right)^2}\right)Y + \alpha\right]}{\cos\alpha}, & \text{when } Q < Q_{crit} \end{cases} \quad (12)$$

where α stands for a constant of integration which cannot be determined from the boundary conditions $\theta(0) = 1$, $\theta(\infty) = 0$ when $Q < 0$. Thus, except for the range $Q \geq 0$, we always have a one-parameter family of non-unique solutions. In the range $Q < Q_{crit}$ of a "*massive heat generation*" the corresponding solution (12) oscillates between positive and negative values, and thus it is always non-physical (in the presence of heat generation, the fluid temperature cannot become smaller than T_∞ when $T_w > T_\infty$).

Obviously, the non-uniqueness of the solutions (12) in the range $Q < 0$ due to the presence of the arbitrary constant α is transferred also in the corresponding Nusselt numbers $Nu = -\theta'(0)$ which read

$$Nu = \begin{cases} \dfrac{Pr}{2} + \sqrt{Pr\,Q + \left(\frac{Pr}{2}\right)^2}, & \text{when } Q \geq 0 \\ \dfrac{Pr}{2} - \sqrt{\mathrm{Pr}\,Q + \left(\frac{Pr}{2}\right)^2}\,\tan h\,\alpha, & \text{when } Q_{crit} < Q < 0 \\ \dfrac{Pr}{2} - \alpha, & \text{when } Q = Q_{crit} \\ \dfrac{Pr}{2} + \sqrt{Pr|Q| - \left(\frac{Pr}{2}\right)^2}\,\tan h\,\alpha, & \text{when } Q < Q_{crit} \end{cases} \quad (13)$$

It is also worth emphasizing here that for vanishing value of the vortex viscosity R, Eqs. (1) and (2) also become decoupled from each other. In this case Eqs. (1) and (2) can be integrated easily, so that the problem further simplifies substantially. On this reason we assume hereinafter in this paper that $R \neq 0$. In this case, Eq. (2) yields.

$$U' = \left(\lambda N'' + N' - 2M\,R\,N\right)/(M\,R) \quad (14)$$

and owing to the second boundary condition (5), the skin-friction coefficient C_f can be expressed in terms of $U'(0)$ alone as

$$C_f = (1 + R - n\,R)U'(0) \quad (15)$$

Substituting Eq. (14) in Eq. (1) we obtain for the micro-rotation N the *non-homogeneous* differential equation of third order.

$$\lambda(1+R)N''' + (1+R+\lambda)N'' + \left(1 - 2M\,R - M\,R^2\right)N' - 2M\,R\,N = -M\,R(Gr_T\,\theta + Gr_C\,\varphi) \quad (16)$$

Once Eq. (16) is solved for N, the velocity solution U results by a simple term-by-term integration of Eq. (14). A comprehensive investigation of this problem would require a discussion of Eq. (16) with respect to all the four cases of the θ-solution (12), combined with the similar structure of the φ-solution. Since such an analysis would be very extensive and even repetitive, we restrict the following discussions to the physical temperature and solutions only. These correspond to the first case of the θ-solution (12) which is unique and holds for $Q \geq 0$. Similarly, we also assume $\gamma > 0$. In this case, substituting in Eq. (16)

$$N\,Y = W\,Y + K_1\,e^{-m_1\,Y} + K_2\,e^{-m_2\,Y} \quad (17)$$

where

$$K_{1,2} = \frac{M\,R\,Gr_{T,C}}{\lambda(1+R)m_{1,2}^3 - (1+R+\lambda)m_{1,2}^2 + \left(1 - 2M\,R - M\,R^2\right)m_{1,2} + 2M\,R} \quad (18)$$

we obtain for the new unknown function $W(Y)$ the homogeneous differential equation.

$$\lambda(1+R)W''' + (1+R+\lambda)W'' + \left(1 - 2M\,R - M\,R^2\right)W' - 2M\,R\,W = 0 \quad (19)$$

To solve Eq. (19) we set $W(Y) = e^{-mY}$ and arrive to the characteristic equation.

$$m^3 + a_2 m^2 + a_1 m + a_0 = 0 \quad (20)$$

where

$$a_2 = -\frac{1+R+\lambda}{\lambda(1+R)}, \quad a_1 = \frac{1 - 2M\,R - M\,R^2}{\lambda(1+R)}, \quad a_0 = \frac{2M\,R}{\lambda(1+R)} \quad (21)$$

Let m_3, m_4, m_5 denote the three roots of the characteristic equation (20). When all three roots are distinct, the general solution of Eq. (19) is a linear combination of $e^{-m_3 Y}$, $e^{-m_4 Y}$ and $e^{-m_5 Y}$, and the general solution of Eq. (16) can be written in the form.

$$N\,Y = \sum_{i=1}^{5} K_i e^{-m_i Y} \tag{22}$$

where K_1 and K_2 are given by Eqs. (18) and the other three coefficients K_i are yet undetermined. The nature of the roots m_3, m_4, m_5 depends on the value of the discriminant

$$\Delta = \left(\frac{a_1}{3} - \frac{a_2^2}{9}\right)^3 + \left(\frac{a_0}{2} - \frac{a_1 a_2}{6} + \frac{a_2^3}{27}\right)^2 \tag{23}$$

It can be shown that for $\lambda = 1$ and any $M > 0$, the discriminant (23) is negative for all $R > 0$, which means that all three roots m_3, m_4, m_5 are real. Moreover, for the parameter values $\lambda = M = 1$ and $R = 1,2,3,4,5$ (used throughout in [32]), two of the roots, m_3 and m_4, say, are positive and m_5 negative (see Table 1). Thus, the asymptotic condition $N(\infty) = 0$ requires $K_5 = 0$, so that Eq. (22) involves in fact only two unknown coefficients, K_3 and K_4. Accordingly, Eq. (14) and (15) become.

$$U'(Y) = -\sum_{i=1}^{4} A_i K_i e^{-m_i Y} \tag{24}$$

$$C_f = -(1 + R - n\,R)\sum_{i=1}^{4} A_i K_i, \tag{25}$$

where

$$A_i = \left(2M\,R + m_i - \lambda\,m_i^2\right)\Big/(M\,R), \quad i = 1,2,3,4 \tag{26}$$

Integrating Eq. (24) once and bearing in mind asymptotic condition $U(\infty) = 0$ we obtain the velocity solution,

$$U(Y) = \sum_{i=1}^{4}\left(\frac{A_i}{m_i} K_i e^{-m_i\,Y}\right) \tag{27}$$

In this way, Eqs (27), (24), (22) and the first two boundary conditions (5) give for determination of K_3 and K_4 the equations

$$\begin{aligned} &\frac{A_3}{m_3}K_3 + \frac{A_4}{m_4}K_4 = 1 - \left(\frac{A_1}{m_1}K_1 + \frac{A_2}{m_2}K_2\right), \\ &(1-nA_3)K_3 + (1-nA_4)K_4 = -(1-nA_1)K_1 - (1-nA_2)K_2 \end{aligned} \tag{28}$$

When its determinant.

$$D = \frac{A_3}{m_3}(1 - n\,A_4) - \frac{A_4}{m_4}(1 - n\,A_3) \tag{29}$$

Table 1
Values of the discriminant Δ and of the roots of the characteristic equation for $\lambda = M = 1$ and the indicated values of the vortex viscosity parameter R.

R	1	2	3	4	5
Δ	−0.1829	−0.9241	−2.4049	−4.8216	−8.3841
m_3	0.6446	0.4857	0.3910	0.3275	0.2817
m_4	1.7446	2.1340	2.4346	2.6893	2.9147
m_5	−0.8892	−1.2864	−1.5756	−1.8167	−2.0297

is non-vanishing, the solution of the system of linear algebraic equations (28) for K_3 and K_4 is unique and reads

$$\begin{aligned} K_3 &= \frac{(1-n\,A_4)\left(1 - \frac{A_1}{m_1}K_1 - \frac{A_2}{m_2}K_2\right) + \frac{A_4}{m_4}[(1-n\,A_1)K_1 + (1-n\,A_2)K_2]}{\frac{A_3}{m_3}(1-n\,A_4) - \frac{A_4}{m_4}(1-n\,A_3)}, \\ K_4 &= -\frac{(1-n\,A_3)\left(1 - \frac{A_1}{m_1}K_1 - \frac{A_2}{m_2}K_2\right) + \frac{A_3}{m_3}[(1-n\,A_1)K_1 + (1-n\,A_2)K_2]}{\frac{A_3}{m_3}(1-n\,A_4) - \frac{A_4}{m_4}(1-n\,A_3)} \end{aligned} \tag{30}$$

In this way, for specified values of the ten physical parameters R, M, λ, Gr_T, Gr_C, Pr, Q, Sc, γ and n, all the K_i's and m_i's are known. The velocity solution $U = U(Y)$ is given by Eq. (27), the micro-rotation solution $N = N(Y)$ by Eq. (22), and the skin-friction coefficient by Eq. (25).

3.2. The symmetry with respect to the thermal and concentration variables

A simple inspection of the governing equations and boundary conditions (1)–(6) shows that our basic boundary value problem remains invariant when θ and φ, Gr_T and Gr_C, Pr and Sc, as well as Q and γ are interchanged, respectively. In other words, the results remain unchanged when the corresponding thermal and concentration variables and parameters are interchanged simultaneously. In particular, the Nusselt and Schmidt numbers change their roles in this case. On this reason it is sufficient to discuss the results with respect to the velocity and micro-rotation on the one hand, and either the temperature or concentration field, on the other hand. This is the objective of the next Section.

3.3. Discussion of the results

Equation (1) shows that the velocity field U is coupled to the temperature and concentration fields by the thermal and chemical buoyancy terms $Gr_T\,\theta$ and $Gr_C\,\varphi$, respectively. When both of these buoyancy effects are negligible, $Gr_T = Gr_C = 0$, the flow is driven only by the wall motion, but the velocity and micro-rotation fields U and N remain still coupled to each other by the cross-terms $R\,N'$ and $2M\,RN$ of Eqs. (1) and (2), as well as by the boundary condition $N(0) = -n\,U'(0)$.

On the other hand, when the buoyancy effects are not negligible, they aid the wall driven flow when both Gr_T and Gr_C are positive, or oppose it when Gr_T and Gr_C are both negative. Obviously, physically also an intermediary situation can occur when the signs of Gr_T and Gr_C are opposite. In this case, one of the buoyancy forces aids the flow and the other one opposes it.

To be specific, let us discuss first the general results obtained in Section 3.1 as functions of the heat absorption parameter Q, setting for the other nine parameters of the problem the values.

$$M = \lambda = Gr_{\mathrm{T}} = Gr_{\mathrm{C}} = 1,\ R = 5,\ \gamma = n = 0.5,\quad Pr = 0.71,\ Sc = 2 \tag{31}$$

These parameter values correspond to the situation in which both buoyancy forces aid the wall driven flow.

The governing parameter Q is involved in the general equations of Section 3.1 via the expression (8) of m_1 directly, and in A_1, K_1, K_3 and K_4 via m_1 indirectly. The roots m_3, m_4, m_5 and the discriminant Δ, which don't depend on Q, are given in Table 1 and the values of the other relevant quantities (which are also independent of Q) are $m_2 = Sh = 2.4142$, $K_2 = -0.1757$, $A_2 = 1.3172$, $A_3 = 2.0405$, $A_4 = 0.8839$ and $D = 4.0481$.

The shapes of the velocity, micro-rotation and temperature profiles as functions of the transverse coordinate Y and, in particular, in the neighborhood of the wall, belong to the basic characteristics of the flow. These features are illustrated in Figs. 1–4. In Fig. 1 the velocity gradient at the wall $U'(0)$ (i.e. the dimensionless wall shear stress), the skin-friction coefficient C_f as well as the micro-rotation at the wall $N(0)$ are plotted as functions of the absorption parameter Q for the fixed values (31) of the other parameters of the problem. While $U'(0)$ and C_f decrease, $N(0)$ increases with increasing values of Q. All three curves intersect the Q-axis and change sign the same point $Q_0 = 2.397419$ (see also Table 2). The effect of this behavior on the full velocity and micro-rotation profiles is shown in Figs. 2 and 3, respectively. For the sake of more transparency, the shapes of $U(Y)$ and $N(Y)$ near to the wall have also been shown in a magnified "windows" of the respective figures. In the range $Q < Q_0$ the velocity profile shows an "overshooting" shape, increasing from the prescribed wall-value $U(0) = 1$ to a maximum which is reached at a certain distance from the wall. In this *overshooting range*, both $U'(0)$ and C_f are positive, i.e. the wall shear stress due to the flow and the external stretching force of the continuous surface point in the same direction. At $Q = Q_0$, the wall shear stress becomes zero and the corresponding velocity curve has a horizontal tangent at the wall (see the window). For larger values than Q_0 of the heat absorption parameter Q, the wall shear stress becomes negative, i.e. opposite to the stretching force of the surface. The effect of Q on the rotation of the micro-rotation vector is shown in Fig. 3. One sees that with increasing distance from the wall the ability of the microelements of the fluid to rotate also increases, and always reaches a maximum value at a certain distance Y. Then, the rotation of the microelements decreases to the vanishing asymptotic value $N(\infty) = 0$ smoothly as $Y \to \infty$. At $Q = Q_0$ where the wall shear stress is zero, the microrotaion also vanishes and the rotation sense of the microelements at the wall changes from negative to positive with increasing Q (see also Table 2). At the same time, the temperature profiles become steeper (increasing Nusselt number, $Nu = -\theta'(0) = m_1$) with increasing values of Q, which is shown in Fig. 4 (see also Table 2).

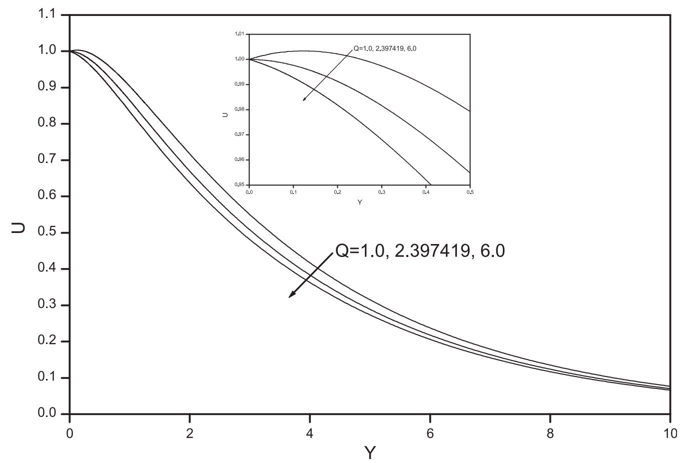

Fig. 2. Effect of the heat absorption parameter Q on the velocity profiles $U(Y)$, for the fixed values (31) of the other parameters of the problem.

When both the thermal and chemical buoyancy effects are negligible, $Gr_T = Gr_C = 0$, Eqs. (18) imply that $K_1 = K_2 = 0$ and thus Eqs. (22) and (30) yield.

$$N(0) = K_3 + K_4 = \frac{n(A_3 - A_4)}{\dfrac{A_3}{m_3}(1 - n\,A_4) - \dfrac{A_4}{m_4}(1 - n\,A_3)} \tag{32}$$

Since this expression of $N(0)$ does not include m_1, it is independent of the heat absorption parameter Q. According to the boundary condition $N(0) = -n\,U'(0)$, the same holds also for $U'(0)$. Taking for the parameters (other than Gr_T and Gr_C) the values (31), we obtain $N(0) = 0.1428$ and $U'(0) = -0.2857$.

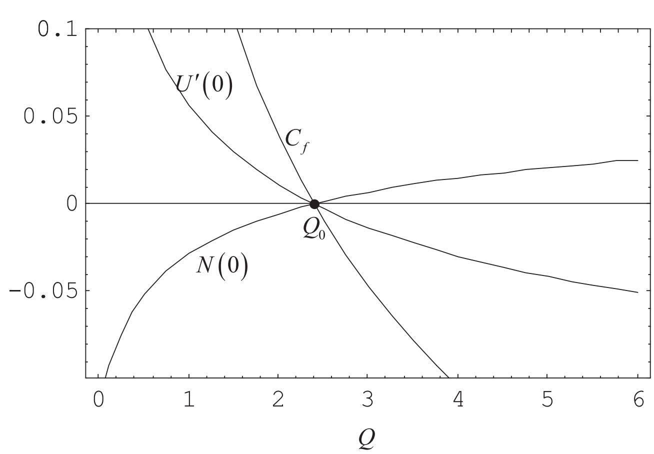

Fig. 1. Shown are the velocity gradient at the wall $U'(0)$, the friction coefficient C_f as well as the wall-value $N(0)$ of the micro-rotation as functions of the absorption parameter Q for the fixed values (31) of the other parameters of the problem. The three curves intersect the Q-axis in the same point $Q_0 = 2.397419$ (see also Table 2).

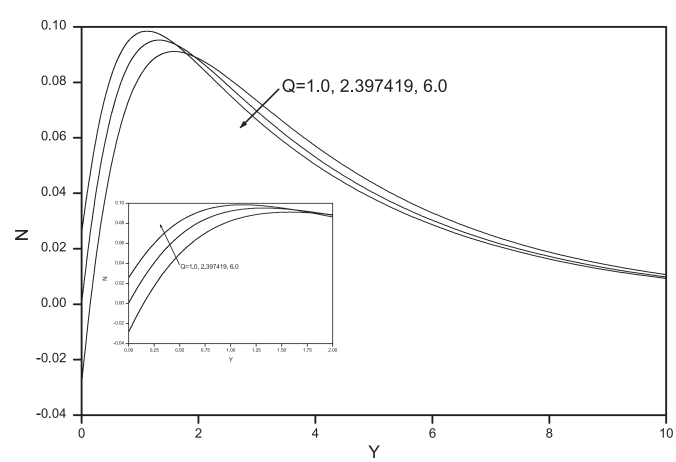

Fig. 3. Effect of the heat absorption parameter Q on the micro-rotation profiles $N(Y)$, for the fixed values (31) of the other parameters of the problem.

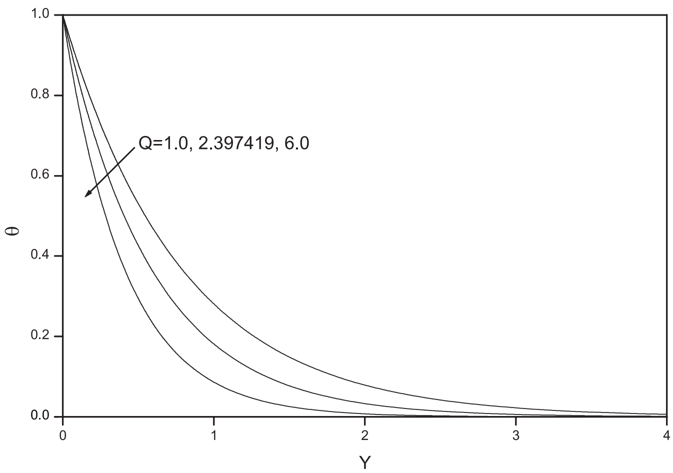

Fig. 4. Effect of the heat absorption parameter Q on the temperature profiles $\theta(Y)$, for the fixed values (31) of the other parameters of the problem.

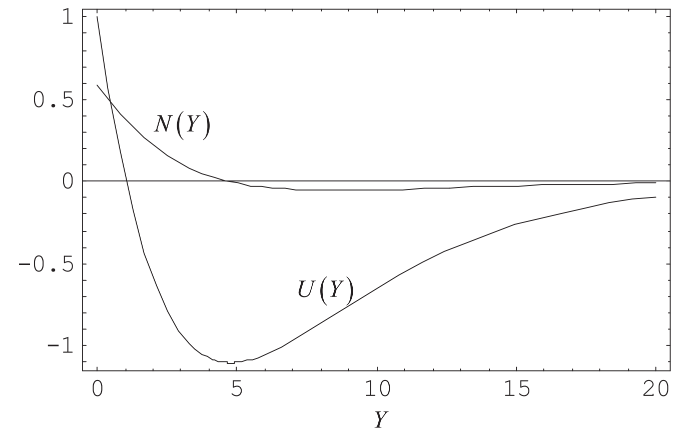

Fig. 5. The opposing effect of the chemical buoyancy leads to an inversion of the wall driven flow and, after a certain distance from the wall, also changes the rotation sense of the microelements. The parameter values used in the calculations are $M = \lambda = Gr_T = 1,\ Gr_C = -1,\ R = 5,\ n = 0.5,\ Pr = 1,\ Q = 0.5, Sc = 0.25, \gamma = 0.1$.

The corresponding velocity and micro-rotation profiles decrease from their wall values $U(0) = 1$ and $N(0) = 0.1428$ monotonically to zero as the coordinate Y increases from zero to infinity. This result shows at the same time, that the occurrence of the maxima in the profiles $U(Y)$ and $N(Y)$ plotted in Figs. 2 and 3 is the joint effect of the aiding thermal and chemical buoyancy forces.

Concerning the possible opposing effect of the buoyancy, we consider here the special case $Gr_T = -Gr_C = 1$ shortly. Obviously, when in addition to this condition also the equalities $\Pr = Sc$ and $Q = \gamma$ hold, then $\theta(Y) = \varphi(Y)$ and the buoyancy terms in Eq. (1) cancel each other. Thus, with respect to the velocity and micro-rotation fields, the present case becomes basically identical to the case $Gr_T = Gr_C = 0$ discussed above. The flow is driven only by the wall motion and, as a consequence, both $U(Y)$ and $N(Y)$ are monotonically decreasing functions of Y. When, on the other hand, at least one of the conditions $\Pr = Sc$ and $Q = \gamma$ are relaxed, in Eq. (1) an effective buoyancy force $Gr_T(\theta - \varphi)$ occurs. When the thickness $\delta_1 = 1/m_1$ of the temperature boundary layer is larger than the thickness $\delta_2 = 1/m_2$ of the concentration boundary layer, the effective buoyancy force $Gr_T(\theta - \varphi)$ aids the wall driven flow and, similarly to Figs. 2 and 3, in the velocity and micro-rotation profiles local maxima occur. When, however, the thickness δ_2 of the concentration boundary layer is larger than the thickness δ_1 of the of the temperature boundary layer, the effective buoyancy force $Gr_T(\theta - \varphi)$ opposes the wall driven flow and in the velocity and micro-rotation profiles local minima occur. At the same time, after some distances from the wall, the flow direction as well as the rotation sense of microelements become reversed. Such a situation is illustrated in Fig. 5, where the thickness of the of the concentration boundary layer is more that five times larger than that of the temperature boundary layer, $\delta_2/\delta_1 = 5.27$.

Table 2
Sign change of $U'(0)$, $N(0)$ and C_f as the heat absorption parameter Q passes the value $Q_0 = 2.397419$ (see also Fig. 1). The values of the other nine parameters are given by Eq. (31). The Nusselt number increases monotonically with Q.

Q	$U'(0)$	$N(0)$	C_f	$Nu = m_1$
1	0.057721	−0.028860	0.202022	1.26934
2.397419	0	0	0	1.70711
6	−0.050715	0.025358	−0.177504	2.44928

4. Summary and conclusions

The numerical study [32] of the combined effect of heat generation or absorption, a first-order chemical reaction as well as of the buoyancy forces on micropolar fluid flows over a uniformly stretched permeable surface has been revisited in the present paper. The exact solution of the problem has been given in a closed analytical form. The new features emerging from this approach can be summarized as follows.

1. The temperature solutions θ are unique only in the presence of the volumetric heat absorption ($Q > 0$) in the fluid. In the case of heat generation ($Q < 0$), a one-parameter family of non-unique temperature solutions exist. Below the critical value $Q_{crit} = -Pr/4$ of Q the solutions become oscillatory and even non-physical. Accordingly, the Nusselt number is only unique for $Q \geq 0$.
2. The same holds for the concentration solutions φ. They are unique only for reactant consuming ($\gamma > 0$) chemical reactions. In the case of reactant producing ($\gamma < 0$) reactions, no unique concentration solutions exists and in the range $\gamma < \gamma_{crit} = -Sc/4$ the solutions become oscillatory and thus non-physical. The Sherwood number is only unique for $\gamma \geq 0$.
3. The governing boundary value problem remains invariant when θ and φ, Gr_T and Gr_C, Pr and Sc, as well as Q and γ are interchanged, respectively. This implies that all the results of the paper remain unchanged when the corresponding thermal and concentration variables and parameters are interchanged simultaneously.
4. For a typical set of parameter values (specified by Eq. (31)) the dimensionless wall shear stress $U'(0)$ and the skin-friction coefficient C_f decrease, while the micro-rotation at the wall $N(0)$ increases with increasing values of Q. At a certain value $Q = Q_0$ of the absorption parameter, the characteristic quantities $U'(0)$, C_f and $N(0)$ become simultaneously vanishing and change sign (see Fig. 1 and Table 2).
5. In the range $Q < Q_0$ of the absorption parameter, the velocity profiles $U(Y)$ show an "overshooting" shape, increasing from the prescribed wall-value $U(0) = 1$ to a maximum which is reached at a certain distance from the wall. At $Q = Q_0$, where the wall shear stress becomes zero and the corresponding velocity curve has a horizontal tangent at the wall

(see the window). For larger values than Q_0 of the heat absorption parameter Q, the wall shear stress becomes negative, i.e. opposite to the stretching force of the surface (see Fig. 2).

6. With increasing distance from the wall the ability of the microelements of the fluid to rotate also increases and the microotation vector $N(Y)$ reaches a maximum value at a certain distance Y from the wall. Then, the rotation of the microelements decreases to the vanishing asymptotic value $N(\infty) = 0$ smoothly as $Y \to \infty$. At $Q = Q_0$ where the wall shear stress is zero, the microrotaion also vanishes and the rotation sense of the microelements at the wall changes from negative to positive with increasing Q (see Fig. 3 and Table 2).
7. With increasing values of Q, the temperature profiles θ become steeper, which implies that the Nusselt number, $Nu = -\theta'(0) = m_1$ increases monotonically (see Fig. 4 and Table 2).
8. The features described under the above points 5–7 correspond to the situation when both the thermal and the chemical buoyancy forces aid the wall driven flow, $Gr_T,\ Gr_C > 0$. When at least one of these forces becomes negative, e.g., $Gr_C < 0$, and in addition the thickness of the concentration boundary layer (in this case) is larger than the thickness of the of the temperature boundary layer, then the effective buoyancy force $Gr_T(\theta - \varphi)$ opposes the wall driven flow and in the velocity and micro-rotation profiles local minima occur. At the same time, after some distances from the wall, the flow direction as well as the rotation sense of microelements become reversed (see Fig. 5).

We may conclude once more that the analytical solution of a problem emphasizes features of the flow behavior which in a numerical approach, although basically accessible, in general remain still hidden.

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Nomenclature

A_i: constants $i = 1, 2, 3, 4$, Eq. (26),
a_0, a_1, a_2: coefficients, Eqs. (21)
C_f: skin-friction coefficient
D: determinant, Eq. (29)
Gr_C: mass Grashof number
Gr_T: thermal Grashof number
K_1, K_2: coefficients, Eq. (18)
K_3, K_4: coefficients, Eqs. (30)
M: dimensionless scaled material parameter
n: dimensionless micro-gyration parameter
m_1, m_2: characteristic roots, Eqs. (8)
m_3, m_4, m_5: roots of characteristic equation (20)
N: dimensionless micro-rotation variable
Nu: Nusselt number
Pr: Prandtl number
Q: dimensionless heat generation or absorption parameter
Q_{crit}: critical heat generation parameter

R: vortex viscosity parameter
T: temperature
Sc: Schmidt number
Sh: Sherwood number
U: dimensionless velocity
W: solution of Eq. (19)
Y: dimensionless transverse coordinate

Greek symbols

α: constant of integration, Eq.(129)
Δ: discriminant, Eq. (23)
λ: dimensionless material parameter
θ: non-dimensional temperature
φ: non-dimensional concentration
γ: dimensionless chemical reaction parameter