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Analytical solutions for hydromagnetic natural convection flow of a particulate suspension through isoflux–isothermal channels in the presence of a heat source or sink

Ali J. Chamkha *, Seham S. Al-Rashidi

Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh 70654, Kuwait

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ABSTRACT

This work considers the problem of steady natural convection hydromagnetic flow of a particulate suspension through an infinitely long channel in the presence of heat generation or absorption effects. The channel walls are maintained at isoflux–isothermal condition. That is, the thermal boundary conditions are such that one of the channel walls is maintained at constant heat flux while the other is maintained at a constant temperature. Various closed-form solutions of the governing equations for different special cases are obtained. A parametric study of the physical parameters involved in the problem is done to illustrate the influence of these parameters on the velocity and temperature profiles of both phases.

1. Introduction

Natural convection flow of a two-phase (fluid/particle) suspension represents one of the most interesting and challenging areas of research in heat transfer. Such flows are found in a wide range of applications including processes in the chemical and food industries, solar collectors where a particulate suspension is used to enhance absorption of radiation, cooling of electronic equipments, and cooling of nuclear reactors. Very little work has been done on natural convection for a two-phase particulate suspension. Most work on natural convection flows within vertical parallel-plate channels are done only for a single phase. The evolution of cooling technology includes the progressive research of using natural convection, which is an inexpensive mode of heat transfer in electronic equipments cooling. Vertical plates and channels are of the most encountered configurations used in natural convection cooling of electronic equipment.

A literature review in general for the historical papers reported in the development of cooling technology for electronic equipments has been presented by Bergles [6]. Later, an extensive review of electronic equipment cooling by different modes of heat transfer has been presented by Incorpera [9]. The review includes natural convection heat transfer in parallel channels, inclined channels and enclosures as well as other configurations with different operating conditions. The importance of heat transfer considerations in the design of electronic equipment has been studied extensively and reported by Aung and Chaimah [4], Jaluria [10], Kraus and Bar-Cohen [12], and Steinberg [17].

Akbari and Borgers [1] studied free convection laminar heat transfer between the channel surfaces of the Trombe wall. The study was done using a line-by-line forward marching implicit finite-difference technique. The study was restricted to laminar flow between two parallel plates, each at some effective uniform temperature. Yao [18] investigated the problem of mixed convection in vertical channel. An analytical solution is developed to study the hydrodynamically and thermally developing laminar flow in a heated channel. The transient effects in natural convection cooling of vertical parallel plates are reported by Joshi [11]. Aung [3] considered fully developed laminar free convection between vertical plates heated asymmetrically. Aung et al. [5] reported on the development of laminar free convection between vertical flat plates with asymmetric heating. A more detailed reference list was given by Muhanna [14] who investigated numerically laminar natural convection flows in obstructed vertical channels. Related references for natural and mixed convection flows of a single phase are given in the book by Gebhart et al. [8].

On the other hand, very little work has been reported on natural convection flow of a particle–fluid suspension over and through different geometries. Chamkha and Ramadan [7] and Ramadan and Chamkha [16] have reported some analytical and numerical results for natural convection flow of a two-phase particulate suspension over an infinite vertical plate. Also, Okada and Suzuki [15] have considered buoyancy-induced flow of a two-phase

* Corresponding author.
E-mail address: achamkha@yahoo.com (A.J. Chamkha).

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Nomenclature

$\vec{B}$	magnetic induction
c	fluid-phase specific heat at constant pressure
c_p	particle-phase specific heat at constant pressure
g	gravitational acceleration
Gr	Grashof number
h	channel width
H	dimensionless buoyancy parameter
k	fluid-phase thermal conductivity
M	Hartmann number
N	interphase momentum transfer coefficient
N_T	interphase heat transfer coefficient
P	fluid-phase hydrostatic pressure
Pr	fluid-phase Prandtl number
Q	heat generation/absorption coefficient
q_1	wall heat flux
r_{qt}	walls thermal ratio
T	fluid-phase temperature
T_p	particle-phase temperature
u	fluid-phase dimensionless velocity
u_p	particle-phase dimensionless velocity
U	fluid-phase velocity
U_p	particle-phase velocity
x, y	cartesian coordinates

Greek symbols

α	velocity inverse Stokes number
γ	specific heat ratio
ε	temperature inverse Stokes number
η	dimensionless y-coordinate
θ	dimensionless fluid-phase temperature
κ	particle loading
μ	fluid-phase dynamic viscosity
ρ	fluid-phase density
ρ_p	particle-phase density
σ	fluid-phase electrical conductivity
ϕ	dimensionless heat generation/absorption coefficient

suspension in an enclosure. Al-Subaie and Chamkha [2] performed an analytical study dealing with natural convection flow of a particulate suspension through a vertical channel with isothermal walls. However, the present authors were unable to locate any theoretical or experimental work in the literature dealing with natural convection laminar flow of a particulate suspension in isoflux–isothermal vertical channels. This is the objective of the present work. In the formulation of the general problem, magnetic effects which affect the flow if the fluid is electrically conducting and heat generation or absorption effects which are important in situations where a heat source or sink may be placed within the flow are included.

2. Problem formulation

Consider steady, laminar, natural convection flow of a particulate suspension in a vertical parallel-plate channel. The channel walls are maintained at the isoflux–isothermal condition. The schematic of the problem is shown in Fig. 1. The fluid phase is assumed to be Newtonian, viscous, electrically conducting, and heat generating or absorbing. The particle phase is assumed to be made up of discrete particles of one size and constant density. The particle phase is assumed to be pressure-less and electrically non-conducting. Both phases are assumed to be interacting continua. The governing equations for this investigation are based on the balance laws of mass, linear momentum and energy for both the fluid and particle phases. For small volume fraction of particles [13], they can be written in vector form as

$$\vec{\nabla}\cdot\left(\rho\vec{V}\right)=0 \quad (1)$$

$$\rho\vec{V}\cdot\vec{\nabla}\vec{V}=-\vec{\nabla}P+\vec{\nabla}\cdot\left(\mu\vec{\nabla}\vec{V}\right)-\rho_pN\left(\vec{V}_p-\vec{V}\right)+\rho\vec{g}+\sigma(\vec{V}\times\vec{B})\times\vec{B} \quad (2)$$

$$\rho c\vec{V}\cdot\vec{\nabla}T=\vec{\nabla}\cdot\left(k\vec{\nabla}T\right)+\rho_pc_pN_T(T_p-T)\pm Q(T-T_o) \quad (3)$$

$$\vec{\nabla}\cdot\left(\rho_pV_p\right)=0 \quad (4)$$

$$\rho_p\vec{V}_p\cdot\vec{\nabla}\vec{V}_p=\rho_pN(\vec{V}_p-\vec{V})+\rho_p\vec{g} \quad (5)$$

$$\rho_pc_p\vec{V}\cdot\vec{\nabla}T_p=-\rho_pc_pN_T(T_p-T) \quad (6)$$

where $\vec{V}$ and $\vec{V}_p$ are the velocity vectors of the fluid and particle phases, respectively. T and T_p are the temperatures of the fluid and particle phases, respectively. $\vec{g}$ is the gravity vector, $\vec{\nabla}P$ is the pressure gradient vector, T_o is the temperature at a reference point "o" in the channel, $\vec{B}$ is the magnetic induction vector and Q is the heat generation or absorption coefficient depending on its sign. The other parameters, namely, ρ, μ, σ, c and k are the density, dynamic viscosity, electrical conductivity, specific heat and thermal conductivity of the fluid phase, while, ρ_p and c_p are the particle-

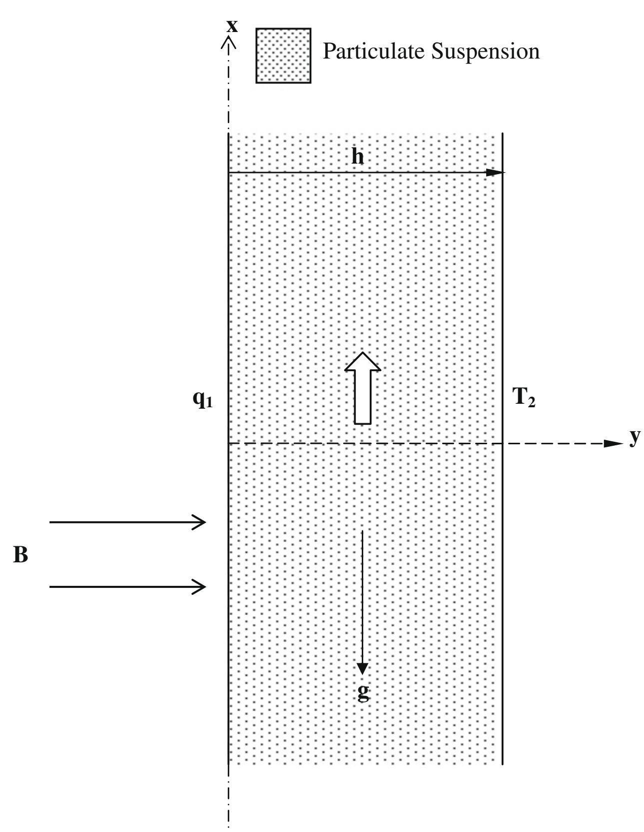

Fig. 1. Schematic of the problem.

phase density and specific heat, respectively. It should be mentioned that in the previous equations, the terms containing the momentum transfer coefficient N represent the interphase drag between the phases while the terms containing the heat transfer coefficient N_T represent the interphase heat transfer between the fluid and the particle phases.

Evaluating the governing equations at a reference point "o" at the channel inlet such that $\vec{V}=0,\ T=T_o,\ \rho=\rho_o,\ \mu=\mu_o,\ \sigma=\sigma_o, \vec{V}_p=\vec{V}_{po}, T_p=T_{po},$ and $\rho_p=\rho_{po}$ yields

- Fluid-phase momentum equation

$$0=-\vec{\nabla}P-\rho_{po}N\left(0-\vec{V}_{po}\right)+\rho_o\vec{g} \tag{7}$$

Rearranging Eq. (7) yields

$$-\vec{\nabla}P=\rho_{po}N\left(0-\vec{V}_{po}\right)-\rho_o\vec{g} \tag{8}$$

- Fluid-phase energy equation becomes

$$\rho_{po}c_{po}N_T(T_{po}-T_o)=0 \tag{9}$$

which gives $T_{po}=T_o$.

- Particle-phase momentum equation

$$\rho_{po}N\left(0-\vec{V}_{po}\right)=-\rho_{po}\vec{g} \tag{10}$$

- Particle-phase energy equation:

$$0=-\rho_{po}c_{po}N_T(T_{po}-T_o) \tag{11}$$

which also yields $T_{po}=T_o$.

Substituting Eq. (10) into (8) gives

$$-\vec{\nabla}P=-\rho_{po}\vec{g}-\rho_o\vec{g} \tag{12}$$

By assuming that all properties of the suspension are constant (except the fluid density in the buoyancy term of the fluid-phase linear momentum equation) the Boussinesq approximation can be adequately employed. Using Eq. (12), eliminating the pressure gradient from the fluid-phase momentum equation and employing the Boussinesq approximation gives:

$$\begin{aligned}\vec{V}.\vec{\nabla}\vec{V}=&-\frac{\rho_{po}}{\rho_o}\vec{g}+\frac{\mu_o}{\rho_o}\vec{\nabla}.\vec{\nabla}\vec{V}-\frac{\rho_{po}}{\rho_o}N\left(\vec{V}-\vec{V}_p\right)\\&-\beta^*(T-T_o)\vec{g}+\frac{\sigma_o}{\rho_{po}}\left(\vec{V}\times\vec{B}\right)\times\vec{B}\end{aligned} \tag{13}$$

where β^* is the volumetric expansion coefficient. The linear momentum equation of the fluid phase, Eq. (2), will be replaced now by Eq. (13) in the governing equations.

The walls of the channel are assumed to be infinitely long. This implies that the dependence of the variables on the x-direction will be negligible compared with that of the y-direction (see Fig. 1). Therefore, all dependent variables in Eqs. (3)–(6) and (12) will only be functions of y as follows:

$$\vec{V}=U(y)\vec{e}_x,\quad \vec{V}_p=U_p(y)\vec{e}_x \tag{14a, b}$$

$$T=T(y),\quad T_p=T_p(y) \tag{14c, d}$$

where $U(y)$ is the fluid phase x-component of velocity, $U_p(y)$ is the particle-phase x-component of velocity, $T(y)$ and $T_p(y)$ are the fluid phase and particle-phase temperatures, respectively and $\vec{e}_x$ is the unit vector in the x-direction. These assumptions also imply that the constant vector $\vec{g}$ will be reduced to $g\vec{e}_x$ which is the magnitude of the acceleration due to gravity component in the x-direction. Also, assuming that the fluid is electrically-conducting and is subjected to a uniform transverse magnetic field which is applied normally to the flow direction (see Fig. 1), the electromotive force $\sigma\vec{V}\times\vec{B}$ in Eq. (13) will provide a current whose interaction with $\vec{B}$ will decelerate the flow. This implies that

$$\sigma\left(\vec{V}\times\vec{B}\right)\times\vec{B}=-\sigma B^2U(y)\vec{e}_x \tag{15}$$

where $\vec{B}$ is the magnitude of magnetic induction. Taking all of the above assumptions into account, the governing equations of the infinite parallel-plate channel can be rewritten as follows:

$$\mu_o\partial_{yy}U-\rho_{po}N(U-U_p)-\sigma_oB^2U+\beta^*\rho_og(T-T_o)+\rho_{po}g=0 \tag{16}$$

$$k\partial_{yy}T+\rho_pc_pN_T(T_p-T)\pm Q(T-T_o)=0 \tag{17}$$

$$\rho_pN(U-U_p)-\rho_pg=0 \tag{18}$$

$$T_p-T=0 \tag{19}$$

It should be noted that the continuity equations of both phases are identically satisfied.

The physical boundary conditions for this problem are:

$$U(0)=U(h)=0 \tag{20a, b}$$

$$\frac{\partial T}{\partial y}(0)=-\frac{q_1}{k},\quad T(h)=T_2 \tag{20c, d}$$

where h is the channel width, T_2 is the channel temperature at $y = h$, q_1 is the wall heat flux at $y = 0$.

Substituting the following parameters:

$$y=h\eta,\ U=\frac{\mu}{\rho h}u,\ U_p=\frac{\mu}{\rho h}u_p,\ T=\frac{q_1h}{k}\theta+T_o,$$
$$T_p=\frac{q_1h}{k}\theta_p+T_o \tag{21}$$

into Eqs. (16)–(19) gives the following dimensionless equations:

$$\partial_{\eta\eta}u-\alpha\kappa(u-up)-M^2u+\mathrm{Gr}\theta+\kappa H=0 \tag{22}$$

$$\frac{1}{\mathrm{Pr}}\partial_{\eta\eta}\theta+\kappa\gamma\varepsilon(\theta_p-\theta)\pm\phi\theta=0 \tag{23}$$

$$\alpha(u-u_p)-H=0 \tag{24}$$

$$\theta_p-\theta=0 \tag{25}$$

where

$$\alpha=h^2\frac{N\rho}{\mu},\quad \kappa=\frac{\rho_p}{\rho},\quad \mathrm{Gr}=\frac{\beta^*q_1h^4\rho^2g}{\kappa\mu^2},\quad M^2=\frac{\sigma B^2h^2}{\mu},$$
$$H=\frac{\rho^2h^3g}{\mu^2},\quad \mathrm{Pr}=\frac{\mu c}{\kappa},\quad \gamma=\frac{c_p}{c},\quad \varepsilon=\frac{\rho N_Th^2}{\mu},\quad \phi=\frac{Qh^2}{\mu c} \tag{26}$$

are the momentum inverse Stokes number, the particle loading, the Grashof number, square of Hartmann number, buoyancy parameter, the Prandtl number, the specific heat ratio, the temperature inverse Stokes number and the heat generation or absorption parameter, respectively.

The dimensionless boundary conditions are

$$u(0)=u(1)=0 \tag{27a, b}$$

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}=-1,\quad \theta(1)=r_{qt} \tag{27c, d}$$

where $r_{qt}=\frac{T_2-T_o}{q_1h/k}$ is the walls thermal ratio.

3. Analytical solutions

In this section, analytical solutions for various special cases of the problem under consideration are reported.

Case 1. This case considers steady natural convection two-phase particle/fluid flow through an isoflux–isothermal vertical channel in the absence of magnetic field ($M = 0$) and heat generation or absorption ($\theta = 0$) effects. The governing equations for this case can be written as follows:

$$D^2u - \alpha\kappa(u - u_p) + \mathrm{Gr}\theta + \kappa H = 0 \tag{28}$$

$$\frac{1}{\mathrm{Pr}}D^2\theta + \kappa\varepsilon\gamma(\theta_p - \theta) = 0 \tag{29}$$

$$\alpha(u - u_p) - H = 0 \tag{30}$$

$$\theta_p - \theta = 0 \tag{31}$$

where the D^2 denotes a second derivative operator with respect to η.

The boundary conditions are:

$$u(0) = u(1) = 0 \tag{32a, b}$$

$$D\theta(0) = -1, \quad \theta(1) = r_{qt} \tag{32c, d}$$

Eq. (30) implies that

$$u_p(\eta) = u(\eta) - \frac{H}{\alpha} \tag{33}$$

which indicates that the particle-phase velocity is the same as the fluid-phase velocity except that it is shifted by the factor $\frac{H}{\alpha}$ below the fluid-phase velocity. Eq. (31) implies that

$$\theta_p(\eta) = \theta(\eta) \tag{34}$$

By substituting Eq. (34) into Eq. (29) one obtains

$$D^2\theta = 0 \tag{35}$$

The solution of this simple second order differential equation, which satisfies the boundary conditions (32c,d) is

$$\theta(\eta) = -\eta + r_{qt} + 1 \tag{36}$$

This indicates that the temperature of both phases has a linear shape of pure conduction.

Now, substituting Eqs. (36) and (33) into Eq. (28) gives

$$D^2u = \mathrm{Gr}\eta - \mathrm{Gr}(r_{qt} + 1) \tag{37}$$

The solution of this second order differential equation subject to the boundary conditions (32a,b) is

$$u(\eta) = \frac{\mathrm{Gr}[\eta^3 - 3(1 + r_{qt})\eta^2 + (2 + 3r_{qt})\eta]}{6} \tag{38}$$

This shows that the fluid-phase velocity profile has a cubic relation with the normal distance. The corresponding solution for $u_p(\eta)$ is obtained by substituting Eq. (38) into Eq. (33) to get

$$u_p(\eta) = \frac{\mathrm{Gr}[\eta^3 - 3(1 + r_{qt})\eta^2 + (2 + 3r_{qt})\eta]}{6} - \frac{H}{\alpha} \tag{39}$$

Case 2. This case considers steady natural convection two-phase particle/fluid flow through an isoflux–isothermal vertical channel subjected to a uniform transverse field which is applied normally to the flow direction and the flow has neither heat generation nor heat absorption effects ($\phi = 0$). The governing equations and the boundary conditions will be the same as Case 1 except the fluid-phase momentum equation, which can be written as

$$D^2u - \alpha\kappa(u - u_p) + \mathrm{Gr}\theta - M^2u + \kappa H = 0 \tag{40}$$

Substituting Eqs. (33) and (36) into Eq. (40) gives

$$D^2u - M^2u = -\mathrm{Gr}(-\eta + r_{qt} + 1) \tag{41}$$

Solving the above equation subject to (32a,b) gives the following fluid-phase velocity:

$$u(\eta) = c_1e^{M\eta} + c_2e^{-M\eta} + \frac{\mathrm{Gr}(-\eta + r_{qt} + 1)}{M^2} \tag{42}$$

where

$$c_1 = \left(\frac{\mathrm{Gr}}{M^2}\right)\left[\frac{(r_{qt} + 1)e^{-M} - r_{qt}}{e^M - e^{-M}}\right] \tag{43}$$

$$c_2 = -\left(\frac{\mathrm{Gr}}{M^2}\right)\left[\frac{(r_{qt} + 1)e^{M} - r_{qt}}{e^M - e^{-M}}\right] \tag{44}$$

The corresponding particle-phase velocity profile $u_p(\eta)$ can be written as

$$u_p(\eta) = c_1e^{M\eta} + c_2e^{-M\eta} + \frac{\mathrm{Gr}(-\eta + r_{qt} + 1)}{M^2} - \frac{H}{\alpha} \tag{45}$$

Fig. 2 shows the linear relationship between the fluid-phase and particle- phase temperature profiles and r_{qt} which is decreasing at the second wall. It is observed that as r_{qt} increases, the temperature profiles increase.

Figs. 3 and 4 present typical velocity profiles for the fluid and particle phases for different values of r_{qt}, respectively. The increase in the fluid-phase temperature due to increases in the value of r_{qt} cause the thermal buoyancy effects to increase. This induces higher flow velocities of both phases in the channel as shown in Figs. 3 and 4.

Figs. 5 and 6 display the effects of increasing the Grashof number Gr on the velocity fields of both the fluid and particle phases, respectively. Increases in the values of Gr increase the thermal buoyancy effect represented by the Grθ term of Eq. (28). This gives rise to an increase in the flow of both phases in the channel. This term represents the driving force of flow such that when Gr = 0 no flow will take place in the channel.

Figs. 7 and 8 present velocity profiles for the fluid and particle phases for various values of the Hartmann number M, respectively. As the strength of the magnetic field increases, the magnetic Lorentz force which is a force that opposes the flow direction increases. This causes the movement of both the fluid and the particle phases to slow down as shown in Figs. 7 and 8.

Case 3. This case considers steady natural convection two-phase flow through an isoflux–isothermal vertical channel in the presence of a heat source ($\phi > 0$) and a magnetic field. The governing equations and the boundary conditions for this case will be the

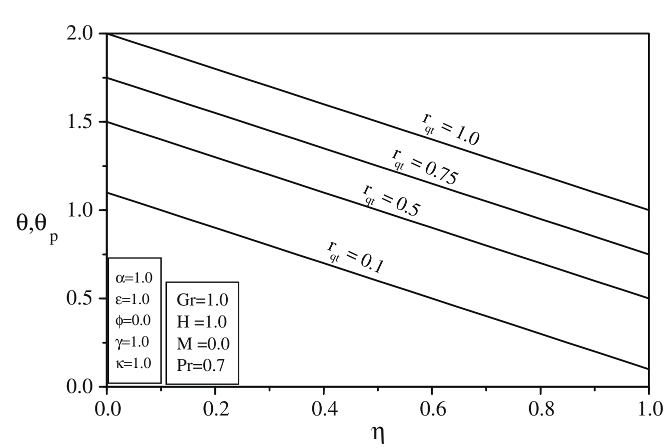

Fig. 2. Effects of r_{qt} on fluid and particle phases temperature profiles.

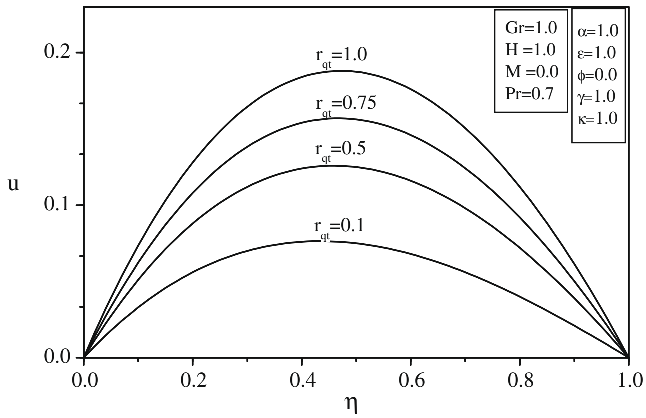

Fig. 3. Effects of r_{qt} on fluid-phase velocity profile.

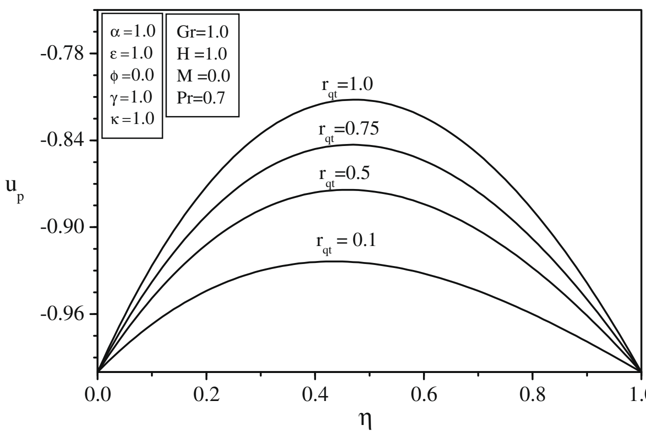

Fig. 4. Effects of r_{qt} on particle-phase velocity profile.

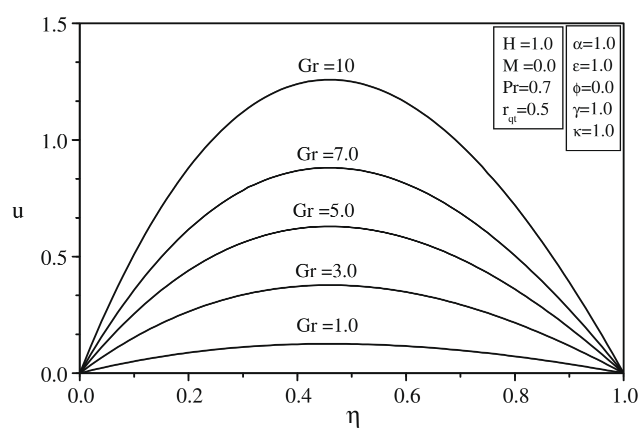

Fig. 5. Effects of Gr on fluid-phase velocity profile.

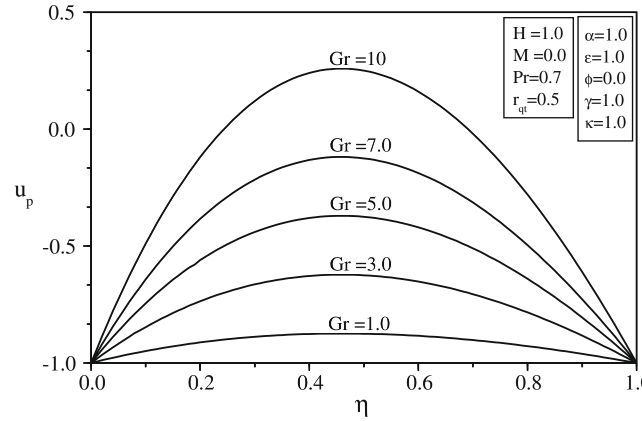

Fig. 6. Effects of Gr on particle-phase velocity profile.

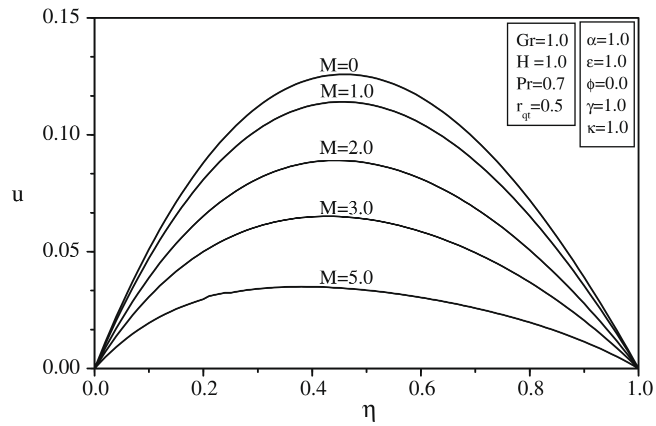

Fig. 7. Effects of M on fluid-phase velocity profile.

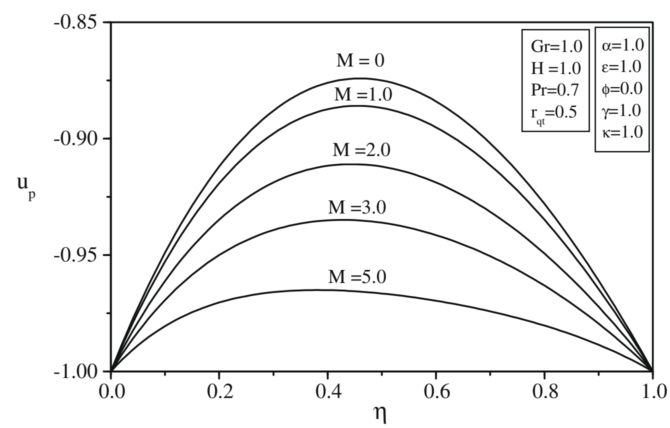

Fig. 8. Effects of M on particle-phase velocity profile.

same as Case 2 except the fluid-phase energy equation, which can be written as follows:

$$\left(\frac{1}{\mathrm{Pr}}\right)D^2\theta + \kappa\gamma\varepsilon(\theta_p - \theta) - \phi\theta = 0 \tag{46}$$

Substituting Eq. (34) into (46) and solving the resulting differential equation subject to (32c,d) gives the following fluid-phase temperature:

$$\theta(\eta) = \theta_p(\eta) = \left(r_{qt} + \frac{\sinh\sqrt{\phi\mathrm{Pr}}}{\sqrt{\phi\mathrm{Pr}}}\right)\frac{\cosh\sqrt{\phi\mathrm{Pr}}\eta}{\cosh\sqrt{\phi\mathrm{Pr}}} - \frac{\sinh\sqrt{\phi\mathrm{Pr}}\eta}{\sqrt{\phi\mathrm{Pr}}} \tag{47}$$

Substituting Eqs. (33) and (47) into (40) yields

$$D^2u - M^2u = -\mathrm{Gr}\theta(\eta) \tag{48}$$

Solving this differential equation subject to the boundary conditions (32a,b) gives the following fluid-phase velocity:

$$u(\eta)=\frac{-\mathrm{Gr}\left(r_{qt}\sqrt{\phi\mathrm{Pr}}+\sinh\sqrt{\phi\mathrm{Pr}}\right)}{\sqrt{\phi\mathrm{Pr}}(\phi\mathrm{Pr}-M^2)}\times\left[\frac{\sinh M(\eta-1)-\cosh\sqrt{\phi\mathrm{Pr}}\sinh M\eta+\sinh M\cosh\sqrt{\phi\mathrm{Pr}}\eta}{\sinh M}\right]+\frac{\mathrm{Gr}}{\sqrt{\phi\mathrm{Pr}}(\phi\mathrm{Pr}-M^2)}\times\left[\frac{\sinh\sqrt{\phi\mathrm{Pr}}\eta\sinh M-\sinh\sqrt{\phi\mathrm{Pr}}\sinh M\eta}{\sinh M}\right] \quad (49)$$

The particle-phase velocity profile is obtained by substituting Eq. (49) into Eq. (33).

This solution can be used to obtain the corresponding problem without a magnetic field ($M = 0$). Thus, the fluid-phase velocity of such problem will be obtained by setting M equals to zero and applying L'Hospital's rule to Eq. (49). If this is done, one obtains

$$u(\eta)=-\mathrm{Gr}\left(r_{qt}\sqrt{\phi\mathrm{Pr}}+\sinh\sqrt{\phi\mathrm{Pr}}\right)\times\frac{\left(\eta\left(1-\cosh\sqrt{\phi\mathrm{Pr}}\right)+\cosh\sqrt{\phi\mathrm{Pr}}\eta-1\right)}{(\phi\mathrm{Pr})^{\frac{3}{2}}}+\mathrm{Gr}\frac{\sinh\sqrt{\phi\mathrm{Pr}}\eta-\eta\sinh\sqrt{\phi\mathrm{Pr}}}{(\phi\mathrm{Pr})^{\frac{3}{2}}} \quad (50)$$

Case 4. This case considers steady natural convection two-phase through an isoflux–isothermal vertical channel in the presence of a heat sink ($\phi < 0$) and a magnetic field. The governing equations and boundary conditions will be the same as in Case 2 except the fluid-phase energy equation, which can be expressed as:

$$\left(\frac{1}{\mathrm{Pr}}\right)D^2\theta+\kappa\gamma\varepsilon(\theta_p-\theta)+\phi\theta=0 \quad (51)$$

Substituting Eq. (34) into Eq. (51) and solving the resulting differential equation gives the following fluid-phase temperature:

$$\theta(\eta)=\left[\frac{r_{qt}+\left(1/\sqrt{\phi\mathrm{Pr}}\right)\sin\sqrt{\phi\mathrm{Pr}}}{\cos\sqrt{\phi\mathrm{Pr}}}\right]\cos\sqrt{\phi\mathrm{Pr}}\eta-\left(1/\sqrt{\phi\mathrm{Pr}}\right)\sin\sqrt{\phi\mathrm{Pr}}\eta \quad (52)$$

By substituting Eqs. (33) and (52) into Eq. (40) and rearranging one obtains

$$D^2u-M^2u=-\mathrm{Gr}\theta(\eta) \quad (53)$$

Eq. (53) can be solved subject to the flow boundary conditions given in Eqs. (32a,b) by the usual method of solving such equation to give the following fluid-phase velocity:

$$u(\eta)=\frac{\mathrm{Gr}\left[r_{qt}+\frac{\sin\sqrt{\phi\mathrm{Pr}}}{\sqrt{\phi\mathrm{Pr}}}\right]}{(\phi\mathrm{Pr}+M^2)\cos\sqrt{\phi\mathrm{Pr}}}\left[\frac{\sinh M(\eta-1)}{\sinh M}+\cos\sqrt{\phi\mathrm{Pr}}\eta-\frac{\cos\sqrt{\phi\mathrm{Pr}}\eta\sinh M\eta}{\sinh M}\right]-\frac{\mathrm{Gr}}{\sqrt{\phi\mathrm{Pr}}(\phi\mathrm{Pr}+M^2)}\left[\sin\sqrt{\phi\mathrm{Pr}}\eta-\frac{\sin\sqrt{\phi\mathrm{Pr}}\sinh M\eta}{\sinh M}\right] \quad (54)$$

Again, this solution can be used to solve the corresponding problem in the absence of a magnetic field. Setting M equals to zero and applying L'Hospital's rule to Eq. (54) will give the fluid-phase velocity for this problem as:

$$u(\eta)=\frac{\mathrm{Gr}\left[r_{qt}+\frac{1}{\sqrt{\phi\mathrm{Pr}}}\sin\sqrt{\phi\mathrm{Pr}}\right]}{\phi\mathrm{Pr}\cos\sqrt{\phi\mathrm{Pr}}}\left(\left(1-\cos\sqrt{\phi\mathrm{Pr}}\right)\eta+\cos\sqrt{\phi\mathrm{Pr}}\eta-1\right)-\mathrm{Gr}\left(\sin\sqrt{\phi\mathrm{Pr}}\eta-\eta\sin\sqrt{\phi\mathrm{Pr}}\eta\right)\Big/(\phi\mathrm{Pr})^{3/2} \quad (55)$$

Some results for θ, θ_p, u and u_p based on the closed-form solutions for the flow through a vertical channel in the presence of heat generation (source) or a heat absorption (sink) are presented in

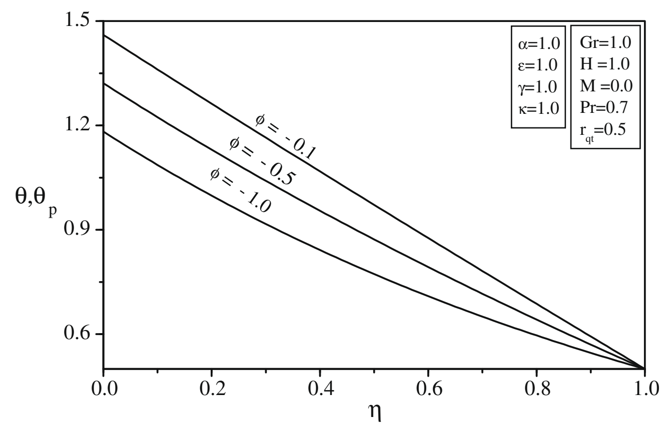

Fig. 9. Effects of ϕ on fluid and particle-phase temperature profiles.

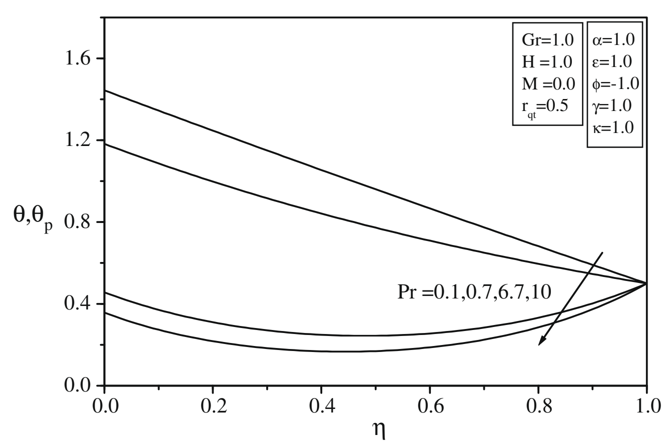

Fig. 10. Effects of Pr on fluid and particle-phase temperature profile.

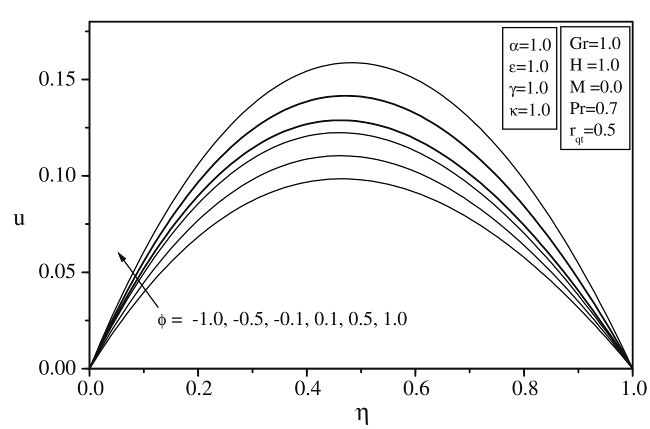

Fig. 11. Effects of ϕ on fluid-phase velocity profile.

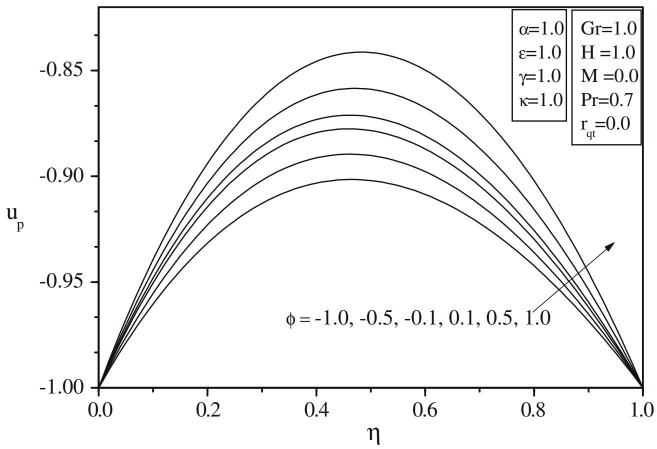

Fig. 12. Effects of ϕ on particle-phase velocity profile.

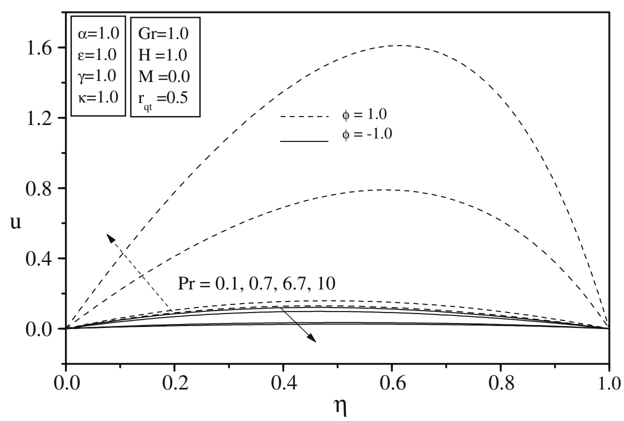

Fig. 13. Effects of Pr on fluid-phase velocity profile.

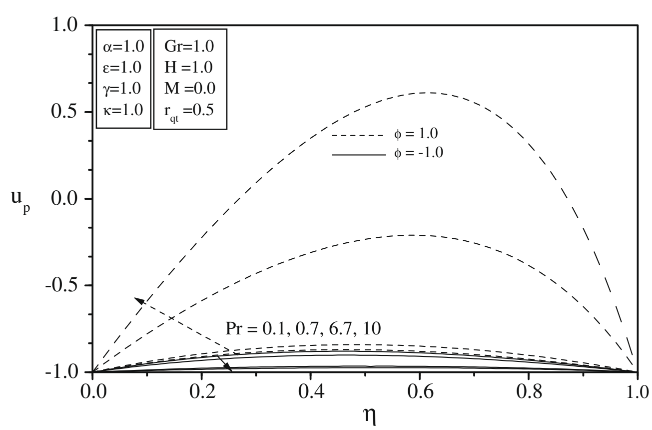

Fig. 14. Effects of Pr on particle-phase velocity profile.

Figs. 9–14. Fig. 9 shows that θ and θ_p increase as ϕ increases such that they increase for heat generation ($\phi > 0$) and decrease for heat absorption effects ($\phi < 0$). Fig. 10 shows that both θ and θ_p decrease as Pr increases for $\phi = -1$.

Figs. 11 and 12 show representative velocity profiles for both phases (u and u_p) for different values of the heat generation or absorption coefficient ϕ. It is depicted that both u and u_p increase as ϕ increases due to the reasons mentioned before.

The effects of the Prandtl number on the velocity profiles of both phases for heat generation or absorption conditions are shown in Figs. 13 and 14, respectively. As observed in Fig. 10 the thermal states of both phases decrease as Pr increases in the presence of heat absorption effects and increase in the presence of heat generation. This causes the velocity profiles of both phases to increase as Pr increases in the presence of heat generation and to decrease in the presence of heat absorption as clearly shown in the figures.

4. Conclusions

The problem of MHD natural convection flow through a vertical parallel-plate isoflux–isothermal channel in the presence of magnetic field and heat generation or absorption effects was considered. The governing equations were non-dimensionalized and solved analytically. Closed-form solutions for four special and general cases were obtained. Representative results were presented graphically to illustrate the influence of the physical parameters on the solutions. The conclusions based on the results of the parametric study performed for the cases mentioned above can be summarized as follows:

1. In the absence of viscous and magnetic dissipation, drag work, and heat generation or absorption, the temperature profiles of both phases were identical and had a linear shape of pure conduction.
2. An increase in the values of wall thermal ratio increased the thermal buoyancy effect which, consequently, increased the flow of both phases in the channel.
3. Increasing in the values of the Grashof number which represents the driving force led to increases in the flow of both phases.
4. The magnetic field had the effect of reducing the velocity of the fluid-phase which, in turn, reduced the particle-phase velocity.
5. Increases in the values of the heat generation (or absorption) coefficient caused the temperature to increase (decrease).
6. The thermal states of both phases decreased as Pr increased in the presence of heat absorption effects and increased in the presence of heat generation.

It is hoped that the analytical results obtained in this project be used as a vehicle for understanding natural convection in two-phase suspensions and a stimulus for experimental work.

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