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EXACT SOLUTIONS OF UNSTEADY SOLUTE DISPERSION OF MAGNETOHYDRODYNAMIC AND VISCOUS FLUIDS BETWEEN TWO PARALLEL PLATES

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ABSTRACT

The dispersion of a solute between two parallel plates consisting of two regions, one electrically conducting and another electrically non-conducting is investigated analytically using Taylor's model [1]. Fluids in both the regions are incompressible and their transport properties are assumed to be constant. The continuity conditions for the velocity and shear stress at the interface between the two fluid layers are assumed. Results are presented graphically for various physical parameters such as pressure gradients, heights of the channel, viscosity, diffusivity and Hartmann number on the concentration. The volumetric flow rate for different values of viscosity and Hartmann number is also found. The validity of the results obtained for two fluid model is verified by comparison with the available one fluid model in the literature and good agreement is found.

Keywords: Taylor dispersion, two fluid model, horizontal channel, MHD.

1. INTRODUCTION

Taylor [1-3] initiated the investigation of the dispersion of a soluble matter in a non-conducting viscous fluid flowing through a circular tube under laminar conditions. His result show that the soluble matter can be regarded as dispersing along the tube with an apparent diffusion coefficient $\left(R^2\,\bar{V}\,x^2\right)/\left(48\,D\right)$, where R is the radius of the circular tube and D is the molecular diffusion coefficient. He has also shown that the condition under which his analysis is valid is $\left(4L\right)/R \gg \left(\bar{V}\,x\,R\right)/D \gg 6.4$, L being the length in the flow direction. Aris [4] extended Taylor's results and established that the rate of growth of variance of the solute distribution is proportional to the sum of the molecular diffusion coefficient and Taylor diffusion coefficient. His analysis removed the restriction imposed by Taylor. Subsequently Taylor's analysis was extended to the case of different models of non-Newtonian fluids by Fan and Hwang [5], Fan and Wang [6], Ghoshal [7], Ghoshal et al. [8]. Gupta and Chatterjee [9] investigated the dispersion of soluble matter in the hydromagnetic laminar flow between two parallel plates.

The flow and heat transfer aspects of immiscible fluids is of special importance in the petroleum extraction and transport. For example, the reservoir rock of an oil field always contains several immiscible fluids in its pores. Part of the pore volume is occupied by water and the rest may be occupied either by oil or gas or both. Crude oils often contain dissolved gasses which may be released into the reservoir rock when the pressure decreases. These examples show the importance of knowledge of the laws governing immiscible multi-phase flows for proper understanding of the processes involved. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. In general, multi-phase flows are driven by gravitational and viscous forces. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe [10-13]. Loharsbi and Sahai [14] studied two-phase MHD flow and heat transfer in a parallel plate channel with one of the fluids being electrically conducting. Two-phase MHD flow and heat transfer in an inclined channel was investigated by Malashetty and Umavathi [15]. Chamkha [16] reported analytical solutions for flow of two-immiscible fluids in porous and non-porous parallel-plate channels. Later on, magnetohydrodynamic two-fluid convective flow and heat transfer in composite porous medium was analyzed by Malashetty et al. [17-19].

The analytical solution for the dispersion of a solute in an electrically conducting fluid flowing between two parallel plates in the presence of transverse magnetic field using Taylor's model was studied by Gupta and Chatterjee [9]. Exact analysis of unsteady magnetohydrodynamics convective diffusion was analyzed by Annapurna and Gupta [20] using Gill and Subramanian model.

The aim of the present paper is to study the dispersion of a solute in a parallel plate channel filled with composite electrically conducting fluid using Taylor's model.

2. MATHEMATICAL FORMULATION

Consider the laminar flow of two immiscible fluids between two parallel plates distant $(h_1 + h_2)$ apart, taking x-axis along the mid-section of the channel and y-axis perpendicular to the walls as shown in Fig. 1. An uniform magnetic field of strength B_0 is applied in the y-direction. Region-1 $(-h_1 \le y_1 \le 0)$ is filled with electrically conducting fluid viscosity μ_1 under a uniform pressure gradient $\frac{dp_1}{dx_1}$. The region-2 $(0 \le y_2 \le h_2)$ is occupied by a different (immiscible) electrically non-conducting viscous fluid viscosity μ_2 under a uniform pressure gradient $\frac{dp_2}{dx_2}$.

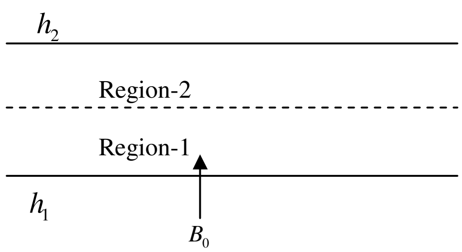

Fig. 1 Physical Configuration

It is assumed that the fluids are incompressible and the flow is steady, laminar, fully developed, and that fluid properties are constant. The flow in both regions is assumed to be driven by a common constant pressure gradients. The transport properties of both the fluids are assumed to be constant. Under these assumptions, the governing equations of motion are (Loharsbi, and Sahai [14])

Region-1

$$\frac{d^2u_1}{dy_1^2} - \frac{\sigma_{e_1} B_0^2}{\mu_1} u_1 = P_1 \qquad (1)$$

Region-2

$$\frac{d^2u_2}{dy_2^2} = \frac{1}{\mu_2}\frac{dp_2}{dx_2} \qquad (2)$$

where u_i is the x-component of the fluid velocity, x_i is the co-ordinate and p_i is the pressure. The subscripts 1 and 2 denote the values for region-1 and region-2 respectively.

The boundary conditions on velocity are no-slip conditions requiring that the velocity must vanish at the walls. In addition, continuity of velocity and shear stress at the interface is assumed. With these assumptions, the boundary and interface conditions on velocity become

$$\begin{aligned} &u_1 = 0 \quad at \quad y_1 = -h_1 \\ &u_2 = 0 \quad at \quad y_2 = h_2 \\ &u_1 = u_2 \quad at \quad y_1 = y_2 = 0 \\ &\frac{\mu_1}{h_1}\frac{du_1}{dy_1} = \frac{\mu_2}{h_2}\frac{du_2}{dy_2} \quad at \quad y_1 = y_2 = 0 \end{aligned} \qquad (3)$$

3. SOLUTIONS

Solutions of "Eq.(1) and (2)" using boundary and interface conditions (3) become

$$u_1 = l_1 \cosh M\eta_1 + l_2 \sinh M\eta_1 - \frac{P_1 h_1^2}{M^2} \qquad (4)$$

$$u_2 = \frac{P_2 h_2^2}{2}\eta_2^2 + l_3 h_2 \eta_2 + l_4 \qquad (5)$$

where $P_1 = \frac{1}{\mu_1}\frac{dp_1}{dx_1}$; $P_2 = \frac{1}{\mu_2}\frac{dp_2}{dx_2}$

Region-1

The average velocity in region-1 is defined by

$$\bar{u}_1 = \int_{-1}^{0} u_1 \, d\eta_1$$

Using the solution of u_1 from "Eq.(4)", the average velocity become

$$\bar{u}_1 = \frac{l_1 \sinh M}{M} + \frac{l_2(1-\cosh M)}{M} - \frac{P_1 h_1^2}{M^2} \qquad (6)$$

Region-2

The average velocity in region-2 is defined by

$$\bar{u}_2 = \int_{0}^{1} u_2 \, d\eta_2$$

Using the solution of u_2 from "Eq.(5)", the above equation become

$$\bar{u}_2 = \frac{P_2 h_2^2}{6} + \frac{l_3 h_2}{2} + l_4 \qquad (7)$$

We assume that a solute diffuses in the absence of first-order irreversible chemical reaction in the liquid under isothermal conditions. The equation for the concentration C_1 of the solute for the first region satisfies

$$\frac{\partial C_1}{\partial t_1} + u_1 \frac{\partial C_1}{\partial x_1} = D_1\left(\frac{\partial^2 C_1}{\partial x_1^2} + \frac{\partial^2 C_1}{\partial y_1^2}\right) \qquad (8)$$

Similarly, the equation for the concentration C_2 of the solute for the second region satisfies

$$\frac{\partial C_2}{\partial t_2} + u_2 \frac{\partial C_2}{\partial x_2} = D_2\left(\frac{\partial^2 C_2}{\partial x_2^2} + \frac{\partial^2 C_2}{\partial y_2^2}\right) \qquad (9)$$

in which D_1 and D_2 are the molecular diffusion coefficients (assumed constants) for the first and second region respectively. We now assume that the longitudinal diffusion is much less than the transverse diffusion i.e.,

$$\frac{\partial^2 C_1}{\partial x_1^2} << \frac{\partial^2 C_2}{\partial y_1^2} \text{ and } \frac{\partial^2 C_2}{\partial x_2^2} << \frac{\partial^2 C_2}{\partial y_2^2}$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane the fluid velocity is given by

$$u_{1x} = u_1 - \bar{u}_1 = l_1 \cosh M\eta_1 + l_2 \sinh M\eta_1 + l_5 \qquad (10)$$

For Region-1 and

$$u_{2x} = u_2 - \bar{u}_2 = P_2\left(\frac{h_2^2\eta_2^2}{2} - \frac{h_2^2}{6}\right) + l_3\left(h_2\eta_2 - \frac{h_2}{2}\right) \qquad (11)$$

For Region-2.

Introducing the dimensionless quantities

$$\theta_1 = \frac{t_1}{\bar{t}_1},\ \bar{t}_1 = \frac{L_1}{u_1},\ \xi_1 = \frac{x_1 - \bar{u}_1 t_1}{L_1},\ \eta_1 = \frac{y_1}{h_1},\ \theta_2 = \frac{t_2}{\bar{t}_2},$$
$$\bar{t}_2 = \frac{L_2}{u_2},\ \xi_2 = \frac{x_2 - \bar{u}_2 t_2}{L_2},\ \eta_2 = \frac{y_2}{h_2},\ M = \sqrt{\frac{\sigma_e}{\mu_1}}\, B_0\, h_1 \tag{12}$$

"Equations (8) and (9)" using the non-dimensional quantities as defined in "Eq.(12)" become

Region-1

$$\frac{1}{t_1}\frac{\partial C_1}{\partial \theta_1} + \frac{u_{1x}}{L_1}\frac{\partial C_1}{\partial \xi_1} = \frac{D_1}{h_1^2}\frac{\partial^2 C_1}{\partial \eta_1^2} \tag{13}$$

Region-2

$$\frac{1}{t_2}\frac{\partial C_2}{\partial \theta_2} + \frac{u_{2x}}{L_2}\frac{\partial C_2}{\partial \xi_2} = \frac{D_2}{h_2^2}\frac{\partial^2 C_2}{\partial \eta_2^2} \tag{14}$$

where L_1 and L_2 are the typical lengths along the flow direction for the first and second regions respectively. Following Taylor [1], we now assume that partial equilibrium is established in any cross-section of the channel so that the variations of C_1 with η_1 and C_2 with η_2 from "Eq.(13)" and "Eq.(14)" become

Region-1

$$\frac{\partial^2 C_1}{\partial \eta_1^2} = \frac{h_1^2}{D_1 L_1} u_{1x} \frac{\partial C_1}{\partial \xi_1} \tag{15}$$

Region-2

$$\frac{\partial^2 C_2}{\partial \eta_2^2} = \frac{h_2^2}{D_2 L_2} u_{2x} \frac{\partial C_2}{\partial \xi_2} \tag{16}$$

Substituting "Eq.(10)" and "Eq.(11)" in "Eq.(15)" and "Eq.(16)" and integrating twice, we get the solutions for C_1 and C_2 after using the conditions $\partial C_1/\partial \eta_1 = 0$ at $\eta_1 = -1$, $\partial C_2/\partial \eta_2 = 0$ at $\eta_2 = 1$ and the entry conditions which are given by

Region-1

$$C_1 = \frac{h_1^2}{D_1 L_1}\frac{\partial C_1}{\partial \xi_1}\left(\frac{l_1 \cosh M\eta_1}{M^2} + \frac{l_2 \sinh M\eta_1}{M^2} + \frac{l_5 \eta_1^2}{2}\right) + l_6 \eta_1 \tag{17}$$

Region-2

$$C_2 = \frac{h_2^2}{D_2 L_2}\frac{\partial C_2}{\partial \xi_2}\left(\frac{P_2 h_2^2}{2}\left(\frac{\eta_2^4}{12} - \frac{\eta_2^2}{6}\right) + l_3 h_2\left(\frac{\eta_2^3}{6} - \frac{\eta_2^2}{4}\right)\right) \tag{18}$$

where

$$l_1 = \frac{(mhM)\dfrac{P_1 h_1^2}{M^2} - \left(\dfrac{P_2 h_2^2}{2} - \dfrac{P_1 h_1^2}{M^2}\right)\sinh M}{(mhM\cosh M + \sinh M)},$$

$$m = \frac{\mu_1}{\mu_2},\ h = \frac{h_2}{h_1},$$

$$l_2 = l_1 \coth M - \frac{P_1 h_1^2}{M^2}\frac{1}{\sinh M},$$

$$l_3 = \frac{\mu_1}{\mu_2}\frac{1}{h_1} l_2 M,\quad l_4 = l_1 - \frac{P_1 h_1^2}{M^2},$$

$$l_5 = \frac{-l_1 \sinh M}{M} - \frac{l_2(1-\cosh M)}{M}$$

$$l_6 = \frac{h_1^2}{D_1 L_1}\frac{\partial C_1}{\partial \xi_1}\left(\frac{l_1 \sinh M}{M} - \frac{l_2 \cosh M}{M} + l_5\right).$$

The volumetric rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) are

$$\begin{aligned} Q_1 = \int_{-1}^{0} h_1 C_1 u_{1x}\, d\eta_1 = \frac{h_1^3}{D_1 L_1}\frac{\partial C_1}{\partial \xi_1}\Bigg[&\frac{l_1^2}{2M^2}\left(1 + \frac{\sinh 2M}{2M}\right) \\ &+ l_1 l_2\left(\frac{1-\cosh 2M}{2M^3}\right) + \frac{l_2^2}{2M^2}\left(\frac{\sinh 2M}{2M} - 1\right) \\ &+ l_5 l_2\left(\frac{2(1-\cosh M)}{M^3} - \frac{\cosh M}{2M} + \frac{\sinh M}{M^2}\right) + \frac{l_5^2}{6} \\ &+ l_1 l_5\left(\frac{2\sinh M}{M^3} - \frac{\cosh M}{M^2} + \frac{\sinh M}{2M}\right)\Bigg] \\ &+ l_6 l_1 h_1\left(\frac{(\cosh M - 1)}{M^2} - \frac{\sinh M}{M}\right) \\ &+ l_6 l_2 h_1\left(\frac{\cosh M}{M} - \frac{\sinh M}{M^2}\right) + \frac{l_5 l_6 h_1}{2} \end{aligned} \tag{19}$$

$$\begin{aligned} Q_2 = \int_{0}^{1} h_2 C_2 u_{2x}\, d\eta_2 = &-\frac{h_2^7 P_2^2}{D_2 L_2}\frac{\partial C_2}{\partial \xi_2}\left(\frac{2}{945}\right) \\ &- \frac{P_2 h_2^6 l_3}{D_2 L_2}\frac{\partial C_2}{\partial \xi_2}\left(\frac{1}{120}\right) - \frac{h_2^5 l_3^2}{D_2 L_2}\frac{\partial C_2}{\partial \xi_2}\left(\frac{1}{120}\right) \end{aligned} \tag{20}$$

The total volumetric rate Q can be computed using $Q = Q_1 + Q_2$.

4. RESULTS AND DISCUSSION

The problem of longitudinal dispersion of a solute in a channel filled with purely viscous and electrically conducting immiscible fluids is studied using Taylor's dispersion model. The average velocity for two different immiscible fluids is obtained using no slip conditions in the boundary and continuity of velocity and shear stress at the interface. The closed form solutions are obtained for the concentration in region-1 and region-2. The volumetric flow rate in both the regions are also analyzed. The effects of governing parameters such as pressure gradient, height of the fluid regions, viscosity of the fluids, diffusivity coefficients and Hartmann number on the solute concentrations are evaluated numerically and shown graphically in Figs. 2-11. The physical parameters are fixed as 1 except the varying parameters in all the graphs.

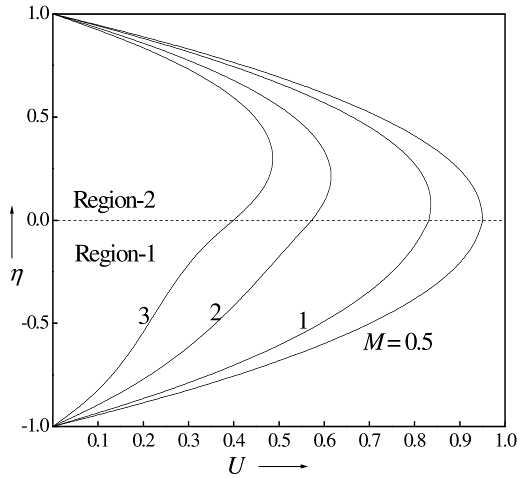

Figure 2: Velocity profile for different values of *M*.

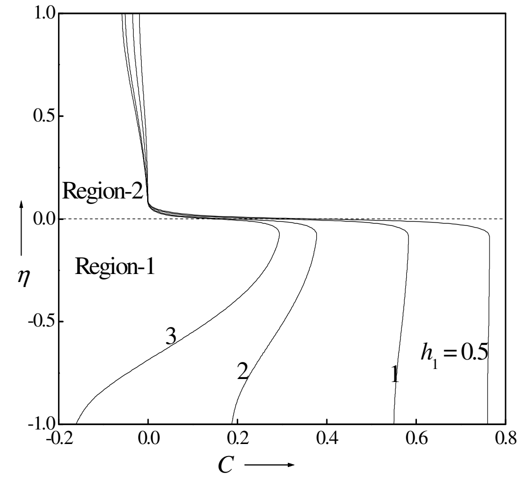

Figure 4: Concentration profile for different values of height h_1.

The effect of Hartmann number M on the velocity is shown in Fig. 2. It is seen that as the Hartmann number increases velocity decreases in clear viscous fluid region and conducting fluid region. It is interesting to note that the Hartmann number is effective in clear viscous fluid region also. This graph is drawn explicitly to understand the interface conditions.

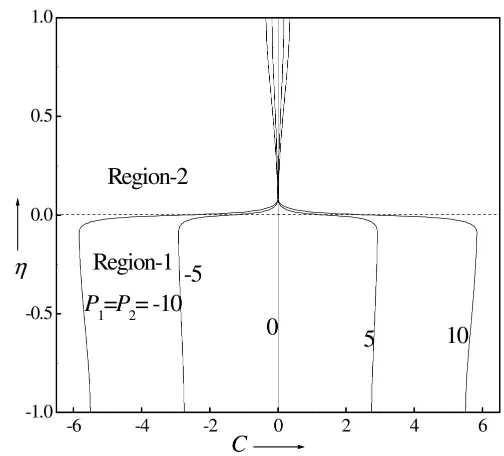

Figure 3: Concentration profiles for different values of pressure P_1 & P_2.

The effect of pressure gradients P_1 and P_2 on the concentration is shown in Fig.3. As the pressure gradient increases, the solute concentration increases in the conducting fluid region and decreases in the clear viscous region. There is no effect of solute concentration in the both the regions for zero pressure gradient.

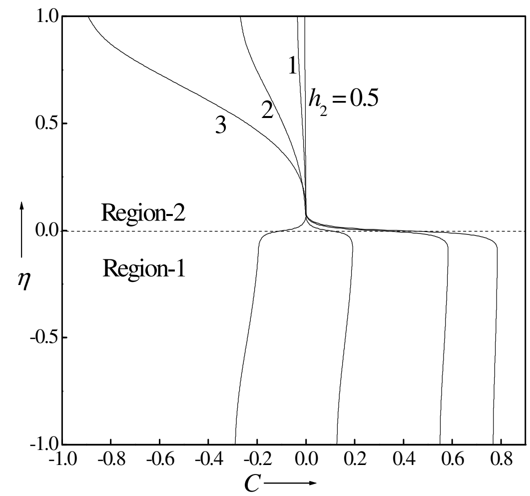

Figure 5: Concentration profiles for different values of hight h_2.

Varying the height of the conducting fluid h_1 in region-1, decreases the solute concentration decreases in both the regions. However the effect is more significant in region-1 compared to region-2 as observed in Fig. 4. Varying the height of clear viscous fluid h_2 in region-2 also reduces the concentrations in the both the regions as seen in Fig.5. It is interesting to observe that the height ratio h_2 as significant in both the regions and it is more operative in region-2 for values of h_2 greater than 1.

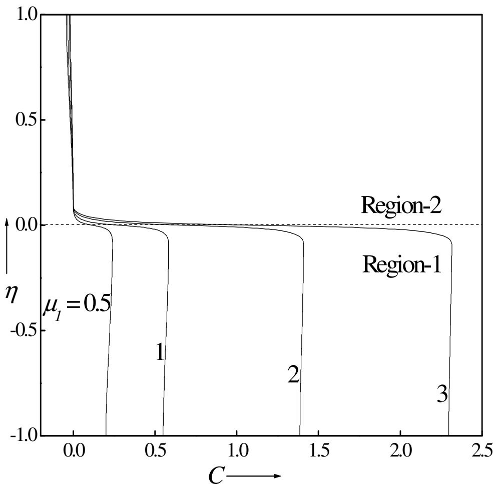

Figure 6: Concentration profiles for different values of viscosity μ_1.

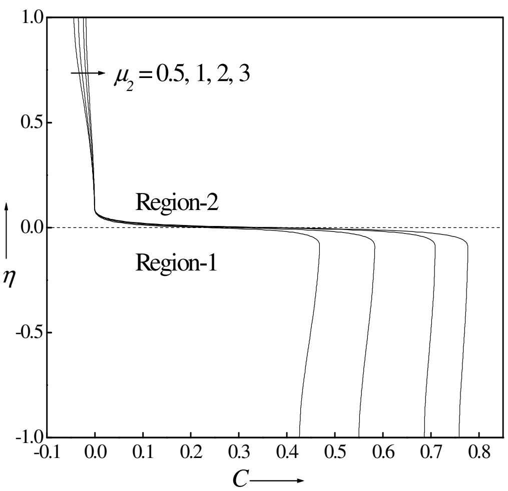

Figure 7: Concentration Profiles for different values of viscosity μ_2.

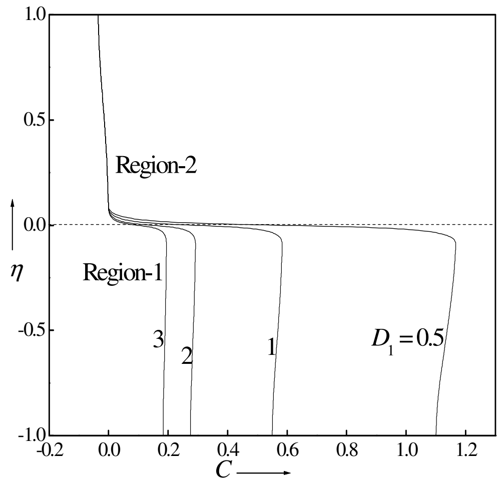

Figure 8: Concentration profiles for different values of diffusivity D_1.

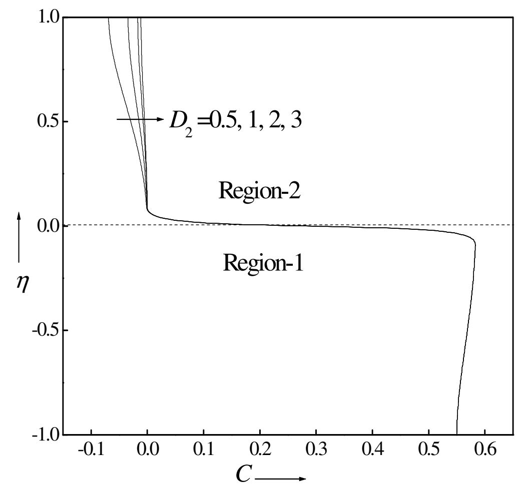

Figure 9: Concentration profiles for different values of diffusivity D_2.

The effect of viscosities μ_1 and μ_2 on the solute concentration is shown graphically in Figs. 6 and 7 respectively. By keeping the viscosity of viscous fluid constant and increasing the viscosity of the electrically conducting fluid, the solute concentration increases substantially in region-1 whereas its effect is insignificant in region-2 as seen in Fig. 6. On the other hand increasing the viscosity of the clear viscous fluid region μ_2 , also increase the solute concentration in both the regions in Fig. 7. It is very interesting to note that the effect of viscosity of the clear fluid is more effective on the solute concentration for electrically conducting fluid when compared to clear fluid.

The effects of diffusivity coefficient D_1 of conducting fluid in region-1 and D_2 of clear fluid in region-2 are shown in Figs. 8 and 9 respectively. Fixing the diffusivity coefficient D_2 and varying the diffusivity coefficient D_1 reduces the solute concentration in region-1 and remains constant in region-2 as observed in Fig.8. Figure 9 depicts that the solute concentration increases with increasing the diffusivity coefficient D_2 of viscous fluid in region-2 and it remains constant in region-1.

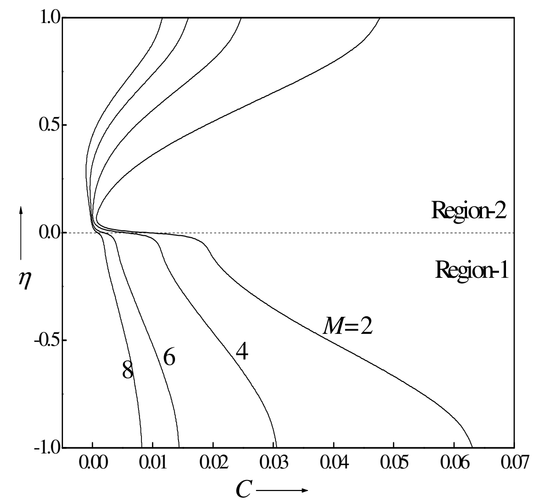

Figure 10: Concentration profile for different values of M.

The effect of Hartmann number M on solute concentration is shown in Fig. 10. As the Hartman umber increases the solute concentration decreases in both the regions. It is interesting to note that there is no effect of Hartmann number M at the interface on the concentration profiles. This is due to that fact that we do not impose any conditions on concentration or diffusion at the interface in finding the solutions of the concentrations. The concentration profiles are almost symmetric at both the boundaries.

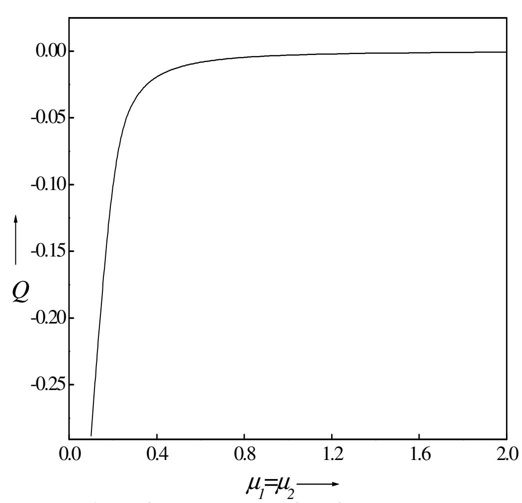

Figure 11: Volumetric rate Q versus viscosity μ_1=μ_2.

The effect of varying the viscosity of the two different fluids on the flow rate is seen in Fig. 11. The viscosity of the both the fluids are taken to be equal for simplicity. It is seen that as the viscosity of the fluid increases in both regions, the flow rate increases in magnitude but its effect is more significant for the values of viscosity varying from 0.1 to 0.5. The flow rate remains almost constant for values of viscosity greater than 0.5.

The effect of Hartman number on the volumetric rate flow rate is shown in Table-1. As the Hartman number M increases the volumetric flow rate increases in magnitude in both the regions. Table-2 is computed for volumetric flow rate in the absence of Hartman number i.e. both the regions are filled with two different immiscible clear viscous fluids. It is seen from Table-2 that the flow rate obtained in the absence of Hartman number for two fluid model are exactly the same values obtained by Gupta and Gupta [21] for one fluid model. It is seen also observed from Table-1 and 2 that the values of flow rate for clear viscous fluid region are exactly the same values.

Table 1. Volumetric flow rate Q versus height $\left(h_1 = h_2 = H\right)$ and Hartman number M

H	M	Q	Q_1	Q_2
0.2	0.2	-0.00000004	-0.00000002	-0.00000007
0.4	0.4	-0.00000323	-0.00000325	-0.00000648
0.6	0.6	-0.00005066	-0.00005147	-0.00010214
0.8	0.8	-0.00033851	-0.00034866	-0.00068718
1.0	1.0	-0.00140888	-0.00147488	-0.00288377
1.2	1.2	-0.00432996	-0.00462356	-0.00895352
1.4	1.4	-0.01079095	-0.01179277	-0.02258372
1.6	1.6	-0.02309019	-0.02590820	-0.04899840
1.8	1.8	-0.04404665	-0.05089994	-0.09494660
2.0	2.0	-0.076882243	-0.09177050	-0.16865274

Table 2. Volumetric flow rate $Q\left(=Q\,X\,10^{-6}\right)$ versus height $\left(h_1 = h_2 = H\right)$ in the absence of Hartman number and comparison of two fluid model with one fluid model (Gupta and Gupta [21]).

Height of the channel	Volumetric rate when $M = 0$			
	One fluid Model	Two fluid model		
H	Q	Q_1	Q_2	Q
0.20	-0.0542	-0.0271	-0.0271	-0.0542
0.21	-0.0762	-0.0381	-0.0381	-0.0762
0.22	-0. 1056	-0.0528	-0.0528	-0. 1056
0.23	-0. 1441	-0.0721	-0.0721	-0. 1441
0.24	-0. 1941	-0.0971	-0.0971	-0. 1941
0.25	-0. 2584	-0. 1292	-0. 1292	-0. 2584
0.26	-0. 3400	-0. 1700	-0. 1700	-0. 3400
0.27	-0. 4428	-0. 2214	-0. 2214	-0. 4428
0.28	-0. 5711	-0. 2856	-0. 2856	-0. 5711
0.29	-0. 7302	-0. 651	-0. 3651	-0. 7302
0.30	-0. 9257	-0. 4629	-0. 4629	-0. 9257

5. CONCLUSION

Analytical solutions are obtained for unsteady solute dispersion of magnetohydrodynamic and viscous fluids between two parallel plates. Separate solutions are obtained for each fluid and these solutions are matched at the interface using suitable matching conditions. The solutions are evaluated numerically and the results are shown graphically. It is found that the Hartmann number suppress the flow in both conducting and non-conducting fluid regions. The pressure gradient increase solute concentration in conducting fluid region and decrease in clear viscous fluid region. The effect of height of the fluid regions reduces the solute concentration in both the regions whereas viscosity increases the solute concentration. The effect of diffusivity coefficient is to reduce the concentration in conducting fluid region and increase in clear viscous fluid region. The effect of Hartman number is to reduce the solute concentration in both the regions. As viscosity increases flow rate decreases for small values of viscosity and remains constant for larger values of viscosity. Hartman number increases the flow rate in magnitude in both the regions.

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Nomenclature

B_0	magnetic field strength $[\mathrm{Weber\,m^{-2}}]$
C_i	concentration of the solute $[\mathrm{Kg\,m^{-3}}]$
D_i	molecular diffusion co-efficient $[\mathrm{m^2\,s^{-1}}]$
h_i	distance between the interface and plates $[\mathrm{m}]$
L_i	typical length along the flow direction $[\mathrm{m}]$
Q_i	volumetric rate $[\mathrm{m^3\,s^{-1}}]$
u_i	velocity $[\mathrm{m\,s^{-1}}]$
$\overline{u}_i$	average velocity $[\mathrm{m\,s^{-1}}]$
$\frac{dp_i}{dx_i}$	pressure gradient [Pa]
h	width ratio (h_2/h_1) [-]
p	pressure [Pa]
m	ratio of the viscosities (μ_1/μ_2) [-]

Greek letters

μ_i	dynamic viscosity $[\mathrm{Pa\,s}]$
σ_e	electrical conductivity $[\mathrm{Ohm\,m^{-1}}]$
M	Hartman number $\left(\sqrt{\sigma_e/\mu_1}\,B_0\,h_1\right)$ [-]

Subscripts

1, 2 refer to the quantities for Region-1 and Region-2, respectively.