Chemical reaction and viscous dissipation effects on Darcy-Forchheimer mixed convection in a fluid saturated porous media

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Abstract
Purpose – The purpose of this work is to study the flow of mixed convection and mass transfer of a steady laminar boundary layer about an isothermal vertical flat plate embedded in a non-Darcian porous medium in the presence of chemical reaction and viscous dissipation effects.

Design/methodology/approach – The governing partial differential equations are converted into ordinary differential equations by similarity transformation, which are solved numerically by employing the fourth-order Runge-Kutta integration scheme with Newton-Raphson shooting technique.

Findings – It was found that the local Nusselt number was predicted to decrease as either of the chemical reaction parameter or the Eckert number increased. On the other hand, the local Sherwood number was predicted to increase as a result of increasing either of the chemical reaction parameter or the Eckert number. Also, in the absence of viscous dissipation, both the local Nusselt and Sherwood numbers increased as the mixed convection increased.

Originality/value – The paper illustrates chemical reaction and viscous dissipation effects on Darcy-Forchheimer mixed convection in a fluid saturated porous media.

Keywords Chemical reactions, Convection, Dissipation factor, Porous materials

Paper type Research paper

Nomenclature

\( C \) \quad \text{species concentration in the fluid}
\( c_f \) \quad \text{Forchheimer coefficient}
\( c_p \) \quad \text{specific heat at constant pressure}
\( D \) \quad \text{mass diffusion coefficient}
\( Ec \) \quad \text{Eckert number}
\( g \) \quad \text{acceleration due to gravity}
\( K \) \quad \text{Darcy permeability of the porous medium}
\( k_f \) \quad \text{thermal conductivity}
\( N \) \quad \text{buoyancy ratio parameter}
\( Nux \) \quad \text{local Nusselt number}
\( Pe_x \) \quad \text{Peclet number}
\( Pr \) \quad \text{Prandtl number}
\( q_w \) \quad \text{heat transfer rate}
\( q_m \) \quad \text{mass transfer rate}
\( Ra_x \) \quad \text{Rayleigh number}
\( Sh_x \) \quad \text{local Sherwood number}
\( Sc \) \quad \text{Schmidt number}
\( T \) \quad \text{temperature of the fluid}
1. Introduction

Transport processes through porous media play important roles in diverse applications such as petroleum industries, chemical catalytic reactors and many others. Moreover, the combined forced and free convection flow (mixed convection flow) is encountered in several industrial and technical applications such as nuclear reactors cooled during emergency shutdown, electronics devices cooled by fans, heat exchangers placed in a low velocity environment, solar central receivers exposed to wind currents, etc. In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies such as polymer production and food processing. In addition, chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first-order, if the rate of reaction is directly proportional to the concentration itself (Cussler, 1988). A few representative fields of interest in which combined heat and mass transfer along with chemical reaction plays an important role is the chemical process industries such as food processing and polymer production. For example, formation of smog is a first-order homogeneous chemical reaction.

Because of its importance and possible applications, combined heat and mass transfer problems with chemical reaction effect received a considerable amount of attention. Yih (1998) presented an analysis of the forced convection boundary-layer flow over a wedge with uniform suction/blowing, whereas Watanabe (1990) investigated the behavior of the boundary layer over a wedge with suction injection in forced flow. Anjali Devi and Kandasamy (2002) studied the effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary-layer flow over a wedge with suction and injection. Recently, Postelnicu (2007) studied the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Seddeek (2005) studied the finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary-layer hydromagnetic flow with heat and mass transfer over a heat surface. The effects of chemical reaction,

Considering the importance of inertia effects for flow in a porous medium and chemical reaction, the problem of mixed convective heat and mass transfer along a vertical surface in a non-Darcian fluid saturated porous medium in the presence of viscous dissipation effects is studied in the present paper.

2. Mathematical description of the problem
Consider the steady, two-dimensional, Darcy-Forchheimer model and mixed convection boundary layer over a vertical flat plate of a constant temperature \( T_w \) and concentration \( C_w \), which is embedded in a fluid saturated porous medium of ambient temperature \( T_\infty \) and concentration \( C_\infty \), respectively. The properties of the fluid and the porous medium are assumed to be constant, isotropic and homogenous. The \( x \)-coordinate is measured along the surface from its leading edge and the \( y \)-coordinate is measured normal to it, as shown in Figure 1. With the Boussinesq approximation and Brownian motion of particles, the governing boundary-layer equations flow from the wall to the fluid saturated porous medium are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial y} + \frac{c_1 \sqrt{K}}{\nu} \frac{\partial u^2}{\partial y} = \frac{K_g}{\nu} \left( \beta \frac{\partial T}{\partial y} + \beta \frac{\partial C}{\partial y} \right) \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma (C - C_\infty) \tag{4}
\]

\[
\rho = \rho_\infty \{ 1 - \beta (T - T_\infty) - \beta^* (C - C_\infty) \}
\]
In the above equations, \( u, v \) are the velocity components in the \( x \)- and \( y \)-directions respectively, \( g \) is the acceleration due to gravity, \( \nu \) is the kinematic viscosity, \( \gamma, \beta \) and \( \beta^* \) are the rate of chemical reaction, the coefficient of volume expansion and the volumetric coefficient of expansion with concentration respectively, \( K \) is the Darcy permeability of the porous medium, \( c_l, c_p \) are the Forchheimer coefficient and the specific heat of the fluid at constant pressure. In addition, \( T \) and \( C \) are the temperature of the fluid inside the thermal boundary layer and the corresponding concentrations. Furthermore, \( \alpha, D \) are the effective thermal diffusivity and the Brownian diffusion coefficient. In Equation (2), the plus sign corresponds to the case where the buoyancy force has a component “aiding” the forced flow and the minus sign refer to the “opposing” case.

The boundary conditions are given by:

\[
\begin{align*}
&v = 0, \ T = T_w, \ C = C_w \quad \text{as} \quad y = 0 \\
&u \to u_\infty, \ T \to T_\infty, \ C \to C_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]

It is convenient to transform the governing equations into a dimensionless form which can be suitable for solution. This can be done by introducing the dimensionless variables for mixed convection:

\[
\begin{align*}
\eta &= \frac{y}{x} \sqrt{Pe_x} , \quad \psi = \alpha \sqrt{Pe_x} F(\eta) \\
\theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} , \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}
\end{align*}
\]

where \( \psi \) is the stream function that satisfies the continuity equation and \( \eta \) is the dimensionless similarity variable. With the change of variables, Equation (1) is identically satisfied and Equations (2)-(4) are transformed, respectively, to:
The corresponding dimensionless boundary conditions take the form:

\[ F(\eta) = 0, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \quad \text{on} \quad \eta = 0 \]
\[ F'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \]  \hspace{1cm} (10)

Here, the primes denote differentiation with respect to \( \eta \), \( \Pr = \nu/\alpha \) is the Prandtl number, \( \Sc = \nu/D \) is the Schmidt number, \( N = \beta'(C_w - C_\infty)/\beta(T_w - T_\infty) \) is the buoyancy ratio parameter, \( Ra_x = Kg\beta'(T_w - T_\infty)x/\nu\alpha \) is the Rayleigh number, \( Pe_x = u_\infty x/\alpha \) is the Peclet number, \( Ec = u_\infty^2/c_p(T_w - T_\infty) \) is the Eckert number, \( R = \gamma x/u_\infty \) is the chemical reaction rate parameter and \( \lambda = c_l\sqrt{K}/\nu \) is the inertia parameter.

Of practical interest in many applications of the problem under consideration are the heat and mass transfer coefficients. The heat and mass transfer coefficients are expressed in terms of the Nusselt and Sherwood numbers respectively, which are given by:

\[ Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D(C_w - C_\infty)} \] \hspace{1cm} (11)

where \( q_w, q_m \) are the heat and mass transfer rate per unit surface area and \( k_f \) is the effective thermal conductivity, and defined by:

\[ q_w = -k_f \frac{\partial T}{\partial y} \bigg|_{y=0}, \quad q_m = -D \frac{\partial C}{\partial y} \bigg|_{y=0} \] \hspace{1cm} (12)

Using Equations (6), (11) and (12), we get the local Nusselt and Sherwood numbers in the following expressions:

\[ Nu_x/\sqrt{Pe_x} = -\frac{\partial \theta}{\partial \eta} \bigg|_{\eta=0} \] \hspace{1cm} (13)
\[ Sh_x/\sqrt{Pe_x} = -\frac{\partial \phi}{\partial \eta} \bigg|_{\eta=0} \] \hspace{1cm} (14)

### 3. Numerical method

The governing Equations (7)-(9) subject to the boundary conditions (10) are solved numerically using the fourth-order Runge-Kutta integration scheme. The numerical procedure used here solves the two-point boundary value problems for a system of \( N \)
ordinary differential equations in the range \((x, x_1)\). The system and the derivatives are written as:

\[
\frac{dy_i}{dx} = f_i(x, y_1, y_2, \ldots, y_N), \quad i = 1, 2, \ldots, N
\]

are evaluated by a procedure that evaluates the derivatives of \(y_1, y_2, \ldots, y_N\) at a general point \(x\). Initially, \(N\) boundary values of the variable \(y_i\) must be specified, some of which will be specified at \(x\) and some at \(x_1\). The remaining \(N\) boundary values are guessed and the procedure corrects them by a form of Newtonian iteration. Starting from the known and guessed values of \(y_i\) at \(x\), the procedure integrates the equations forward to a matching point \(X\) using Merson’s method. Similarly, starting from \(x_1\) it integrates backwards to \(X\). The difference between the forward and backward values of \(y_i\) at \(X\) should be zero for a true solution. The procedure uses a generalized Newton method to reduce these differences to zero, by calculating corrections to the estimated boundary values. This process is repeated iteratively until convergence is obtained to a specified accuracy. The tests for convergence and the perturbation of the boundary conditions are carried out in a mixed form, e.g. if the error estimate for \(y_i\) is \(ERROR_i\), we test whether \(ABS(ERROR_i) < ERROR_i [1 + ABS(y_i)]\)

In the present problem, a solution was considered to be converged if the newly calculated values of \(F, F', \theta\) and \(\phi\) differed from their previous guessed values within a tolerance of \(E < 10^{-5}\). The numerical results were found to depend on \(\eta_\infty\) and the step size \(\Delta \eta\). We have used \(\Delta \eta = 0.001\) and \(\eta_\infty = 6.0\) without causing numerical oscillations in the values \(F, F', \theta, \theta', \phi\) and \(\phi'\).

4. Results and discussion

In order to gain physical insight into the heat and mass transfer problem, the system of ordinary differential Equations (7)-(9) along with the boundary conditions (10) are integrated numerically by means of the Runge-Kutta method with systematic estimate of \(F'(0), \theta'(0)\) and \(\phi'(0)\) with Newton-Raphson shooting technique. The step size \(\Delta \eta = 0.001\) is used to obtain the numerical solution and the boundary condition \(\eta \rightarrow \infty\) is approximated by \(\eta_{\max} = 6\) which is sufficiently large for the velocity to approach the relevant free stream velocity within five-decimal accuracy as the criterion for convergence. A parametric study of the physical parameters is performed to illustrate interesting features of the numerical solutions. The results of the parametric study are displayed graphically in Figures 2-23 and are discussed below.

Figures 2-4 present typical profiles for the velocity, temperature and concentration for various values of the mixed convection parameter \(Ra_x/Pe_x\), respectively. When \(Ra_x/Pe_x \gg 1\), the flow is dominated by natural convection, whereas when \(Ra_x/Pe_x \ll 1\) forced convection takes the leading role. Therefore, when \(Ra_x/Pe_x = 1\), the effects of natural and forced convection achieve equal importance and the flow is truly under mixed convection conditions. In addition, Figure 2 shows that the fluid velocity in the boundary layer is larger than the external velocity as \(Ra_x/Pe_x > 1\), due to the buoyancy force. When the free stream and the buoyancy force are in opposite directions (opposing flow conditions), the buoyancy force retards the fluid in the boundary layer. Figures 3 and 4 show that the temperature and concentration profiles as well as the thermal and solutal boundary-layer thicknesses decrease with increasing values of the mixed convection parameter \(Ra_x/Pe_x\).
Figure 2. Effects of the mixed convection parameter $Ra_x / Pe_x$ on the velocity distribution.

Figure 3. Effects of the mixed convection parameter $Ra_x / Pe_x$ on the temperature distribution.

Figure 4. Effects of the mixed convection parameter $Ra_x / Pe_x$ on the concentration distribution.
Effects of the chemical reaction parameter $R$ on the velocity distribution.

Figure 5.

Effects of the chemical reaction parameter $R$ on the temperature distribution.

Figure 6.

Effects of the chemical reaction parameter $R$ on the concentration distribution.

Figure 7.
Figure 8. Effects of the inertia parameter $\lambda$ on the velocity distribution

Figure 9. Effects of the inertia parameter $\lambda$ on the temperature distribution

Figure 10. Effects of the inertia parameter $\lambda$ on the concentration distribution
Figure 11. Effects of the Eckert number $Ec$ on the velocity distribution

Figure 12. Effects of the Eckert number $Ec$ on the temperature distribution

Figure 13. Effects of the Eckert number $Ec$ on the concentration distribution
Figure 14. Effects of the Schmidt number $Sc$ on the velocity distribution.

Figure 15. Effects of the Schmidt number $Sc$ on the temperature distribution.

Figure 16. Effects of the Schmidt number $Sc$ on the concentration distribution.
Figure 17.
Effects of the buoyancy ratio $N$ on the velocity distribution

Figure 18.
Effects of the buoyancy ratio $N$ on the temperature distribution

Figure 19.
Effects of the buoyancy ratio $N$ on the concentration distribution
Figure 20.
Local Nusselt number as a function of $Ra_x / Pe_x$ for various values of the chemical reaction parameter.

Figure 21.
Local Sherwood number as a function of $Ra_x / Pe_x$ for various values of the chemical reaction parameter.

Figure 22.
Local Nusselt number as a function of $Ra_x / Pe_x$ for various values of the Eckert number.
The effects of the chemical reaction parameter R on the fluid velocity, temperature and concentration profiles are illustrated in Figures 5-7, respectively. Increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as R increases. Consequently, less flow is induced along the surface resulting in decreases in the fluid velocity in the boundary layer. Also, increasing the chemical reaction parameter leads to an increase in the temperature profiles. These behaviors are clearly depicted in Figures 5-7.

Figures 8-10 depict the influence of the inertia parameter λ on the velocity, temperature and concentration profiles, respectively. It is well known that the presence of the porous medium in the flow presents an obstacle to flow causing the flow to move slower and the fluid temperature and concentration to increase. It is clear from Figure 8 that an increase in the value of the porous medium inertia parameter leads to a decrease in the fluid velocity peak, which occurs at the surface. This decrease in the fluid velocity takes place because when the porous medium inertia effect is increased, the form drag of the porous medium is increased. Moreover, we observe that the concentration and temperature distributions in the boundary layer increase owing to the increase in the value of the inertia parameter and as a result, both of the thermal and velocity boundary layers become thicker.

Figures 11-13 illustrate the effect of the Eckert number (viscous dissipation effect) on the velocity, temperature and concentration profiles, respectively. Increases in the value of the Eckert number Ec have the tendency to increase the fluid temperature and decrease the solute concentration. The increase in the fluid temperature tends to increase the thermal buoyancy effects which induce more flow along the plate. It is clear from these Figures 11-13 that both of the velocity and temperature increase while the concentration decreases with increasing values of the Eckert number Ec.

The effects of the Schmidt number Sc on the velocity, temperature and concentration distributions are plotted in Figures 14-16, respectively. The Schmidt number is an important parameter in heat and mass transfer processes as it characterizes the ratio of thicknesses of the viscous and concentration boundary layers. Its effect on the species concentration has similarities to the Prandtl number effect on the temperature. That is, increases in the values of Sc cause the species concentration and its boundary-layer thickness to decrease significantly with a slight increase in the fluid temperature. This decrease in the solute concentration causes a reduction in the...
solutal buoyancy effects resulting in less induced flow along the surface. This is evident in the decreases in the velocity and concentration profiles and the slight increase in the temperature profile as the value of the Schmidt number Sc increases shown in Figures 14-16.

Figures 17-19 display the influence of the buoyancy ratio parameter $N$ on the fluid velocity, temperature and concentration distributions, respectively. It is clearly shown in these figures that the concentration and the temperature profiles decrease with increasing values of the buoyancy ratio parameter whereas, the velocity increases near the plate as $N$ increases.

The heat and mass transfer results in terms of $N_{u_x}/\sqrt{P_e_x}$ and $S_{h_x}/\sqrt{P_e_x}$ as a function of the mixed convection parameter $Ra_x/P_e_x$ for different values of the chemical reaction parameter are depicted in Figures 20 and 21. It is observed that as the chemical reaction parameter increases, the local Nusselt number decreases whereas the local Sherwood number increases. Figures 22 and 23 illustrate the effects of the Eckert number on the local Nusselt and Sherwood numbers, respectively. It is observed that as the Eckert number increases, the local Nusselt number decreases while the local Sherwood number increases. Again, it is clear from Figures 20-23 that increasing $Ra_x/P_e_x$ leads to increases of the momentum transport in the boundary layer and more heat and mass species are carried out of the surface, thus decreasing the thickness of the thermal and concentration boundary layers and hence increasing the heat and mass transfer rates.

5. Conclusions
The problem of mixed convection and mass transfer of a steady laminar boundary layer about an isothermal vertical flat plate embedded in a non-Darcian porous medium in the presence of chemical reaction and viscous dissipation effects was investigated. The governing partial differential equations were converted into ordinary differential equations by using a suitable similarity transformation, which were solved numerically by employing a fourth-order Runge-Kutta integration scheme with Newton-Raphson shooting technique. It was found that the local Nusselt number decreased as either of the chemical reaction parameter or the Eckert number increased. On the other hand, the local Sherwood number increased as a result of increasing either of the chemical reaction parameter or the Eckert number. Also, in the absence of viscous dissipation, both the local Nusselt and Sherwood numbers increased as the mixed convection increased. It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

References


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