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This work is focused on the numerical solution of steady boundary-layer stagnation-point flow of a polar fluid towards a stretching surface embedded in porous media in the presence of the effects of Soret and Dufour numbers and first-order homogeneous chemical reaction. The governing boundary-layer equations of the problem are formulated and transformed into a self-similar form. The obtained equations are solved numerically by an efficient, iterative, tri-diagonal, implicit finite-difference method. Both assisting and opposing flow conditions are considered. Comparisons of the present numerical results with previously published work under limiting cases are performed and found to be in excellent agreement. Representative results for the fluid velocity, angular velocity, temperature, and solute concentration profiles as well as the local heat and mass transfer rates for various values of the physical parameters are displayed in both graphical and tabular forms.

Keywords Chemical reaction; Porous media; Soret and Dufour effects; Stagnation point; Stretching surface

Introduction

Polar fluids are defined as fluids that sustain couple stresses. They model those fluids with microstructures that are mechanically significant when the characteristic dimension of the problem is of the same order of magnitude as the size of the microstructure. Extensive reviews of the theory can be found in the review article by Cowin (1974). Since the microstructure size is the same as the average pore size, it is pertinent to study the flow of polar fluids through a porous medium. Examples of fluids that can be modeled as polar fluids are mud, crude oil, body fluids, and lubricants with polymer additives. Patil and Hiremath (1992) have studied the effects of couple stresses on the flow of polar fluids through a porous medium. Raptis and Takhar (1999) have discussed the problem of steady flow of a polar fluid through a porous

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medium by using Forchheimer’s model. Ogulu (2005) has reported analytical solutions for the problem of the flow of a polar fluid past a vertical porous plate in the presence of couple stresses and radiation, where the temperature of the plate is assumed to oscillate about a mean value. Patil and Kulkarni (2008) have examined the effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation.

The flow in the neighborhood of a stagnation line has attracted many investigations during the past several decades. Hiemenz (1911) was the first to study the two-dimensional stagnation-point flow and obtained an exact similarity solution of the governing Navier-Stokes equations. Eckert (1942) extended this work by including the energy equation and obtained an exact similar solution for the thermal field. Hassanien and Gorla (1990) have considered both assisting and opposing flows and found that a reversed flow was developed in the buoyancy opposing flow region, and dual solutions were found to exist for a certain range of the buoyancy parameter to micropolar fluids.

The study of steady flows of a viscous incompressible fluid driven by a linearly stretching surface through a quiescent fluid are encountered in a number of industrial processes, e.g., the cooling of metallic plates in a cooling bath, the aerodynamic extrusion of plastic sheets, polymer sheets extruded continuously from a dye, and heat-treated materials that travel between feed and wind-up rolls or on a conveyor belt possess the characteristics of a moving continuous surface. Mahapatra and Gupta (2002) have studied the heat transfer in steady two-dimensional stagnation-point flow of an incompressible fluid over a flat deformable sheet that is stretched in its own plane with a velocity proportional to the distance from the stagnation-point by taking into consideration the viscous dissipation effect of the fluid. Abel et al. (2007) have discussed the viscoelastic boundary layer flow and heat transfer over a stretching sheet in the presence of viscous dissipation and nonuniform heat source. They have obtained analytical solutions of a highly nonlinear momentum equation and confluent hypergeometric similarity solution of heat transfer equations. Nazar et al. (2004) analyzed the unsteady two-dimensional boundary layer flow of a viscous and incompressible fluid in a region of the stagnation point on a stretching flat sheet, where the unsteadiness is caused by the sudden stretching of the surface and by the impulsive motion of the free stream velocity. Recently, Ishak et al. (2006) have analyzed mixed convection boundary layer flow near the two-dimensional stagnation-point flow of an incompressible fluid over a stretching vertical sheet. Dulal (2009) investigated heat and mass transfer in two-dimensional stagnation-point flow of an incompressible viscous fluid over a stretching vertical sheet in the presence of buoyancy force and thermal radiation.

The heat and mass transfers simultaneously affecting each other will cause the cross-diffusion effect. The mass transfer caused by the temperature gradient is called the Soret effect, and it has been utilized for isotope separation and in a mixture between gases with very light molecular weight (H₂, He) and medium molecular weight (H₂, air), while the heat transfer caused by the concentration gradient is called the Dufour effect. Alam and Rahman (2005) have studied the Dufour and Soret effects on steady magnetohydrodynamic (MHD) free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium. Tsai and Huang (2009) have studied steady stagnation-point flow over a flat stretching surface in the presence of species concentration and mass diffusion under Soret and Dufour effects.
The objective of the present work is to study steady two-dimensional stagnation-point flow of a polar fluid impinging on a flat stretching wall embedded in a porous medium and to investigate the behavior of the polar fluids near the stagnation point, considering the effects of material parameters characterizing the polar fluids for both assisting flow and opposing flow. In addition, the effects of Soret and Dufour numbers and first-order homogeneous chemical reaction are considered.

Mathematical Analysis

Consider steady, two-dimensional, double-diffusive convective boundary-layer stagnation-point flow of an incompressible, viscous, and electrically conducting polar fluid towards a stretching surface in a porous medium in the presence of a homogeneous chemical reaction, and Soret and Dufour effects (see Figure 1). The fluid properties are assumed to be constant and the chemical reaction is assumed to take place in the flow. The physical properties and the rate of the chemical reaction are assumed constant throughout the fluid. Under the boundary-layer assumptions and considering the Boussinesq approximation, the governing equations are given by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = U \frac{dU}{dx} + (\nu + \nu_r) \frac{\partial^2 u}{\partial y^2} + 2\nu \frac{\partial \omega}{\partial y} \pm g \beta_T (T - T_\infty) \pm g \beta_C (C - C_\infty) \\
\quad + \left( \frac{\nu + \nu_r}{k_1} \right) (U - u),
\]

\[
\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y} = \frac{\gamma}{I} \frac{\partial^2 \omega}{\partial y^2}
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m k T}{C_s C_p} \frac{\partial^2 C}{\partial y^2},
\]

![Figure 1. Physical model and coordinate system.](image-url)
\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_c (C - C_\infty) + \frac{D_m k_T}{T_M} \frac{\partial^2 T}{\partial y^2}, \]  

(5)

where \( x, y \) are the Cartesian coordinates; \( u, v, \omega, T, \) and \( C \) are the fluid x-component of velocity, y-component of velocity, angular velocity of particle rotation, temperature, and solute concentration, respectively; \( k_1 \) is the permeability of the porous medium; \( \rho \) is the fluid density; \( \nu \) is the kinematic viscosity; \( \nu_r \) is the rotational kinematic viscosity; \( k \) is the fluid thermal conductivity; \( C_p \) is the specific heat at constant pressure; \( k_1 \) is the rate of chemical reaction; \( g \) is the acceleration due to gravity; \( b_T \) is the volumetric coefficient of thermal expansion; \( b_C \) is the volumetric coefficient of concentration expansion; \( D_m \) is the coefficient of mass diffusivity; \( U, T_\infty, \) and \( C_\infty \) are the free stream velocity, temperature, and concentration, respectively; \( T_M \) is the mean fluid temperature; \( k_T \) is the thermal diffusion ratio; \( C_S \) is the concentration susceptibility; \( \gamma \) is the spin gradient parameter; and \( I \) is a constant of the same dimension as that of the moment of inertia of unit mass. In Equation (2), the “+” sign corresponds to an assisting flow, while the “−” sign refers to an opposing flow.

The physical boundary conditions of the problem are:

\[ u = u_W(x), \quad v = 0, \quad \frac{\partial \omega}{\partial y} = -m \frac{\partial^2 u}{\partial y^2} \]

\[ T = T_W(x) = T_\infty + S x^n, \quad C = C_W(x) = C_\infty + S_1 x^{n_1} \quad \text{on} \ y = 0 \]

\[ u = U(x), \quad \omega \to 0, T \to T_\infty, \quad C \to C_\infty \quad \text{as} \ y \to \infty \]

(6)

where \( T_W \) and \( C_W \) are the wall temperature and concentration, respectively, and \( S, S_1, n, \) and \( n_1 \) are constants. The wall boundary condition for the angular velocity assumes that the couple stresses are dominant during the rotation of the particles. Further, \( m \) is a constant and \( 0 \leq m \leq 1 \). The case \( m = 0 \), which corresponds to \( \frac{\partial \omega}{\partial y} = 0 \) at the wall, represents concentrated particle flows in which the microelements close to the wall surface are not able to rotate. This case is known as strong concentration of microelements. The case \( m = 0.5 \) corresponds to the vanishing of anti-symmetric part of the stress tensor and indicates weak concentration of microelements. The case \( m = 1.0 \) corresponds to the modeling of turbulent boundary-layer flows.

Substituting the following dimensionless variables:

\[ \psi = \sqrt{au x f(\eta)}, \quad \eta = \frac{a}{\nu} y, \quad \omega = ax \sqrt{\frac{a}{\nu} \Omega(\eta)}, \quad u_W(x) = ax, \quad U(x) = bx, \]

\[ \theta = \frac{T - T_\infty}{T_W - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_W - C_\infty}, \]

(7)

(where \( a \) and \( b \) are constants) into Equations (1)–(5) yields the following dimensionless equations:

\[(1 + \alpha) f'' + \beta f' - f'^2 + \frac{b^2}{a^2} + 2 \alpha \Omega' \pm Gr \theta \pm Gc \phi + (1 + \alpha) K \left( \frac{b}{a} - f' \right) = 0 \]

(8)

\[ \beta \Omega' - f' \Omega + f \Omega' = 0, \]

(9)
where \( Gr = \frac{g \beta_r (T_w - T_\infty)}{a \nu_w} \) and \( Gc = \frac{g \beta_c (C_w - C_\infty)}{a \nu_w} \) are the thermal Grashof number and modified concentration Grashof number, respectively. \( K = \frac{\nu_r}{\nu} \) is the dimensionless permeability parameter, \( \alpha = \frac{\nu_r}{\nu} \) is the ratio between the rotational kinematic viscosity \( \nu_r \) and kinematic viscosity \( \nu \), and \( \beta = \frac{c_p}{\nu} \) is the ratio parameter between coefficients of couple stress viscosities and the kinematic viscosity, \( Du = \frac{D_m \kappa T}{\nu C P C_w} \left( \frac{C_w - C_\infty}{T_w - T_\infty} \right) \) is the Dufour number, \( Sr = \frac{D_m \kappa T}{\mu C_P C_w} \left( \frac{T_m - T_w}{T_w - T_\infty} \right) \) is the Soret number, and \( \Delta = \frac{k}{a} \) is the chemical reaction parameter.

The corresponding transformed dimensionless boundary conditions become:

\[
\begin{align*}
f' & = 1, \quad f = 0, \quad \Omega = -f'', \quad \theta = 1, \quad \phi = 1 \quad \text{at} \ \eta = 0 \\
f' & = b/a, \quad \Omega \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{at} \ \eta \to \infty
\end{align*}
\tag{12}
\]

Of special significance for this type of flow and heat and mass transfer situation are the local skin-friction coefficient \( C_f \), the local heat transfer rate or local Nusselt number \( Nu \), and the local mass transfer rate or local Sherwood number \( Sh \). These physical quantities are defined, respectively, as follows:

\[
C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu = \frac{\chi q_w}{k(T_w - T_\infty)}, \quad \text{and} \quad Sh = \frac{\chi m_w}{\rho D_m(C_w - C_\infty)}
\tag{13}
\]

where \( \tau_w = (\mu + \mu_r) \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=0} \), \( q_w = -k \left( \frac{\partial T}{\partial \eta} \right)_{\eta=0} \), and \( m_w = -\rho D_m \left( \frac{\partial C}{\partial \eta} \right)_{\eta=0} \) (with \( \mu \) and \( \mu_r \) being the dynamic viscosity and rotational dynamic viscosity, respectively) are the shear stress, surface heat flux, and the surface mass flux, respectively. Using the new variables in Equation (7) gives

\[
C_f \text{Re}_{\chi}^{1/2} = (1 + \alpha) \left( \frac{\partial^2 \chi}{\partial \eta^2} \right)_{\eta=0}, \quad Nu/\text{Re}_{\chi}^{1/2} = [-\theta'(\eta)]_{\eta=0}, \quad Sh/\text{Re}_{\chi}^{1/2} = [-\phi'(\eta)]_{\eta=0}
\tag{14}
\]

where \( \text{Re}_{\chi} = \frac{u_x \chi}{\nu} \) is the local Reynolds number.

The heat and mass transfer problem for the stagnation-point flow represented by Equations (8)–(11) are nonlinear and, therefore, must be solved numerically. The standard implicit finite-difference method discussed by Blottner (1970) has proven to be adequate and gives accurate results for such equations. For this reason, it is employed in the present work, and graphical and tabular results based on this method will be presented subsequently.

Equations (8)–(11) are discretized using three-point central difference formulae with \( f' \) replaced by another variable \( V \). The \( \eta \) direction is divided into 196 nodal points, and a variable step size is used to account for the sharp changes in the variables in the region close to the surface where viscous effects dominate. The initial step size used is \( \Delta \eta_1 = 0.001 \) and the growth factor \( K = 1.037 \) such that
\[ \Delta \eta_i = K \Delta \eta_{i-1} \] (where the subscript “i” is the number of nodes minus one). This gives \( \eta_{\text{max}} \approx 35 \), which represents the edge of the boundary layer at infinity. The ordinary differential equations are then converted into linear algebraic equations that are solved by the Thomas algorithm discussed by Blottner (1970). The equation \( V = f' \) is solved by the trapezoidal rule. Iteration is employed to deal with the nonlinear nature of the governing equations. The convergence criterion employed in this work was based on the relative difference between the current and the previous iterations. When this difference or error reached \( 10^{-5} \), then the solution was assumed converged and the iteration process was terminated. It is possible to compare the present results obtained by this numerical method with the previously published work of Mahapatra and Gupta (2002), Nazar et al. (2004), Ishak et al. (2006), and Dulal (2009). Table I shows the results of this comparison for the values of \( C_f \) \( \text{Re}^{1/2} \) for different values of \( b/a \) in the absence of the thermal buoyancy parameter \( Gr \) and concentration buoyancy parameter \( Gc \). It is evident from this table that these results are in excellent agreement.

### Results and Discussion

In order to get clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. A representative set of results is shown in Figures 2–18. Computations were carried out for various values of physical parameters such as the chemical reaction parameter \( \Delta \), permeability parameter \( K \), polarity parameters \( \alpha \) and \( \beta \), Schmidt number \( Sc \), Soret number \( Sr \), and the Dufour number \( Du \). The values of the local skin-friction coefficient, local Nusselt number, and the local Sherwood number are shown in Tables II and III for various values of the chemical reaction parameter, Schmidt number, polarity parameters, and Soret and Dufour numbers.

Figures 2–4 show the influence of the chemical reaction parameter \( \Delta \) on the velocity, temperature, and concentration profiles in the boundary layer for assisting and opposing flow conditions, respectively. Increasing the chemical reaction parameter produces a decrease in the species concentration for both assisting and opposing flow conditions. This, in turn, causes the concentration buoyancy effects to decrease as \( \Delta \) increases. Consequently, less flow is induced along the plate, resulting in decreases in the fluid velocity in the boundary layer for assisting flow conditions. However, for opposing flow conditions, the velocity profiles tend to increase as the chemical reaction parameter increases. In addition, increasing the chemical reaction parameter

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leads to increases in the temperature profiles for both assisting and opposing flow conditions. These behaviors are clear from Figures 2–4.

Figures 5–7 present the effects of the free stream to wall velocities $b/a$ on the velocity, temperature, and concentration profiles for polar and Newtonian fluid conditions, respectively. It is noticed that the boundary layer is formed when the stretching velocity is less than the free stream velocity, i.e., $b/a > 1$ for polar and Newtonian fluid conditions.

**Figure 2.** Effects of chemical reaction parameter on the velocity profiles.

**Figure 3.** Effects of chemical reaction parameter on the temperature profiles.
Newtonian fluids. For $b/a > 1$, the straining motion near the stagnation region increases so that the acceleration of the external stream increases, which leads to a decrease in the thickness of the boundary layer with the increase in the value of the ratio $b/a$, and the velocity profiles tend to increase. This fact was experimentally confirmed by Flachsbart (Schlichting, 1968). In addition, as $b/a$ increases, both the temperature and concentration profiles decrease for both polar and Newtonian fluids.

Figure 4. Effects of chemical reaction parameter on the concentration profiles.

![Figure 4](image1.png)

Figure 5. Effects of $b/a$ on the velocity profiles.

![Figure 5](image2.png)
Figures 8–11 display the effects of the combined Soret and Dufour numbers on the velocity, temperature, concentration, and angular velocity profiles for assisting and opposing flow conditions, respectively. As the Soret number increases and the Dufour number decreases, the fluid velocity and solute concentration profiles tend to increase, while the fluid temperature and angular velocity profiles decrease for assisting flow conditions. However, for opposing flow conditions, the velocity...
profiles tend to decrease, while the angular velocity increases as the Soret number increases and Dufour number decreases.

Figures 12–15 display the effects of the pair of the polarity parameters $\alpha$ and $\beta$ on the velocity, temperature, concentration, and angular velocity profiles for assisting and opposing flow conditions, respectively. It is seen that as the polarity parameters $\alpha$ and $\beta$ increase, the velocity, temperature, concentration, and angular velocity profiles tend to decrease.

**Figure 8.** Effects of Soret and Dufour numbers on the velocity profiles.

**Figure 9.** Effects of Soret and Dufour numbers on the temperature profiles.
parameters increase, all of the velocity, temperature, and concentration profiles decrease, while the angular velocity profiles increase for assisting flow conditions. However, these behaviors remain unchanged for opposing flow conditions except for the velocity profiles, which increase as the polarity parameters increase.

Figure 10. Effects of Soret and Dufour numbers on the concentration profiles.

Figure 11. Effects of Soret and Dufour numbers on the angular velocity profiles.
Figures 16–18 show the velocity, temperature, and concentration profiles in the boundary layer for various values of the permeability parameter $K$ and polar and Newtonian fluid conditions, respectively. Increasing the value of $K$ has the tendency to resist the flow, causing its velocity to decrease and its temperature and concentration species to increase. These behaviors are clearly depicted in Figures 16–18.

Figures 19 and 20 show the velocity and temperature profiles in the boundary layer for various values of the wall temperature power index $n$ for assisting and

![Graph showing velocity profiles](image)

**Figure 12.** Effects of polarity parameters $\alpha$ and $\beta$ on the velocity profiles.

Figures 16–18 show the velocity, temperature, and concentration profiles in the boundary layer for various values of the permeability parameter $K$ and polar and Newtonian fluid conditions, respectively. Increasing the value of $K$ has the tendency to resist the flow, causing its velocity to decrease and its temperature and concentration species to increase. These behaviors are clearly depicted in Figures 16–18.

Figures 19 and 20 show the velocity and temperature profiles in the boundary layer for various values of the wall temperature power index $n$ for assisting and

![Graph showing temperature profiles](image)

**Figure 13.** Effects of polarity parameters $\alpha$ and $\beta$ on the temperature profiles.
opposing flow conditions, respectively. It is seen that as the wall temperature power index \( n \) increases, the temperature profiles for assisting and opposing flow conditions decrease, while the velocity profiles decrease for assisting flow conditions and increase for opposing flow conditions.

Figures 21 and 22 depict the influence of the wall concentration power index \( n_1 \) on the velocity and concentration profiles in the boundary layer for assisting and opposing flow conditions.

**Figure 14.** Effects of polarity parameters \( \alpha \) and \( \beta \) on the concentration profiles.

**Figure 15.** Effects of polarity parameters \( \alpha \) and \( \beta \) on the angular velocity profiles.
opposing flow conditions, respectively. Similar to the behaviors shown in Figures 19 and 20, it is observed that as the wall concentration power index $n_1$ increases, the concentration profiles decrease for both assisting and opposing flow conditions, while the fluid velocity decreases for assisting flow conditions and increases for opposing flow conditions.

**Figure 16.** Effects of permeability parameter on the velocity profiles.

**Figure 17.** Effects of permeability parameter on the temperature profiles.
Figures 23 and 24 illustrate the effects of the thermal Grashof number $Gr$ and the concentration Grashof number $Gc$ on the velocity profiles for assisting and opposing flow conditions, respectively. As expected, it is observed that as either the thermal buoyancy parameter represented by $Gr$ or concentrational buoyancy parameter represented by $Gc$ increases, the velocity profiles increase for assisting flow conditions, while they decrease for opposing flow conditions.

**Figure 18.** Effects of permeability parameter on the concentration profiles.

**Figure 19.** Effects of wall temperature power index $n$ on the velocity profiles.
Table II presents the values of $C_f$ Re$_1^{1/2}$, Nu/Re$_1^{1/2}$, and Sh/Re$_1^{1/2}$ for different values of the Schmidt number $Sc$, chemical reaction $\Delta$, and the polarity parameters $\alpha$ and $\beta$ for assisting and opposing flow conditions. It is observed that the values of the local skin-friction coefficient increase as either the chemical reaction parameter or the Schmidt number increases for opposing flow conditions. However, the local

![Figure 20. Effects of wall temperature power index $n$ on the temperature profiles.](image)

![Figure 21. Effects of wall concentration power index $n_1$ on the velocity profiles.](image)
skin-friction coefficient decreases as the polarity parameters, chemical reaction parameter, and Schmidt number increase for assisting flow conditions. The local Nusselt number is predicted to decrease as the polarity parameters, Schmidt number, and the chemical reaction increase for assisting flow conditions, while it increases as the polarity parameters, Schmidt number, and the chemical reaction increase for opposing flow conditions. Finally, the local Sherwood number decreases as the polarity parameters, chemical reaction parameter, and Schmidt number increase for assisting flow conditions.

Figure 22. Effects of wall concentration power index \( n_1 \) on the concentration profiles.

Figure 23. Effects of thermal buoyancy parameter \( Gr \) on the velocity profiles.
parameters increase for assisting flow. On the other hand, the local Sherwood number increases as the polarity parameters increase for opposing flow conditions, and it increases as the Schmidt number and the chemical reaction increase for both assisting and opposing flow conditions.

Table III shows the values of $C_f \frac{Re_1}{2}$, $Nu \frac{Re_1}{2}$, and $Sh \frac{Re_1}{2}$ for different values of the Soret and Dufour numbers. It is observed that as the Soret number increases and the Dufour number decreases, the local skin-friction coefficient and

Table II. Values of $C_f \frac{Re_x}{2}$, $Nu \frac{Re_x}{2}$, and $Sh \frac{Re_x}{2}$ for different values of Schmidt number $Sc$, chemical reaction $\Delta$, and polarity parameters $\alpha$ and $\beta$

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<td>-0.47805</td>
<td>0.78847</td>
<td>1.366901</td>
<td></td>
<td></td>
<td>-2.784929</td>
<td>0.52722</td>
<td>1.300835</td>
</tr>
</tbody>
</table>

Figure 24. Effects of concentration buoyancy parameter $Gc$ on the velocity profiles.
the local Nusselt number increase, while the local Sherwood number decreases for both assisting and opposing flow conditions.

Table IV displays the values of $C_f \sqrt{Re_x^{1/2}}$, $Nu/Re_x^{1/2}$, and $Sh/Re_x^{1/2}$ for different values of wall temperature power index $n$, wall concentration power index $n_1$, thermal buoyancy parameter (or thermal Grashof number) $Gr$, and the concentration buoyancy parameter (or concentration Grashof number) $Gc$ for assisting and opposing flow conditions. It is seen that the values of the local skin-friction coefficient increase as either the thermal buoyancy parameter $Gr$ or the concentration buoyancy parameter $Gc$ increases for assisting flow conditions and as either the wall temperature power index $n$ or the wall concentration power index $n_1$ increases for opposing flow conditions. However, the opposite is true; the values of the local skin-friction coefficient decrease as either $Gr$ or $Gc$ increases for opposing flow conditions and as either $n$, or $n_1$ increases for assisting flow conditions. In addition, the local Nusselt number increases as either the wall temperature power index $n$, the

Table III. Values of $C_f \sqrt{Re_x^{1/2}}$, $Nu/Re_x^{1/2}$, and $Sh/Re_x^{1/2}$ for different values of Soret and Dufour numbers

<table>
<thead>
<tr>
<th>$Sr$</th>
<th>$Du$</th>
<th>$C_f \sqrt{Re_x^{1/2}}$</th>
<th>$Nu/Re_x^{1/2}$</th>
<th>$Sh/Re_x^{1/2}$</th>
<th>$C_f \sqrt{Re_x^{1/2}}$</th>
<th>$Nu/Re_x^{1/2}$</th>
<th>$Sh/Re_x^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>-0.40375</td>
<td>0.72869</td>
<td>1.057013</td>
<td>-2.915450</td>
<td>0.43391</td>
<td>0.85816</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>-0.40882</td>
<td>0.86972</td>
<td>0.94984</td>
<td>-2.895419</td>
<td>0.57535</td>
<td>0.79081</td>
</tr>
<tr>
<td>1</td>
<td>0.12</td>
<td>-0.38735</td>
<td>0.94707</td>
<td>0.7913</td>
<td>-2.907669</td>
<td>0.64117</td>
<td>0.67228</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td>-0.31756</td>
<td>0.98463</td>
<td>0.48156</td>
<td>-2.960267</td>
<td>0.63956</td>
<td>0.44775</td>
</tr>
</tbody>
</table>

Table IV. Values of $C_f \sqrt{Re_x^{1/2}}$, $Nu/Re_x^{1/2}$, and $Sh/Re_x^{1/2}$ for different values of wall temperature power index $n$, wall concentration power index $n_1$, thermal buoyancy parameter $Gr$, and concentration buoyancy parameter $Gc$
thermal buoyancy parameter $Gr$, or the concentration buoyancy parameter $Gc$ increases for assisting flow conditions and as the wall temperature power index $n$ increases for opposing flow conditions. On the other hand, the opposite behavior is observed; the local Nusselt number decreases as either $n$, $Gr$, or $Gc$ increases for opposing flow conditions and as $n_1$ increases for assisting flow conditions. Finally, the local Sherwood number is predicted to increase as either the wall concentration power index $n_1$, the thermal buoyancy parameter $Gr$, or the concentration buoyancy parameter $Gc$ increases for assisting flow conditions and as the wall temperature power index $n$ increases for opposing flow conditions. However, the opposite behavior occurs; the local Sherwood number decreases as either $n_1$, $Gr$, or $Gc$ increases for opposing flow conditions and as $n$ increases for assisting flow conditions.

**Conclusions**

The problem of double-diffusive natural convective flow of a polar fluid near the stagnation point along a stretching surface embedded in a porous medium in the presence of a homogeneous chemical reaction and Soret and Dufour effects was investigated. The governing boundary-layer equations are formulated and then transformed into a set of similarity equations. The obtained similarity equations were solved numerically by an efficient, iterative, tri-diagonal, implicit finite-difference method. Comparison with previously published work was performed and the results were found to be in excellent agreement. It was found that

1. In general, the fluid velocity increased as either the ratio of free stream to stretching velocity or the Soret number increased and the Dufour number decreased. On the other hand, the velocity decreased as either the chemical reaction parameter, polarity parameters, or the permeability parameter increased for assisting flow conditions.

2. For assisting flow conditions, the local skin-friction coefficient increased as either the thermal buoyancy parameter or the concentration buoyancy parameter increased, and it decreased as either the polarity parameters, chemical reaction parameter, Schmidt number, wall temperature power, or the wall concentration power index increased. For opposing flow conditions, the local skin-friction increased as either the chemical reaction parameter, Schmidt number, wall temperature power index, or the wall concentration power index increased, and it decreased as either the thermal buoyancy parameter or the concentration buoyancy parameter increased.

3. For assisting flow conditions, the local Nusselt number increased as either the wall temperature power index, thermal buoyancy parameter, or the concentration buoyancy parameter increased, and it decreased as the polarity parameters, Schmidt number, wall concentration power index, and the chemical reaction parameter increased. For opposing flow conditions, the local Nusselt number increased as either the polarity parameters, Schmidt number, wall concentration power index, or the chemical reaction parameter increased, and it decreased as the wall temperature power index, thermal buoyancy parameter, or the concentration buoyancy parameter increased.

4. Finally, for assisting flow conditions, the local Sherwood number increased as either the wall concentration power index, thermal buoyancy parameter, or the concentration buoyancy parameter increased, and it decreased as either the
polarity parameters or the wall temperature power index increased. For opposing flow conditions, the local Sherwood number increased as either the wall temperature power index or the polarity parameters increased, and it decreased as either the wall concentration power index, thermal buoyancy parameter, or the concentration buoyancy parameter increased.

References


