EFFECTS OF SORET AND DUFOUR NUMBERS ON FREE CONVECTION OVER ISOTHERMAL AND ADIABATIC STRETCHING SURFACES EMBEDDED IN POROUS MEDIA

A. M. Aly,[1,*] *M. A. Mansour,*[2] *& Ali Chamkha*[3]

[1]*Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt*

[2]*Department of Mathematics, Faculty of Science, Assuit University, Assuit, Egypt*

[3]*Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh, 70654, Kuwait*

*Address all correspondence to A. M. Aly E-mail: abdelreheam.abdallah@sci.svu.edu.eg

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A numerical solution is presented for double-diffusive free convective flow over a vertical stretching surface embedded in a porous medium in the presence of a homogeneous first-order chemical reaction, radiation, and Soret and Dufour effects. Two different types of flows have been studied, namely, an isothermal surface and an adiabatic surface. The governing partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically using a fourth-order Runge–Kutta scheme with the shooting method. The results obtained show that the flow field is influenced appreciably by the presence of material parameters.

KEY WORDS: *chemical reaction, MHD, Dufour effect, Soret effect, isothermal surface, adiabatic surface*

1. INTRODUCTION

Natural convection in fluid saturated in porous media arises in a large number of natural sciences as well as several branches of technology. These include geophysics, soil mechanics, metal casting, ceramic engineering, paper technology, and insulating materials. The mass transfer caused by the temperature gradient is called the Soret effect, and it has been utilized for isotope separation and in a mixture of gases with very light molecular weight (H_2, He) and medium molecular weight (H_2, air), while the heat transfer caused by the concentration gradient is called the Dufour effect.

Eckert and Drake (1972) presented several cases of Dufour effect. Weaver et al. (1991) pointed out that when the differences in temperature and concentration are large or when the difference in molecular mass of the two elements in a binary mixture is large, the coupled interaction is significant. Due to the importance of Soret (thermal diffusion) and Dufour (diffusion-thermo) effects for fluids with very light molecular weight as well as medium molecular weight, many investigators (see, for example, Dursunkaya and Worek, 1992, Anghel et al., 2000, and Postelnicu, 2004) have studied these effects. Erickson et al. (1966) studied the problem of heat and mass transfer in the laminar boundary-layer flow of a moving flat surface with constant surface velocity and temperature, focusing on the effect of suction/injection. The problem of heat and mass transfer on a stretching sheet with suction or blowing was investigated by Gupta and Gupta (1977).

Recently, Alam and Rahman (2005) studied the Dufour and Soret effects on steady magnetohydrodynamics (MHD) free convective heat and mass-transfer flow past a semi-infinite vertical porous plate embedded in a

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porous medium. Mansour et al. (2008) studied the effects of chemical reaction, Soret number, and Dufour number on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous medium. Tsai and Huang (2009) studied the steady stagnation point flow over a vertical stretching surface in the presence of species concentration and mass diffusion under the Soret and Dufour effects.

In the present work we study the effects of chemical reaction, Soret number, Dufour number, and radiation on MHD free convective heat and mass transfer over a vertical stretching surface embedded in porous media using a self-similar transformation for two conditions, namely, an isothermal surface and a vertical adiabatic surface.

2. MATHEMATICAL ANALYSIS

Two-dimensional, steady, nonlinear MHD boundary-layer flow of an incompressible, viscous, electrically conducting fluid in the presence of a uniform magnetic field has been considered with heat, mass transfer, and chemical reaction in porous medium considering Soret and Dufour effects. The x axis is chosen parallel to the vertical surface, and the y axis is taken normal to it. A transverse magnetic field of strength B_0 is applied parallel to the y axis (see Fig. 1). The fluid properties are assumed to be constant, and the chemical reaction is taking place in the flow. The physical properties ρ, μ, D and rate of chemical reaction k_c are constant throughout the fluid. Under the boundary-layer assumptions, the governing equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

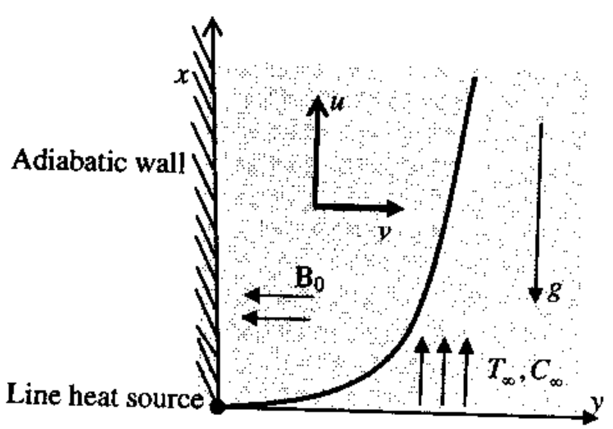

FIG. 1: The physical model and coordinate system

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T\left(T - T_\infty\right) + g\beta_C\left(C - C_\infty\right) - \left(\frac{\nu}{k_1} + \frac{\sigma B_0^2}{\rho}\right)u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_P}\frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + \upsilon\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} - k_c\left(C - C_\infty\right), \tag{4}$$

where B_0, σ, and T are the strength of magnetic field, the electrical conductivity of the fluid, and the temperature; C is the concentration; T_∞ and C_∞ are the free-stream temperature and concentration, respectively; k_c is the rate of chemical reaction; ρ density; g acceleration due to gravity; β_T the coefficient of volume expansion; β_C the volumetric coefficient of expansion with concentration; ν, k_1 and D_m the kinematics viscosity, permeability of porous media, and coefficient of mass diffusivity, respectively; q_r the radiative heat flux; C_p the specific heat at constant pressure; T_m the mean fluid temperature; k_T the thermal-diffusion ratio; and C_s is the concentration susceptibility.

The radiative heat flux term is simplified using the Rosseland approximation (see Sparrow and Cess, 1962) as

$$q_r = -\frac{4\sigma_0}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ_0 and k^* are the Stefan–Boltzman constant and mean absorption coefficient, respectively. The obtained Taylor series expansion for T^4, neglecting higher order terms, is

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Using Eqs. (5) and (6) in energy equation (3), we obtain

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\left(1 + \frac{4}{3}R\right)\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_P}\frac{\partial^2 C}{\partial y^2}, \tag{7}$$

where $\Pr = \frac{\rho\nu C_P}{k}$ is the Prandtl number, $R = \frac{4\sigma_0 T_\infty^3}{kk^*}$ is the radiation parameter, and ν is the kinematics viscosity. The boundary conditions for this problem can be written as the following:

(a) Isothermal surface with horizontal leading edge

$$u = U(x), \quad \upsilon = 0, \quad T = T_W, \quad C = C_W \text{ at } y = 0; \qquad u = 0, \quad C = C_\infty \text{ as } y \to \infty \tag{8}$$

(b) Adiabatic surface with a concentrated heat source along the horizontal leading edge

$$
\begin{aligned}
&u = U(x), \quad \upsilon = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \\
&\text{at } y = 0 \\
&u = 0, \quad C = C_\infty \quad \text{as} \quad y \to \infty
\end{aligned} \tag{9}
$$

The following transformation can be introduced:

$$
\begin{aligned}
&\psi = [\nu x U(x)]^{1/2} f(\eta), \ \ \eta = [U(x)/\nu x]^{1/2} y, \\
&U(x) = ax, \quad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \\
&(T_W - T_\infty) = T^* = s x^n, \quad \varphi = \frac{C - C_\infty}{C_W - C_\infty}, \\
&(C_W - C_\infty) = s_1 x^{n_1}
\end{aligned} \tag{10}
$$

where T_W and C_W are the wall temperature and concentration, respectively; T^* is the characteristic temperature; and n is the thermal stratification parameter constant such that $0 \leq n < 1$. The n defined as the thermal stratification parameter is equal to $m_1/(1 + m_1)$ of Nakayama and Koyama (1989), where m_1 is a constant.

It can be easily verified that the continuity Eq. (1) is identically satisfied. Here a is the dimensional constant, and the values of s, n_1, and s_1 are constants. Then, by introducing the relation (10) into Eqs. (2), (4), and (7), we obtain the following dimensionless ordinary differential equations:

$$
\begin{aligned}
&f''' + \mathrm{Gr}_x \mathrm{Re}_x \theta + \mathrm{Gc}_x \mathrm{Re}_x \varphi - \Big(M\mathrm{Re}_x + \mathrm{Re}_x \\
&+ \mathrm{Re}_x / K \Big) f' - f'^2 + f f'' = 0,
\end{aligned} \tag{11}
$$

$$
\left(1 + \frac{4}{3} R\right) \theta'' + \Pr (\mathrm{Du} \varphi'' - n f' \theta + f \theta') = 0, \tag{12}
$$

$$
\varphi'' + \mathrm{Sc} \, (\mathrm{Sr} \, \theta'' - \gamma \mathrm{Re}_x \varphi - n_1 f' \varphi + f \varphi') = 0, \tag{13}
$$

where primes denote differentiation with respect to η and $\mathrm{Re}_x = U_x/\nu$ is the Reynolds number, $\mathrm{Gr}_x = \nu g \beta_T (T_w - T_\infty)/U_3$ is the Grashof number, $\mathrm{Gc}_x = \nu g \beta_C (C_w - C_\infty)/U^3$ is the modified Grashof number, $\Pr = (\mu C_p)/k$ is the Prandtl number, $\mathrm{Sc} = \nu / D_m$ is the Schmidt number, $M = \frac{\sigma B_0^2 \nu}{\rho U^2}$ is the magnetic parameter, $K = \frac{k_1 U^2}{\nu^2}$ is the permeability parameter, $\gamma = \frac{\nu k_c}{U^2}$ is the chemical reaction parameter, $\mathrm{Du} = \frac{D_m k_T}{\nu C_S C_p} \left(\frac{C_w - C_\infty}{T_w - T_\infty} \right)$ is the Dufour number, and $\mathrm{Sr} = \frac{D_m k_T}{\nu T_m} \left(\frac{T_w - T_\infty}{C_w - C_\infty} \right)$ is the Soret number.

The corresponding boundary conditions are given by

(a) Isothermal surface

$$
\begin{aligned}
&f = \frac{\partial f}{\partial \eta} = 0, \ \theta = 1, \ \phi = 1 \ \text{ on } \ \eta = 0 \\
&\frac{\partial f}{\partial \eta} = 0, \ \theta = 0, \ \phi = 0 \quad \text{as} \quad \eta \to \infty
\end{aligned} \tag{14}
$$

(b) Adiabatic surface

$$
\begin{aligned}
&f = \frac{\partial f}{\partial \eta} = 0, \ \frac{\partial \theta}{\partial \eta} = 0, \ \frac{\partial \phi}{\partial \eta} = 0 \ \text{ on } \ \eta = 0 \\
&\frac{\partial f}{\partial \eta} = 0, \ \theta = 0, \ \phi = 0 \quad \text{as} \quad \eta \to \infty
\end{aligned} \tag{15}
$$

Also, the quantities of physical interest in this problem are the local Nusselt and Sherwood numbers.

The surface heat flux $q_w(x)$ and local Nusselt number Nu are defined by

$$
\begin{aligned}
q_w &= -k \frac{\partial T}{\partial y}\bigg|_{y=0} - \frac{4\sigma_0}{3k^*} \left(\frac{\partial T^4}{\partial y} \right)_{y=0} \\
&= [-\theta'(0)] \ (T_w - T_\infty) \ \frac{k}{x} \, (\mathrm{Re}_x)^{1/2} \left(1 + \frac{4}{3} R\right),
\end{aligned} \tag{16}
$$

$$
\mathrm{Nu} = \frac{q_w}{(T_w - T_\infty)} \left(\frac{x}{k} \right) = -\theta'(0) \, (\mathrm{Re}_x)^{1/2} \left(1 + \frac{4}{3} R\right). \tag{17}
$$

The surface mass flux, m_w, and local Sherwood number, Sh, are determined as

$$
\begin{aligned}
m_w &= -\rho D \frac{\partial C}{\partial y}\bigg|_{y=0} = [-\varphi'(0)] \, (C_w - C_\infty) \, \frac{\rho D}{x} \\
&\times (\mathrm{Re}_x)^{1/2},
\end{aligned} \tag{18}
$$

$$
\mathrm{Sh} = \frac{m_w}{(C_w - C_\infty)} \left(\frac{x}{\rho D} \right) = -\varphi'(0) \, (\mathrm{Re}_x)^{1/2} \tag{19}
$$

The governing Eqs. (11)–(13) are solved numerically subject to the boundary conditions given in Eqs. (14) and (15) using the fourth-order Runge–Kutta method along with the shooting method. The obtained results are validated by direct comparison with the work of Mansour et al. (2008). Table 1 shows the results of this comparison. It can be seen that good agreement exists between the results.

TABLE 1: Results of $f''(0)$, $\theta'(0)$, and $\varphi'(0)$ for various values of γ with $M^2 = 1$, $\mathrm{Re}_x = 1$, $\mathrm{Pr} = 0.71$, $\mathrm{Sc} = 0.62$, $\mathrm{Gr}_x = 1$, $\mathrm{Gc}_x = 2$, $n = 0.5$, $K = 1$, $\mathrm{Sr} = 0.5$ and $\mathrm{Du} = 0.3$ without radiation for isothermal surface compared with those obtained by Mansour et al. (2008)

γ	Mansour et al. (2008)			Present work		
	$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$	$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.3	0.57160	0.87710	0.67940	0.54695	0.87805	0.65110
0.5	0.59349	0.89609	0.75115	0.57358	0.89515	0.73067
0.7	0.61335	0.91408	0.81819	0.59681	0.91135	0.80323
1.0	0.63997	0.93950	0.91135	0.62676	0.93429	0.90189

3. RESULTS AND DISCUSSION

Numerical calculations were carried out for different values of material parameters characterizing the problem and for fixed values of $n_1 = 0.5$, $n = 0.5$, $K = 1$, $\mathrm{Re}_x = 1$, $\mathrm{Gr}_x = 1$, and $\mathrm{Gc}_x = 2$. Figures 2–4 show the effects of the chemical reaction parameter with two values of Soret number on the velocity, temperature, and concentration profiles for an isothermal surface. Increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as γ increases. Consequently, less flow is induced along the surface, resulting in a decrease in fluid velocity in the boundary layer. Also, increasing the chemical reaction parameter leads to a decrease in the temperature profiles. In addition, increasing the Soret number leads to an increase in both the velocity and concentration profiles, while the temperature profiles decrease. Figure 5 shows the influence of the magneticfield parameter on the velocity profiles for an adiabatic increases, the velocity profiles increase. Figures 6 and 7 present the effects of the Soret and Dufour numbers on the temperature profiles for an adiabatic surface. It is seen that as the Soret number increases and the Dufour number decreases, both the temperature and concentration profiles decrease.

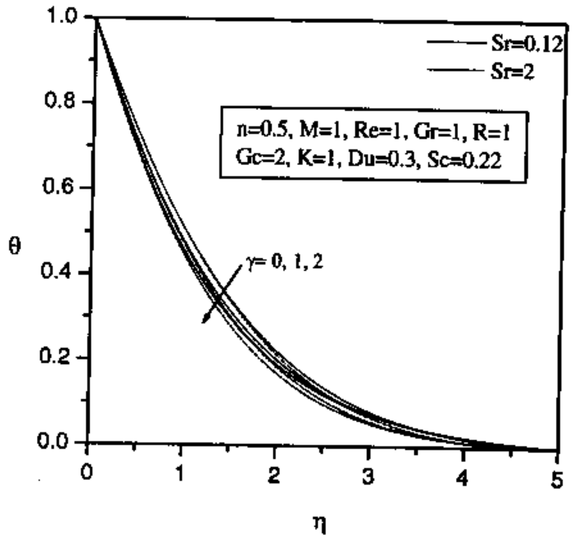

FIG. 3: Effects of chemical reaction with two values of Soret number on temperature profiles for the isothermal surface case

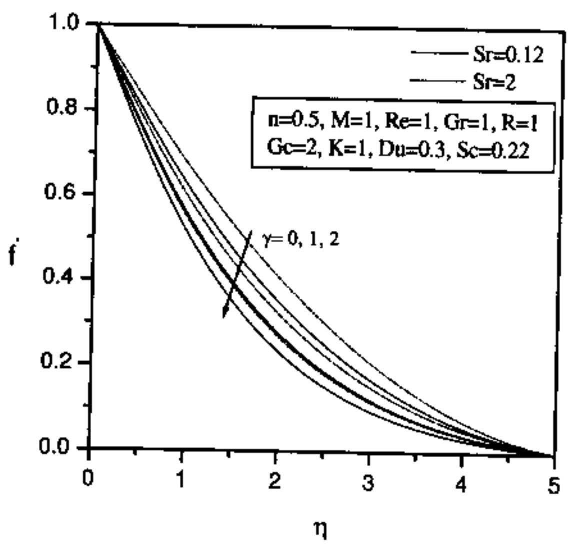

FIG. 2: Effects of chemical reaction with two values of Soret number on velocity profiles for the isothermal surface case

Table 2 illustrates the influence of the Soret and Dufour numbers on the values of, $\theta'(0)$, and $\varphi'(0)$ for the isothermal surface case and $f''(0)$, $\theta(0)$, and $\varphi(0)$ for the adiabatic surface case. It is observed that as the Soret

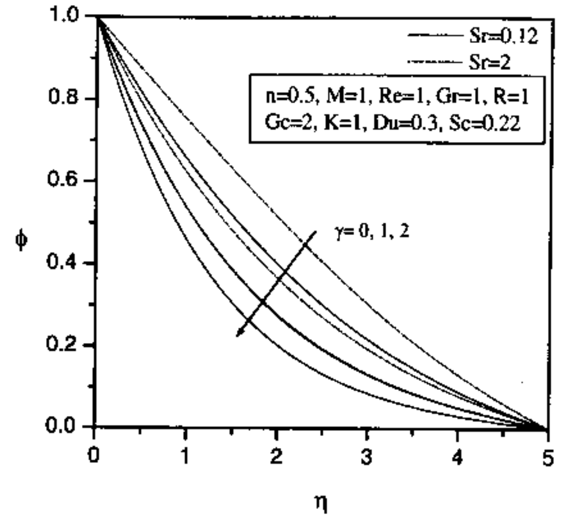

FIG. 4: Effects of chemical reaction with two values of Soret number on the concentration profiles for the isothermal surface case

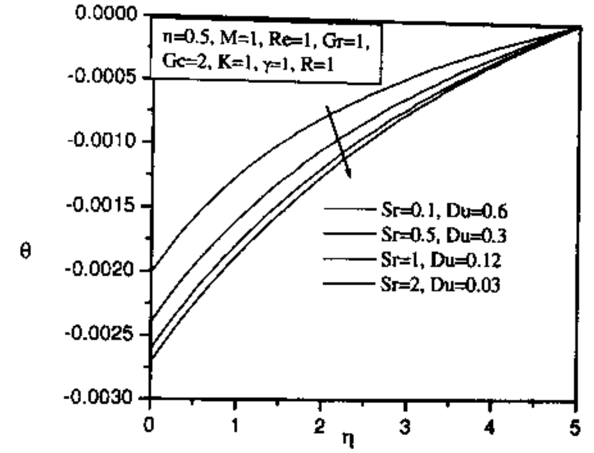

FIG. 6: Effects of Soret and Dufour numbers on temperature profiles for the adiabatic surface case

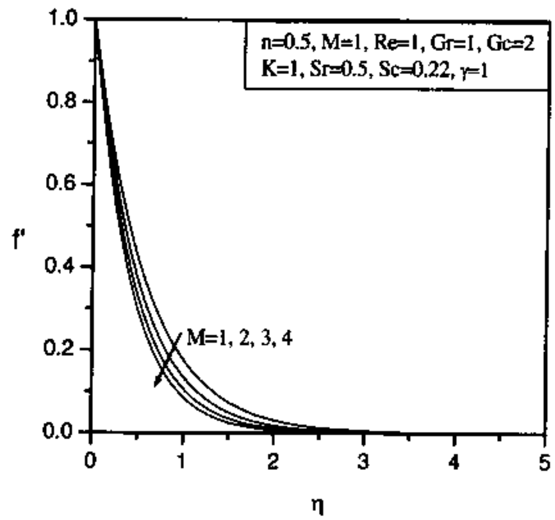

FIG. 5: Effects of magnetic field parameter on the velocity profiles for the adiabatic surface case

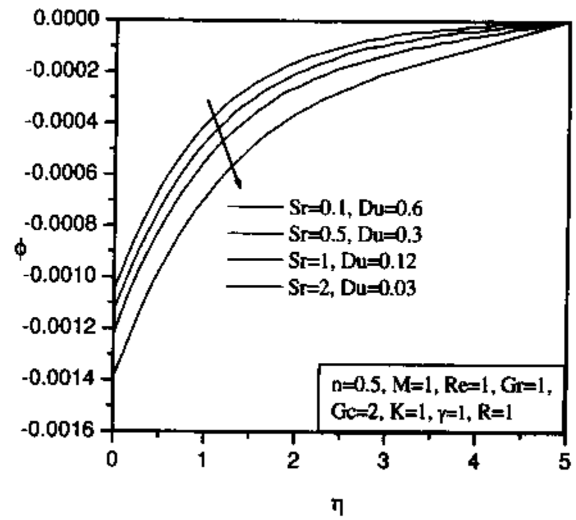

FIG. 7: Effects of Soret and Dufour numbers on the concentration profiles for the adiabatic surface case

number increases and the Dufour number decreases, the values of all of the local skin-friction coefficient $-f''(0)$, Nusslet number, and Sherwood number increase for the isothermal surface case. However, as the Soret number increases and the Dufour number decreases, the local skin-friction coefficient $-f''(0)$ decreases, while the wall temperature $\theta(0)$ and the wall concentration $\varphi(0)$ increase for the adiabatic surface condition.

4. CONCLUSION

The problem of double-diffusive natural convective flow along a vertical stretching surface embedded in a porous medium in the presence of a homogeneous chemical reaction, radiation, and Soret and Dufour effects was investigated. Two cases of surface conditions, namely, isothermal and adiabatic thermal, were considered. The governing boundary-layer equations were formulated, nondimensionalized, and then transformed into a set of similarity equations which were solved numerically using

TABLE 2: Results of $f''(0)$, $\theta'(0)$, $\varphi'(0)$, $\theta(0)$, and $\varphi(0)$ for various values of Sr and Du with $M^2 = 1$, $\mathrm{Re}_x = 1$, $\gamma = 1$, Pr = 0.71, Sc = 0.62, $n = 0.5$, $\mathrm{Gr}_x = 1$, $\mathrm{Gc}_x = 2$, $K = 1$, and $R = 1$ for isothermal surface and adiabatic surface

Sr	Du	**Isothermal surface**			**Adiabatic surface**		
		$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$	$-f''(0)$	$-\theta(0)$	$-\varphi(0)$
2	0.03	0.52093	0.50444	0.84865	1.73529	0.00436	0.00113
1	0.12	0.55790	0.52998	0.93717	1.73522	0.00434	0.00108
0.5	0.3	0.58472	0.59824	0.97777	1.73516	0.00428	0.00105
0.1	0.6	0.62328	0.73126	1.02644	1.73506	0.00416	0.00103

a fourth-order Runge–Kutta scheme with the shooting method. Comparisons with previously published work were made, and the results were found to be in good agreement. It was found that as the Soret number increases and the Dufour number decreases, the local skin-friction coefficient increases for the isothermal surface case, while it decreases for the adiabatic surface case. The rate of heat and mass transfer increase as the Soret number increases and the Dufour number decreases for the isothermal surface case. Also, the wall temperature and concentration increase for the adiabatic surface case as the Soret number increases and the Dufour number decreases.

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