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Effect of chemical reaction on heat and mass transfer by mixed convection flow about a sphere in a saturated porous media

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Abstract

Purpose – The purpose of this paper is to study the effects of chemical reaction on mixed convection flow along a sphere in non-Darcian porous media.

Design/methodology/approach – The sphere surface is maintained at uniform temperature and species concentration for both cases of heated (assisting flow) and cooled (opposing flow) sphere. An appropriate transformation is employed and the transformed equations are solved numerically using an efficient implicit iterative tri-diagonal finite difference method.

Findings – It is found that chemical reactions have significant effect on heat and mass transfer. Comparisons with previously published work are performed and the results are found to be in excellent agreement.

Originality/value – The paper is original and describes how a parametric study of the physical parameters was conducted and illustrates graphically a representative set of numerical results for the velocity, temperature, and concentration profiles, as well as the local skin-friction coefficient, local wall temperature, and local wall concentration, to show interesting features of the solutions.

Keywords Heat transfer, Mass transfer, Porous materials, Convection, Chemical reactions

Paper type Research paper



1. Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy

and species transport. Also, the convective heat and mass transfer in a saturated porous medium has many important applications in geothermal and geophysical engineering such as the extraction of geothermal energy, the migration of moisture in fibrous insulation, underground disposal of nuclear waste, and the spreading of chemical pollutants in saturated soil. However, representative studies in this area may be found in the monographs by Nield and Bejan (1999), Vafai (2000), Pop and Ingham (2001) and Ingham and Pop (2002). Although Darcy model has been considered by most of the researchers in their studies on convection in a porous media, but it is now being realized that this model is applicable only under special circumstances, and therefore a generalized model, which includes Forchheimer's inertia term and Brinkman's viscous term has to be considered for the accurate prediction of convection in a porous medium. Vafai and Tien (1982) have summarized the importance of both boundary and inertia effects in porous media. The problems of heat and mass transfer by mixed convection on bodies embedded in a non-Darcian porous medium have been extensively studied by many authors (see, for example, Lai, 1991, has investigated the coupled heat and mass transfer by mixed convection from an isothermal vertical plate in a porous medium). Lai and Kulacki (1990a, b) have reported similarity solutions for mixed convection flow over horizontal and inclined plates embedded in fluid-saturated porous media in the presence of surface mass flux. Yih (1998) investigated the coupled heat and mass transfer in mixed convection over a vertical flat plate embedded in saturated porous media. Yih (1999) has also analyzed coupled heat and mass transfer in mixed convection about an inclined surface in saturated porous medium. The Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the coupled effects of thermal and mass diffusion is analyzed by Jumah *et al.* (2001). The study of Dufour and Soret effects on heat and mass transfer by Magneto-Hydro-Dynamics (MHD) mixed convection from a radiate vertical permeable plate embedded in porous media were considered by Chamkha and Ben-Nakhi (2008).

On other hand, the combined heat and mass transfer problems with chemical reactions in porous medium are of importance in many processes, and therefore, have received a considerable amount of attention in recent years. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, the heat and mass transfer occurs simultaneously. Chemical reactions can be classified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be the first order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glassware, food processing, tubular reactors, oxidation of solid materials, and synthesis of ceramic materials. Das *et al.* (1994) considered the effects of a first-order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan (2001) have studied the first-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Chamkha *et al.* (2004) have investigated the double-diffusive convective flow of a micropolar fluid over a vertical plate embedded in a porous medium with a

chemical reaction. Recently, the heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction has been analyzed numerically by Postelnicu (2007). Non-Darcian and chemical reaction effects on mixed convective heat and mass transfer past a porous wedge are presented by Kandasamy *et al.* (2008). Rashad et El-Kabeir (2010) have studied the coupled heat and mass transfer in transient flow by mixed convection past a vertical stretching sheet embedded in a fluid-saturated porous medium in the presence of a chemical reaction effect. The effects of permeability and chemical reaction on heat and mass transfer over an infinite moving permeable plate in a saturated porous medium were reported by Modather *et al.* (2009).

Motivated by the investigations mentioned above, the purpose of the present work is to consider simultaneous heat and mass transfer by mixed convection flow about a solid sphere embedded in a porous medium subjected with uniform wall temperature and species concentration, considering Brinkman-Forchheimer extended Darcy model in the presence chemical reaction effect. The order of chemical reaction in this work is taken as first-order reaction.

2. Governing equations

Consider the problem of combined heat and mass transfer in mixed convection, flow of a viscous, incompressible fluid about a solid sphere of radius a which is embedded in a non-Daracian porous medium in the presence of chemical reaction effect. The flow model and physical coordinate system are shown in Figure 1. The sphere is

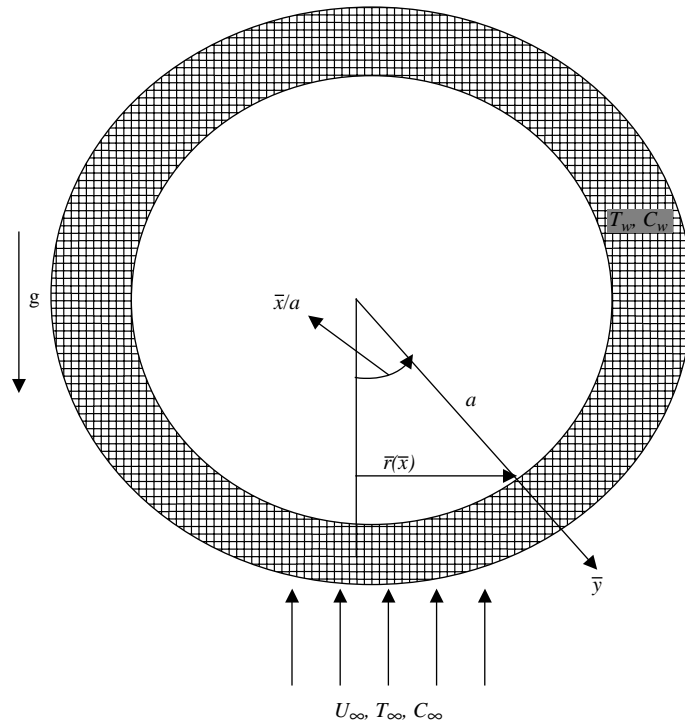


Figure 1.
Physical model and coordinate system

placed in a flow field with the undisturbed free stream velocity U_∞ , the ambient porous medium temperature T_∞ and the ambient porous medium species concentration C_∞ . The convective forced flow is assumed to be moving upward, while the gravity vector g acts downward in the opposite direction, where the local orthogonal coordinates \bar{x} measures the distance along the surface of the sphere from the stagnation point and \bar{y} measures the distance normal to the surface in the fluid, respectively. Also, it is considered that the surface of the sphere is maintained at a constant temperature T_w with $T_w > T_\infty$ and constant species concentration C_w with $C_w > C_\infty$ corresponding to a heated sphere (aiding flow). However, the system can be also for downward flow (opposing flow) by changing the inequality in the wall temperatures, i.e. $T_w < T_\infty$, $C_w < C_\infty$ corresponding to a cooled sphere (opposing flow). All fluid properties are assumed constant except the density in the buoyancy term of the x -momentum equation and a first-order homogeneous chemical reaction is assumed to take place in the flow. Taking all of the above assumptions into consideration and invoking the boundary layer and Boussinesq approximations, the governing equations can be written in dimensional form as (Nazar *et al.*, 2002):

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}} = 0, \quad (1)$$

$$\bar{u} \frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v} \frac{\partial\bar{u}}{\partial\bar{y}} = \bar{u}_e \frac{\partial\bar{u}_e}{\partial\bar{x}} + \nu \frac{\partial^2\bar{u}}{\partial\bar{y}^2} + g \sin\left(\frac{\bar{x}}{a}\right) (\beta(T - T_\infty) + \beta^*(C - C_\infty)) + \frac{\nu\varepsilon}{K} (\bar{u}_e - \bar{u}) + \frac{F\varepsilon^2}{K^{1/2}} (\bar{u}_e^2 - \bar{u}^2), \quad (2)$$

$$\bar{u} \frac{\partial T}{\partial\bar{x}} + \bar{v} \frac{\partial T}{\partial\bar{y}} = \frac{\nu}{\text{Pr}} \frac{\partial^2 T}{\partial\bar{y}^2}. \quad (3)$$

$$\bar{u} \frac{\partial C}{\partial\bar{x}} + \bar{v} \frac{\partial C}{\partial\bar{y}} = \frac{\nu}{\text{Sc}} \frac{\partial^2 C}{\partial\bar{y}^2} - k(C - C_\infty), \quad (4)$$

where $\bar{r}(\bar{x}) = a \sin(\bar{x}/a)$, $\bar{u}_e(\bar{x}) = (3/2)U_\infty \sin(\bar{x}/a)$.

The boundary conditions for this problem are defined as follows:

$$\begin{aligned} \bar{u} = \bar{v} = 0, \quad T = T_w, \quad C = C_w \quad \text{on } \bar{y} = 0 \\ \bar{u} \rightarrow \bar{u}_e(\bar{x}), \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{on } \bar{y} \rightarrow \infty, \end{aligned} \quad (5)$$

where \bar{u} and \bar{v} are the velocity components along \bar{x} and \bar{y} axes, respectively, $\bar{r}(\bar{x})$ is the radial distance from symmetric axis to surface of the sphere. $\bar{u}_e(\bar{x})$ is the local free stream velocity. g , β , β^* , and ν are the gravitational acceleration, thermal diffusivity, thermal expansion coefficient, concentration expansion coefficient, and kinematic viscosity, respectively. T and C are the temperature and concentration of the fluid. K , F , and ε are the dimensional permeability parameter of the porous medium, inertia term in second-order matrix and the porosity of the medium, respectively. ρ , C_p , and k are the fluid density, specific heat at constant pressure and the dimensional chemical reaction parameter, respectively. Pr and Sc are the Prandtl and Schmidt numbers for a porous medium, respectively. The above equations are further non-dimensionalised using the new variable:

$$\xi = \frac{\bar{x}}{a}, \eta = \text{Re}^{1/2} \left(\frac{\bar{y}}{a} \right), r(\xi) = \frac{\bar{r}(\bar{x})}{a}, u = \frac{\bar{u}}{U_\infty}, v = \text{Re}^{1/2} \left(\frac{\bar{v}}{U_\infty} \right), \quad (6)$$

$$u_e(\xi) = \frac{\bar{u}_e(\bar{x})}{U_\infty}, \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\xi, \eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where $\text{Re} = U_\infty a / \nu$ is the Reynolds number.

Substituting equation (6) into equations (1)-(5) leads the following non-dimensional equations:

$$\frac{\partial(ru)}{\partial \xi} + \frac{\partial(rv)}{\partial \eta} = 0, \quad (7)$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} = u_e \frac{\partial u_e}{\partial \xi} + v \frac{\partial^2 u}{\partial \eta^2} + \Lambda \sin \xi (\theta + N \phi) + K_1 (u_e - u) + K_2 (u_e^2 - u^2), \quad (8)$$

$$u \frac{\partial \theta}{\partial \xi} + v \frac{\partial \theta}{\partial \eta} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial \eta^2}, \quad (9)$$

$$u \frac{\partial \phi}{\partial \xi} + v \frac{\partial \phi}{\partial \eta} = \frac{1}{\text{Sc}} \frac{\partial^2 \phi}{\partial \eta^2} - \gamma \phi. \quad (10)$$

The dimensionless boundary conditions become:

$$u = v = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on } \eta = 0$$

$$u \rightarrow \frac{3}{2} \sin \xi, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (11)$$

where $\Lambda = Gr/\text{Re}^2$ is the mixed convection parameter, $Gr = g\beta(T_w - T_\infty)a^3/\nu^2$ is the Grashof number, Re is the Reynolds numbers, respectively. ξ is the circumferential position parameter, $N = \beta^*(C_w - C_\infty)/\beta(T_w - T_\infty)$ is the ratio of the buoyancy force due to mass diffusion to the buoyancy force due to the thermal diffusion, $K_1 = (\varepsilon\nu a)/(U_\infty K)$, $K_2 = (F\varepsilon^2 a)/K^{1/2}$ being the dimensionless permeability parameter and the inertial parameter (which correspond to the Darcian and Forchheimer flows) and $\gamma = ka/U_\infty$ is the dimensionless of chemical reaction parameter. It is worth mentioning that $\Lambda > 0$ is for assisting flow (heated sphere) and $\Lambda < 0$ is for opposing flow (cooled sphere). It should be noted that for small values of $|\Lambda|$ the forced convection effects dominate while for large values of $|\Lambda|$, natural convection is important, so that the value of Λ is of $O(1)$, where both effects are comparable, are of most interest.

To solve equations (6)-(10) subject to the boundary conditions (equation (11)), we assume the form proposed by Nazar *et al.* (2002) as: $\psi = \xi r(\xi) f(\xi, \eta)$, where ψ is the non-dimensional stream function defined in the usual way as: $u = (1/r)\partial\psi/\partial y$ and $v = -(1/r)\partial\psi/\partial x$. Then we have the following transformed equations:

$$f''' + (1 + \xi \cot \xi)ff'' - f'^2 + \frac{9}{8} \frac{\sin 2\xi}{\xi} + \Lambda \frac{\sin \xi}{\xi} (\theta + N\phi) + K_1 \left(\frac{3}{2} \frac{\sin \xi}{\xi} - f' \right) + K_2 \xi \left(\left(\frac{3}{2} \frac{\sin \xi}{\xi} \right)^2 - f'^2 \right) = \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right), \quad (12)$$

$$\frac{1}{\text{Pr}} \theta'' + (1 + \xi \cot \xi)f\theta' = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right), \quad (13)$$

$$\frac{1}{\text{Sc}} \phi'' + (1 + \xi \cot \xi)f\phi' - \gamma\phi = \xi \left(f' \frac{\partial \phi}{\partial \xi} - \phi' \frac{\partial f}{\partial \xi} \right). \quad (14)$$

Subject to the boundary conditions:

$$\begin{aligned} f = f' = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on } \eta = 0 \\ f' \rightarrow \frac{3}{2} \frac{\sin \xi}{\xi}, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (15)$$

It can be seen from the above equations that at the lower stagnation point of the sphere, $\xi \approx 0$, equations (12)-(14) reduce to:

$$f''' + 2ff'' - f'^2 + \Lambda(\theta + N\phi) + K_1 \left(\frac{3}{2} - f' \right) + \frac{9}{4} = 0, \quad (16)$$

$$\frac{1}{\text{Pr}} \theta'' + 2f\theta' = 0, \quad (17)$$

$$\frac{1}{\text{Sc}} \phi'' + 2f\phi' - \gamma\phi = 0, \quad (18)$$

and the boundary conditions (equation (15)) become:

$$\begin{aligned} f(0) = f'(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ f'(\infty) = \frac{3}{2}, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \end{aligned} \quad (19)$$

where the primes denote differentiation with respect to η .

In practical applications, the physical quantities of principal interest are the skin-friction coefficient and the Nusselt and Sherwood numbers. These can be computed from the following relations:

$$\begin{aligned} (\text{Re}^{1/2} U_\infty) C_f &= \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}, \\ Nu &= - \frac{a}{k^* (T_w - T_\infty)} \text{Re}^{-1/2} \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0}, \\ Sh &= - \frac{a}{D(C_w - C_\infty)} \text{Re}^{-1/2} \left(\frac{\partial C}{\partial \bar{y}} \right)_{\bar{y}=0} \end{aligned} \quad (20)$$

Using the non-dimensional variables (equation (6)), one obtains the following:

$$C_f = \xi f''(\xi, 0), \quad Nu_x = -\theta'(\xi, 0), \quad Sh = -\phi'(\xi, 0). \quad (21)$$

3. Numerical method

The non-similar equations (12)-(14) are linearized and then discretized using three-point central difference quotients with variable step sizes in the η direction and using two-point backward difference formulae in the ξ direction with a constant step size. The resulting equations form a tri-diagonal system of algebraic equations that can be solved by the well-known Thomas algorithm (Blottner, 1970). The solution process starts at $\xi = 0$ where equations (16)-(18) are solved and then marches forward using the solution at the previous line of constant ξ until it reaches the desired value of ξ ($= 120^\circ$ in this case). Owing to the non-linearities of the equations, an iterative solution with successive over or under relaxation techniques is required. The convergence criterion required that the maximum absolute error between two successive iterations be 10^{-6} . The computational domain was made of 196 grids in the η direction and 1,000 grids in the ξ direction. A starting step size of 0.001 in the η direction with an increase of 1.0375 times the previous step size and a constant step size in the ξ direction of 0.0021 were found to give very accurate results. The maximum value of η (η_∞) which represented the ambient conditions was assumed to be 35. The step sizes employed were arrived at after performing numerical experimentations to assess grid independence and ensure accuracy of the results. The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by Nazar *et al.* (2002) for various values of Λ at $K_1 = K_2 = \gamma = 0$ (in the absence of permeability of porous medium, inertia effects and mass transfer) at the lower stagnation point of the sphere, i.e. $\xi \approx 0$. Table I presents the results of this comparison. It can be seen from this table that excellent agreement between the results exists. This favorable comparison lends confidence in the numerical results to be reported in the next section.

4. Results and discussions

A comprehensive numerical parametric study is conducted and the results are reported in terms of graphs. Numerical calculations have been carried out for different values of the dimensionless permeability parameter K_1 , the mixed parameter Λ , the chemical reaction parameter γ , inertial parameter K_2 , and the circumferential position parameter ξ . Throughout the calculations, we have fixed the values of Pr and Sc as Pr = 0.68, Sc = 0.22. The values of Prandtl number Pr and Schmidt number Sc are chosen to represent hydrogen at 25°C and 1 atmospheric pressure and the corresponding buoyancy ratio parameter N is taken to be 1.0 for low concentration.

The effects of the permeability parameter K_1 and the mixed convection parameter Λ for the buoyancy aiding/opposing flow on the velocity and temperature and concentration profiles are shown in Figures 2-4, respectively. For the buoyancy opposing flow condition ($\Lambda < 0$), decreasing the permeability parameter K_1 has the tendency to resist the flow. This in turn, produces decreases in the maximum velocity and increases in both of the fluid temperature and concentration. In contrast, increasing the mixed convection parameter for the buoyancy aiding flow condition ($\Lambda > 0$) has an effect opposite to that of K_1 on all of the velocity, temperature, or concentration profiles. However, increasing the values of the mixed convection parameter

Λ	Nazar <i>et al.</i> (2002)		Present results	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
-4.7	-0.0081	0.5892	-0.00842	0.58933
-4.6	0.0770	0.6011	0.07705	0.60097
-4.5	0.1566	0.6117	0.15641	0.61140
-4.0	0.5028	0.6534	0.50294	0.65272
-3.0	1.0700	0.7108	1.07027	0.71098
-2.0	1.5581	0.7529	1.55801	0.75182
-1.0	2.0016	0.7870	2.00157	0.78693
0.0	2.4151	0.8162	2.41518	0.81604
1.0	2.8064	0.8463	2.80593	0.84643
2.0	3.1804	0.8648	3.18042	0.86461
3.0	3.5401	0.8857	3.54026	0.88544
4.0	3.8880	0.9050	3.88737	0.90481
5.0	4.2257	0.9230	4.22527	0.92271
6.0	4.5546	0.9397	4.55427	0.93845
7.0	4.8756	0.9555	4.87542	0.95470
8.0	5.1896	0.9704	5.18856	0.97039
9.0	5.4974	0.9846	5.49538	0.98463
10.0	5.7995	0.9981	5.79747	0.99869
20.0	8.5876	1.1077	8.58663	1.10765

Table I.
Comparison of $f''(0)$ and $-\theta'(0)$ for different values of Λ for $K_1 = Q = \gamma = 0$ and $Pr = 0.7$

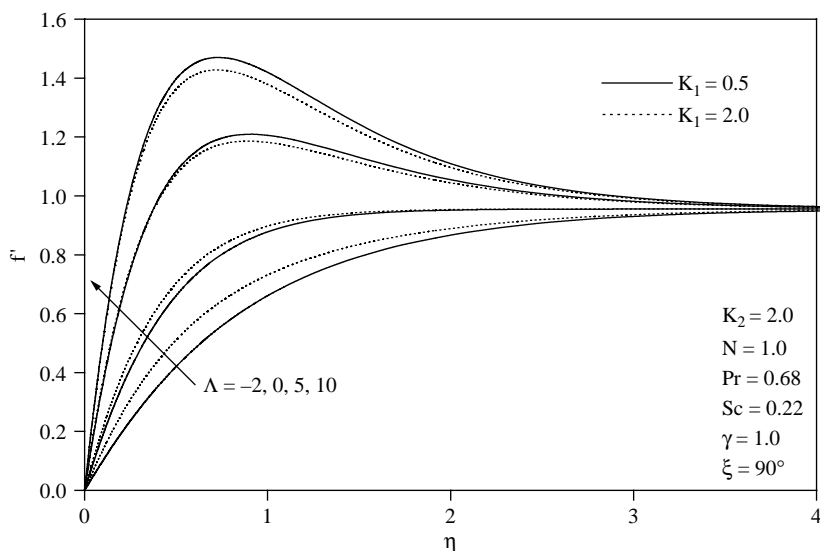


Figure 2.
Effects of Λ and K_1 on velocity profiles

Λ increases the velocity inside the boundary layer due to favorable buoyancy effects and consequently, increases both the temperature and concentration profiles. It is interesting to note that the distinctive peaks in the velocity profiles move toward the sphere surface as Λ increases.

Figures 5-7 show the variations of the local skin-friction coefficient and the local Nusselt and Sherwood numbers against the circumferential position ξ for different

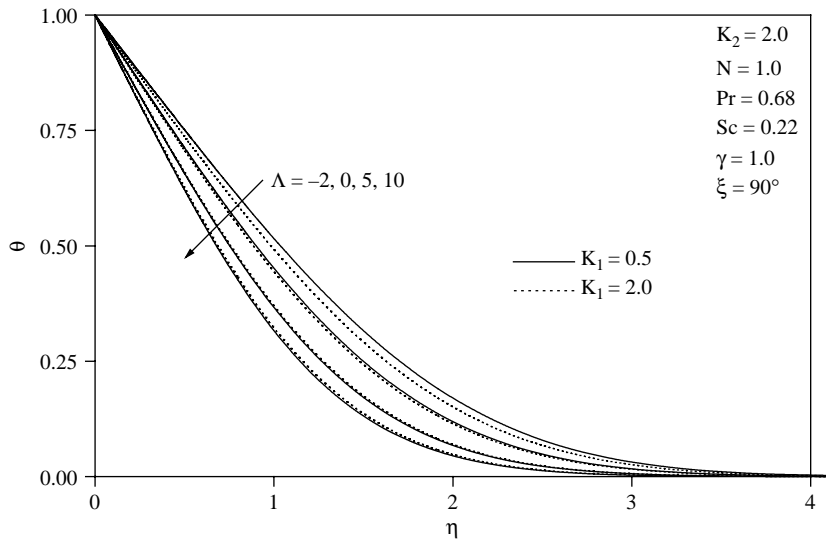


Figure 3.
Effects of Λ and K_1 on temperature profiles

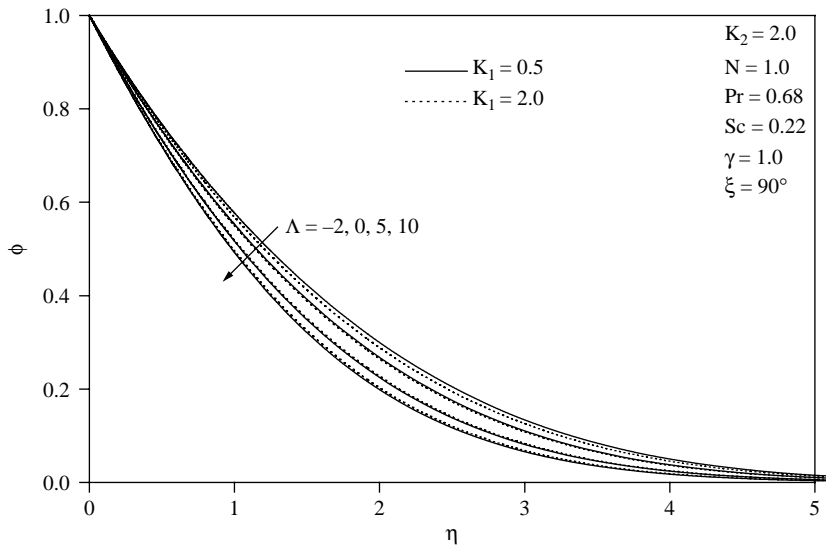


Figure 4.
Effects of Λ and K_1 on concentration profiles

values of the permeability parameter and the mixed convection parameter, respectively. It is found that increasing the permeability parameter leads to increases in both the local Nusselt and Sherwood numbers for opposing flow conditions and decreases the local Nusselt and Sherwood numbers for aiding flow conditions. Moreover, as K_1 increases, the local skin-friction coefficient increases for both aiding and opposing flow conditions. It is also seen that the mixed convection parameter has pronounced effects on the local skin-friction coefficient, and the local Nusselt and Sherwood numbers. Furthermore,

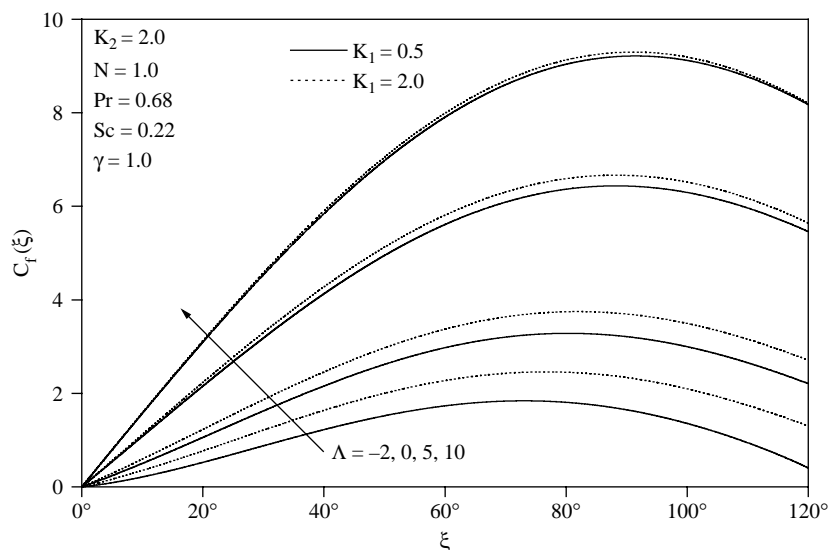


Figure 5.
Effects of Λ and K_1
on local skin-friction
coefficient

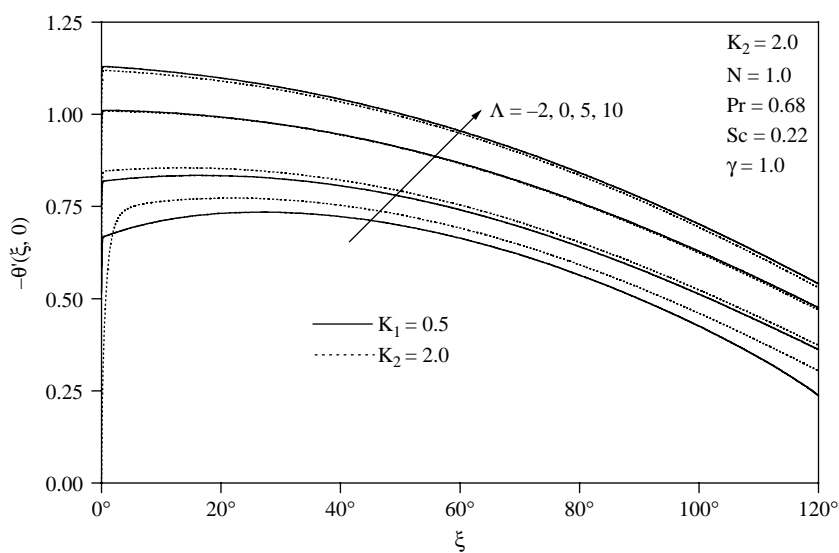


Figure 6.
Effects of Λ and K_1
on local Nusselt number

increasing the mixed convection parameter enhances the local skin-friction coefficient as well as the local the Nusselt and Sherwood numbers. It is also observed that the permeability parameter has a more pronounced effect on the local skin-friction coefficient as well as local heat and mass transfer rates for opposing flow conditions than for aiding flow conditions. It should be mentioned that Nazar *et al.* (2002) found that both the local skin-friction coefficient and the local Nusselt number increase with increasing buoyancy forces for aiding flow conditions and decrease with increasing buoyancy forces for

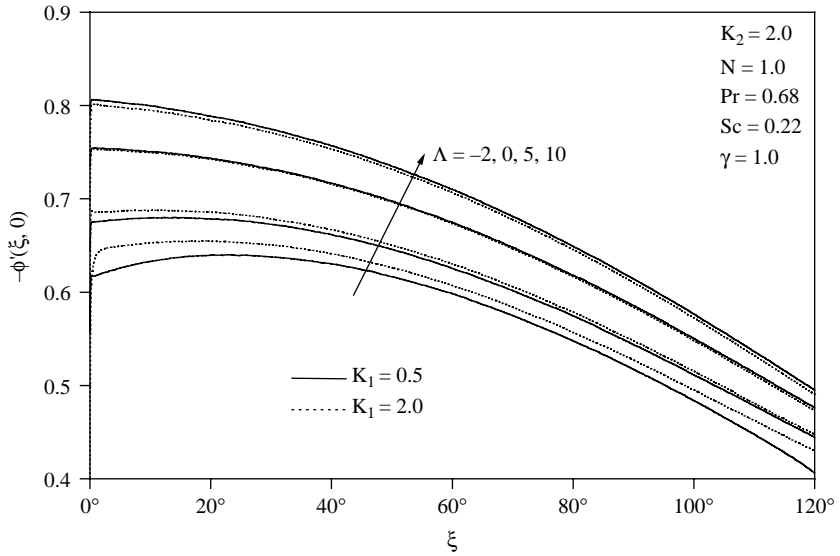


Figure 7.
Effects of Λ and K_1 on
local Sherwood number

opposing flow conditions. They also found that the effect of the buoyancy forces on the forced convection becomes significant for $\Lambda > 1.67$ and $\Lambda < -1.33$, for aiding and opposing flow conditions, respectively.

The effects of the chemical reaction parameter γ and the inertial parameter K_2 on the velocity and temperature and concentration profiles are shown in Figures 8-10, respectively. It is seen from these figures that the velocity of the fluid and solute concentration decrease with increasing values of $\gamma > 0$ (destructive chemical reaction)

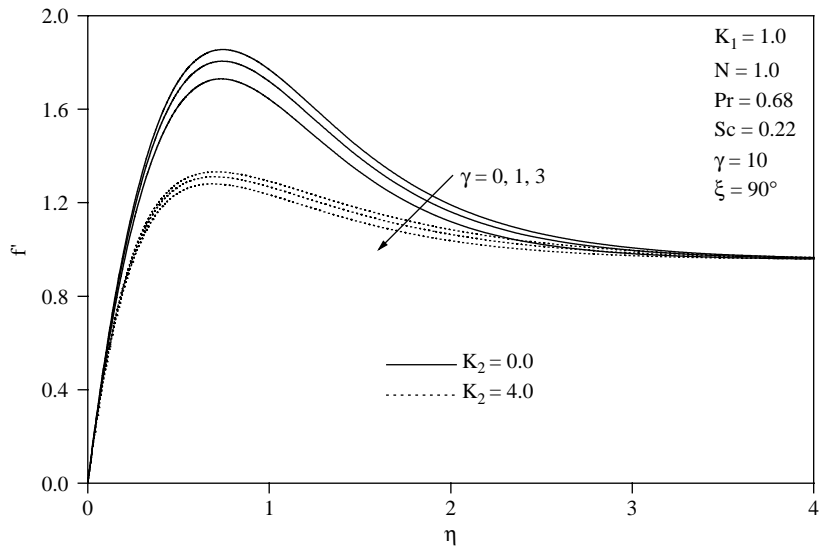


Figure 8.
Effects of γ and K_2 on
velocity profiles

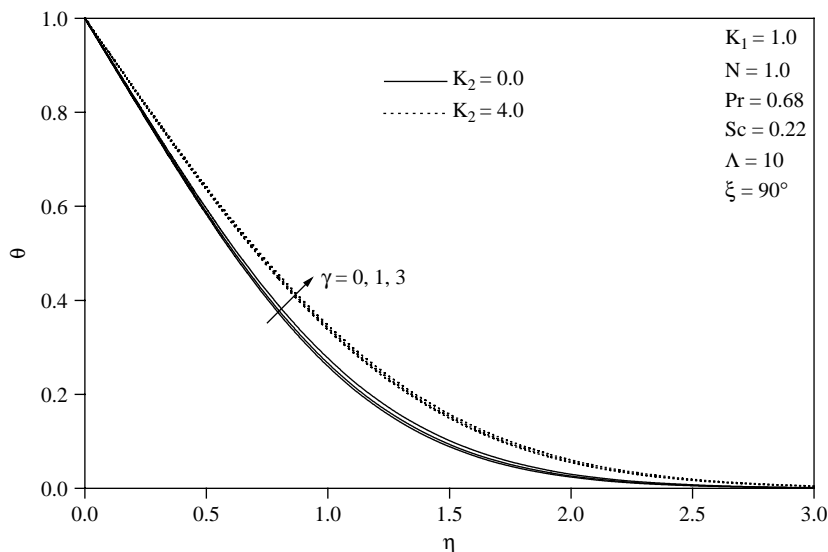


Figure 9.
Effects of γ and K_2 on
temperature profiles

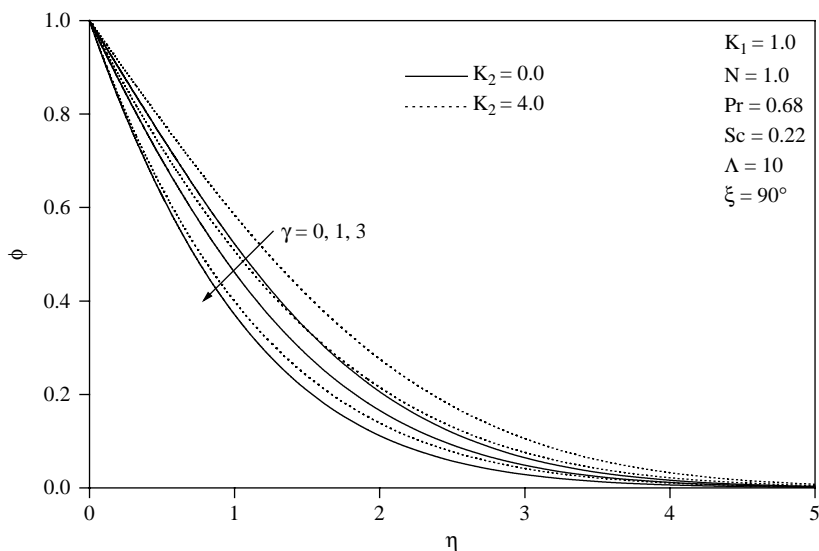


Figure 10.
Effects of γ and K_2 on
concentration profiles

whereas the changes in the temperature of the fluid are not that significant due to increases in the chemical reaction parameter. Also, it is observed that the concentration of the solute in fluid decreases uniformly near the wall of the sphere. Further, it is obvious that the velocity increases slightly as the inertial parameter K_2 increases (due to the increase in the Forchheimer drag in the porous medium). The reason for this behavior is that the inertia of the porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with

enhanced temperature throughout the boundary layer adjacent to the sphere surface and the concentration profiles.

Finally, the variations of the local skin-friction coefficient and the local Nusselt and Sherwood numbers against the circumferential position ξ for different values of the chemical reaction parameter γ and inertial parameter K_2 are shown in Figures 11-13, respectively. The porous medium inertia effects constitute a strong resistance to flow. Increasing the inertial parameter K_2 boosts the Forchheimer inertial drag which slows

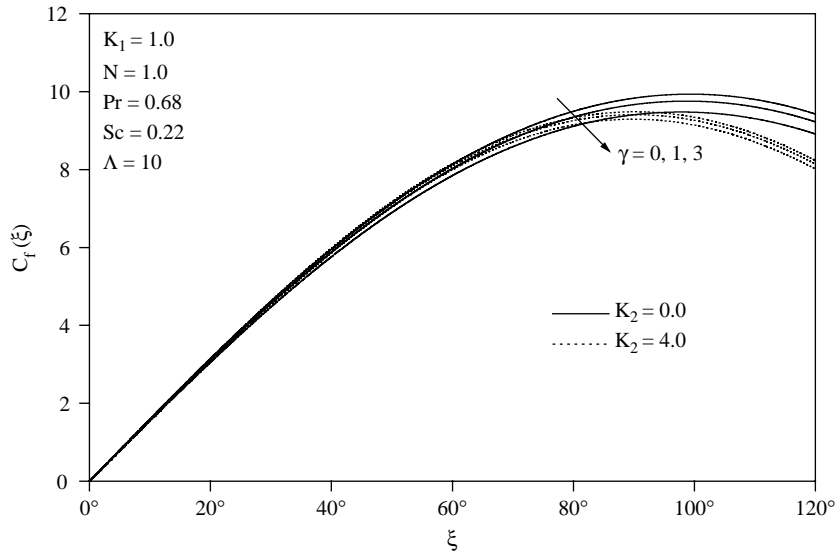


Figure 11.
Effects of γ and K_2 on local skin-friction coefficient

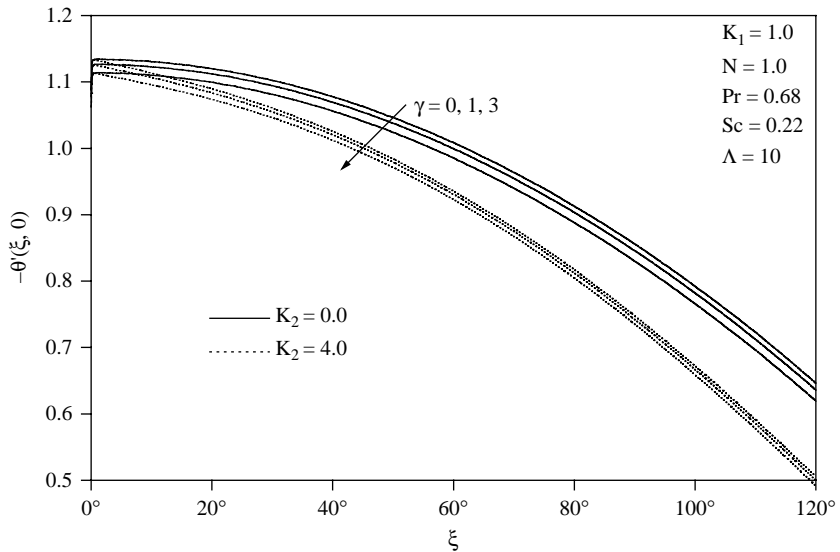


Figure 12.
Effects of γ and K_2 on local Nusselt number

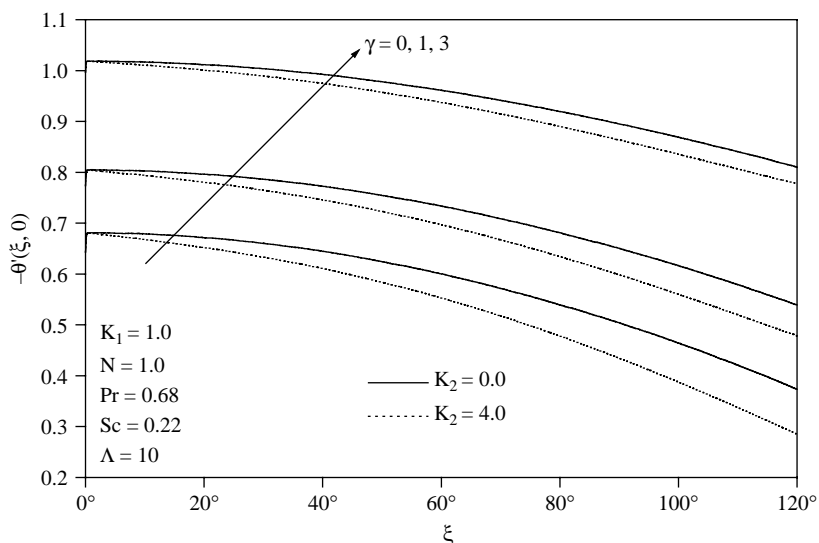


Figure 13.
Effects of γ and K_2 on
local Sherwood number

down the flow in the porous medium. Therefore, the local skin-friction coefficient is seen to be reduced considerably as K_2 increases, and consequently, both the negative wall slopes of the temperature and concentration profiles are reduced. Moreover, it can be seen that as γ increases, the local Sherwood number increases, while an opposite effect is found for both of the local skin-friction coefficient and the local Nusselt number. This is because as γ increases, the concentration difference between the surface and the fluid decreases and so the rate of mass transfer at the surface must increase, while both of the wall local skin-friction coefficient and rate of heat transfer decrease as a result of the decrease in the flow velocity and fluid temperature, respectively. Finally, the magnitude of the local skin-friction coefficient is minimum at the circumferential position $\xi = 0$ (lower stagnation point of the sphere) whereas the magnitude of the local Nusselt number and the local Sherwood number are maximum there.

5. Conclusions

The effect of chemical reaction on coupled heat and mass transfer by mixed convection boundary layer flow of a chemically reacting fluid past a solid sphere with constant surface temperature and species concentration saturated porous medium for both cases of assisting flow and opposing flow using the Brinkman-Forchheimer extended Darcy model was investigated. The problem was formulated and the governing equations were transformed into a set of non-similar equations which were then solved by an accurate implicit finite difference method. Comparisons with previously published work were performed and the results were found to be in excellent agreement. Numerical results for the velocity, temperature, and concentration profiles as well as the local skin-friction coefficient, local wall temperature, and local wall concentration were reported graphically. It was found that both the local Nusselt and Sherwood numbers decreased due to increases in either of the permeability parameter or the inertial parameter for aiding flow conditions. However, they both increased due to increases in the mixed convection

parameter. Also, increases in the values of the chemical reaction parameter produced decreases in the local Nusselt number and increases in the local Sherwood number. Finally, the local skin-friction coefficient was increased as either of the mixed convection, the permeability and inertial parameters or the circumferential position increased, and it was decreased due to increases in the chemical reaction parameter. Furthermore, it was observed that the permeability parameter had a more pronounced effect on the local skin-friction coefficient as well as local Nusselt and Sherwood numbers for opposing flow conditions than for aiding flow conditions. Moreover, the mixed convection parameter had a comparable effect on the local Sherwood number as that on the local Nusselt number. It is hoped that the present work will serve as a motivation for future experimental work which seems to be lacking at the present time.

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