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FULLY DEVELOPED MIXED CONVECTION IN A VERTICAL CHANNEL IN THE PRESENCE OF HEAT SOURCE OR HEAT SINK

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ABSTRACT

The objective of this paper is to analyze the effect of heat and mass transfer on mixed convective flow of a viscous incompressible fluid past an infinite vertical plate in the presence of heat source or sink. Through proper of choice of non-dimensional variables and parameters the governing equations are developed and three types of thermal boundary conditions are presented. These thermal boundary conditions are isothermal-isothermal, isoflux-isothermal and isothermal-isoflux for the left-right walls of the channel. The velocity and temperature fields are obtained analytically by perturbation series method and numerically by finite difference method. Results are presented graphically for various parameters such as the ratio of Grashof number to Reynolds number, Brinkman number and heat source or sink. The viscous dissipation enhances the flow reversal in the case of downward flow while it counters the flow in the case of upward flow.

Keywords: Mixed convection, finite difference method, heat source or sink.

1. INTRODUCTION

Heat transfer in mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices, chemical processing equipment, micro-electronic cooling and in the field of solar energy collection (see Laviene [1] and Barletta [2]). Some of the published papers such as Aung [3], Aung et al. [4], Aung and Worku [5, 6], Barletta [7], and Boulama and Galanis [8], deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime.

It is well known that buoyancy place an important role on the forced fluid flow and heat transfer in a heated vertical channel. For an aiding flow with a sufficient high Gr/Re^2, the fluid near the heated walls is accelerated to a very high speed, causing the flow reversal in the central portion of a channel in order to maintain mass conservation. On the other hand, in general, a recirculating flow is observed nearby the heated walls when the opposing buoyancy force is strong enough to reverse the forced flow locally. Consequently understanding of mixed convection heat transfer becomes important and necessary. Tao [9] and Quintiere and Mueller [10] studied the steady fully developed and developing mixed convection. Habchi and Acharya [11] numerically investigated the aiding mixed convection of air. Their results shows that the air temperature increases with Gr/Re^2 and the Nusselt number decreases monotonically. A similar study was performed by Aung and Worku [5], indicating that the buoyancy force can cause a severe distortion in the velocity profiles especially under asymmetric heat condition. The mixed convection with a low Peclet number in a short channel was examined by Cho et al. [12]. Various axial length scales to distinguish regions of different convective mechanisms were discussed by Yao [13]. Cebeci et al. [14] investigated the recirlculating flow and heat transfer in steady laminar opposing mixed convection in a vertical flat duct. Aung and Worku [6] and Lavine [1] proposed the criteria for the presence of reverse flow in vertical and inclined ducts respectively. Ingham et al. [15] observed that poor heat transfer results for flow retarded by an opposing buoyancy force, but for a large and negative Gr/Re^2, heat transfer is rather effective. Actually, heat transfer may be greatly enhanced over the section containing a strong reverse flow. Recently Prathap [16-18] studied mixed convection of two fluid flows in a vertical channel.

There appears to be a little information available on the important problem of coupled heat and mass transfer with internal heat generation (Chion and El-Wakil [19], Lyons and Hatcher [20], Melese and Wilkins [21]). The study of mixed convection in a vertical channel heat generation or absorption in moving fluids is important in view of several physical problems such as those dealing with chemical reactions and those concerned with dissociating fluids. Other investigations dealing internal heat generation or absorption can be found in the works of Chamka [22] and Mamun et al. [23]. Taneja and Jain [24] have discussed the problem of MHD free convection flow in the presence of a temperature-dependent heat source in a viscous incompressible fluid between a long vertical wavy wall and parallel flat wall with constant heat flux and slip-flow boundary condition. Recently Srinivas and Muthuraj [25] studied MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space.

Keeping in view of several applications of heat generation and absorption as mentioned above, the aim of the present work is to analyze laminar fully developed mixed convection in a vertical parallel plate channel in the presence of internal heat generation or absorption. This will be done for three types of left-right walls and these are the isothermal-isothermal, isoflux-isothermal and isothermal-isoflux wall conditions.

2. MATHEMATICAL FORMULATION

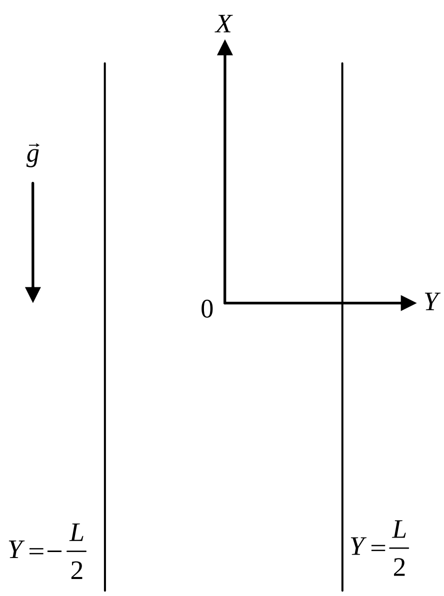

Fig. 1 Physical model and the co-ordinate system.

The flow geometry of interest is depicted in Fig. 1 which shows an open ended vertical channel filled with a clear viscous fluid. A Cartesian co-ordinate system is chosen with the transverse co-ordinate Y and the co-ordinate in the direction parallel to the walls is X. The origin of the axes is such that the channel walls are at positions $Y=-\frac{L}{2}$ and $Y=\frac{L}{2}$. The fluid is assumed to have constant properties except the density in the buoyancy term. A fluid rises in the channel driven by buoyancy forces. We consider the fluid to be incompressible and the flow is steady, laminar, and fully developed. It is assumed that the only non-zero component of the velocity is X-component U. Thus, as a consequence of the mass balance equation, one obtain

$$\frac{\partial U}{\partial X}=0 \quad (1)$$

so that U depends only on Y.

The stream wise and the transverse momentum balance equation using Brinkman model yield (Arpaci and Lassn [26]).

$$g\beta\left(T-T_0\right)-\frac{1}{\rho_0}\frac{dP}{dX}+\nu\frac{d^2U}{dY^2}=0 \quad (2)$$

and the Y-momentum balance equation can be expressed as

$$\frac{\partial P}{\partial Y}=0 \quad (3)$$

where $P=p+\rho_0 g X$ is the difference between the pressure and the hydrostatic pressure. We assume that the walls of the channel are isothermal. In particular, the temperature of the boundary at $Y=-L/2$ is T_1, and the temperature at $Y=L/2$ is T_2 with $T_2\geq T_1$. These boundary conditions are compatible with Eq. (2) if and only if $\frac{dP}{dX}$ is independent of X. Therefore, there exists a constant A such that

$$\frac{dP}{dX}=A \quad (4)$$

Differentiating Eq. (2) with respect to X, by using Eq. (3), we get

$$\frac{dT}{dX}=0 \quad (5)$$

so that the temperature also depends only on Y.

By taking into account the effect of viscous dissipation in the presence of heat generation or heat absorption, the energy balance equation becomes

$$\alpha\frac{d^2T}{dY^2}+\frac{\nu}{C_p}\left(\frac{dU}{dY}\right)^2\pm\frac{Q\left(T-T_0\right)}{\rho_0 C_p}=0 \quad (6)$$

Equations (2) and (6) gives a differential equation for U, namely

$$\frac{d^4U}{dY^4}=\frac{\beta g}{\alpha C_P}\left(\frac{dU}{dY}\right)^2\mp\frac{Q}{K}\frac{d^2U}{dY^2}\pm\frac{QA}{K\mu} \quad (7)$$

The boundary conditions of U are both the no slip conditions

$$U\left(-\frac{L}{2}\right)=U\left(\frac{L}{2}\right)=0 \quad (8)$$

and the thermal boundary conditions are given by using Eqs. (2) and (4) are

$$\left.\frac{d^2U}{dY^2}\right|_{y=-L/2}=\frac{A}{\mu}+\frac{\beta g\left(T_1-T_0\right)}{\nu} \quad (9)$$

$$\left.\frac{d^2U}{dY^2}\right|_{y=L/2}=\frac{A}{\mu}-\frac{\beta g\left(T_2-T_0\right)}{\nu} \quad (10)$$

The following non-dimensional parameters are used for writing Eqs. (6) to (10) in the dimensionless form

$$u=\frac{U}{U_0};\theta=\frac{T-T_0}{\Delta T};y=\frac{Y}{D};\ Gr=\frac{g\,\beta\,\Delta T\,D^3}{\nu^2}\ ;\ \lambda=\frac{Gr}{\text{Re}}$$
$$R_T=\frac{T_2-T_1}{\Delta T};\ \text{Re}=\frac{u_0 D}{\nu};\ \text{Pr}=\frac{\nu}{\alpha};\ Br=\frac{\mu U_0^2}{K\Delta T};\ \phi=\frac{QD^2}{K} \quad (11)$$

where $D=2L$ is the hydraulic diameter. The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0=-\frac{AD^2}{48\mu};\ \ T_0=\frac{T_1+T_2}{2} \quad (12)$$

Moreover, the temperature difference ΔT is given by $\Delta T=T_2-T_1$ if $T_1<T_2$. As a consequence, the dimensionless parameter R_T can only take the values 0 and 1. That is R_T is 1 for asymmetric heating i. e. $T_1<T_2$, while R_T is 0 for symmetric heating i. e. $T_1=T_2$, respectively. Equation (4) implies that A can be either positive or negative. If $A>0$, then U_0, Re and λ are negative, i. e. the flow is downward. On the contrary, if $A<0$, the flow is upward, so that U_0, Re and λ are positive.

Using the variables in Eq. (11), Eqs. (6) to (10) become

$$\frac{d^2\theta}{dy^2}=-Br\left(\frac{du}{dy}\right)^2\mp\phi\theta \quad (13)$$

$$\frac{d^4u}{dy^4}=\lambda\,Br\left(\frac{du}{dy}\right)^2\mp\phi\frac{d^2u}{dy^2}\mp 48\phi \quad (14)$$

$$u\left(-\frac{1}{4}\right)=u\left(\frac{1}{4}\right)=0 \quad (15)$$

$$\left.\frac{d^2u}{dy^2}\right|_{y=-\frac{1}{4}}=-48+\frac{R_T\lambda}{2},\qquad \left.\frac{d^2u}{dy^2}\right|_{y=\frac{1}{4}}=-48-\frac{R_T\,\lambda}{2} \quad (16)$$

Substituting the Eqs. (11) and (12) in Eq. (2), we obtain the temperature field as

$$\theta=-\frac{1}{\lambda}\left(48+\frac{d^2u}{dy^2}\right) \quad (17)$$

3. SOLUTIONS

3.1 Special cases

We shall solve first Eq. (14) subject boundary conditions (15) and (16) for the case when the viscous dissipation is absent, that is, when the parameter $Br=0$. Thus in this case, the dimensionless velocity component becomes

$$u=\frac{3}{2}-24y^2-\frac{2\lambda R_t}{\phi}\left(y-\frac{\sin(\sqrt{\phi}y)}{4\sin(\sqrt{\phi}/4)}\right) \quad (18)$$

for the case of heat generation and

$$u=\frac{3}{2}-24y^2+\frac{2\lambda R_t}{\phi}\left(y-\frac{\sinh(\sqrt{\phi}y)}{4\sinh(\sqrt{\phi}/4)}\right) \quad (19)$$

for the case of heat absorption.

By substituting the Eqs. (18) and (19) in Eq. (17), we obtain the temperature field as

$$\theta=\frac{R_T}{2}\frac{\sin(\sqrt{\phi}y)}{\sin(\sqrt{\phi}/4)} \quad (20)$$

for the case of heat generation and

$$\theta=\frac{R_T}{2}\frac{\sinh(\sqrt{\phi}y)}{\sinh(\sqrt{\phi}/4)} \quad (21)$$

for the case of heat absorption.

If heat generation or heat absorption is negligible then Eqs. (14) and (17) becomes

$$u=\left(\frac{R_T\lambda\,y}{3}+24\right)\left(\frac{1}{16}-y^2\right) \quad (22)$$

$$\theta=2R_T\,y \quad (23)$$

which corresponds to the velocity and temperature profiles determined by Aung and Worku [6].

In the case of asymmetric heating, when buoyancy forces are dominated i.e., when $\lambda\to\pm\infty$, Eqs. (18) and (22) for heat generation and heat absorption gives

$$\frac{u}{\lambda}=-\frac{2}{\phi}\left(y-\frac{\sin(\sqrt{\phi}\,y)}{4\sin(\sqrt{\phi}/4)}\right) \quad (24)$$

$$\frac{u}{\lambda}=\frac{2}{\phi}\left(y-\frac{\sinh(\sqrt{\phi}\,y)}{4\sinh(\sqrt{\phi}/4)}\right) \quad (25)$$

In the absence of heat generation or heat absorption, the above equations for clear viscous fluid reduces to

$$\frac{u}{\lambda}=\frac{y}{3}\left(\frac{1}{16}-y^2\right) \quad (26)$$

which is Batchelor's [27] velocity profile for free convection.

When buoyancy forces are negligible and viscous dissipation is relevant, i.e., $\lambda=0$, so that a purely forced convection occurs, solutions of Eqs. (13) and (14) becomes

$$u=\frac{3}{2}-24y^2 \quad (27)$$

$$\theta=C_1\cos(\sqrt{\phi}y)+C_2\sin(\sqrt{\phi}y)+\frac{2304Br}{\phi^2}\left(y^2\phi-2\right) \quad (28)$$

for the case of heat generation and

$$\theta=C_1\cosh(\sqrt{\phi}y)+C_2\sinh(\sqrt{\phi}y)+\frac{2304Br}{\phi^2}\left(y^2\phi+2\right) \quad (29)$$

for the case of heat absorption.

Solutions of Eqs. (13) and (14) for clear viscous fluid in the absence of buoyancy force, heat generation or heat absorption leads to the Hagen-Poiseuille velocity profile

$$u=24\left(\frac{1}{16}-y^2\right)\ \text{and}\ \theta=-192\ Br\,y^4+2R_T\,y+\frac{3Br}{4} \quad (30)$$

which agree with the results obtained by Cheng and Wu [28] in the case of forced convection with asymmetric heating.

3.2 Perturbation solution

Equation (14) along with boundary conditions (15) and (16) are nonlinear and hence it is difficult to find the closed form solutions. However, to obtain an approximate analytical solution of Eq. (14), we employ the series perturbation method by using the small dimensionless parameter $\varepsilon(<1)$, which is defined as

$$\varepsilon = Br\ \lambda = \text{Re Pr}\ \frac{\beta g D}{C_p}$$

as the perturbation parameter. Then the temperature field is obtained by using Eq. (17).

The solution of Eq. (14) can be expressed by the perturbation series method as

$$u(y) = u_0(y) + \varepsilon\, u_1(y) + \varepsilon^2 u_2(y) + \cdots\cdots = \sum_{n=0}^{\infty} \varepsilon^n u_n(y) \tag{31}$$

The second and higher order terms of ε gives a correction to u_0, u_1 accounting for the viscous and Darcy dissipation effects.

Isothermal-isothermal walls $(T_1 - T_2)$:

Substituting Eq. (31) in Eq. (14) and equating the coefficients of like powers of ε on both side, we obtains the boundary value problem for $n=0$ and $n=1$ as

$$\frac{d^4u_0}{dy^4} = \mp\phi\frac{d^2u_0}{dy^2} \mp 48\phi \tag{32}$$

$$\frac{d^4u_1}{dy^4} = \left(\frac{du_0}{dy}\right)^2 \mp \phi\frac{d^2u_1}{dy^2} \tag{33}$$

for the case of heat generation and heat absorption respectively.

$$u_0\left(-\frac{1}{4}\right) = u_0\left(\frac{1}{4}\right) = 0 \tag{34}$$

$$\left.\frac{d^2u_0}{dy^2}\right|_{y=-\frac{1}{4}} = -48 + \frac{\lambda\, R_T}{2}, \qquad \left.\frac{d^2u_0}{dy^2}\right|_{y=\frac{1}{4}} = -48 - \frac{\lambda\, R_T}{2} \tag{35}$$

$$u_1\left(-\frac{1}{4}\right) = u_1\left(\frac{1}{4}\right) = 0 \tag{36}$$

$$\left.\frac{d^2u_1}{dy^2}\right|_{y=-\frac{1}{4}} = \left.\frac{d^2u_1}{dy^2}\right|_{y=\frac{1}{4}} = 0 \tag{37}$$

Equation (32) is an ordinary linear differential equation and its exact solution can be found. These solutions obviously coincides with the solution of Eq. (14) in the case *Br=0*. The solution of Eq. (32) subjected to the boundary conditions (34) and (35) is

$$u_0 = C_1 + C_2 y - 24y^2 + C_3\cos(\sqrt{\phi}y) + C_4\sin(\sqrt{\phi}y) \tag{38}$$

for the case of heat generation and

$$u_0 = C_1 + C_2 y - 24y^2 + C_3\cosh(\sqrt{\phi}y) + C_4\sinh(\sqrt{\phi}y) \tag{39}$$

for the case of heat absorption.

Equation (33) can also be solved exactly by using Eqs. (38) and (39) with boundary conditions given by Eqs. (36) and (37) and the solutions are

$$\begin{aligned} u_1 &= C_5 + C_6 y + C_7\cos(\sqrt{\phi}y) + C_8\sin(\sqrt{\phi}y) + l_1\cos(2\sqrt{\phi}y) \\ &+ l_2\sin(2\sqrt{\phi}y) + l_3 y^2\cos(\sqrt{\phi}y) + l_4 y^2\sin(\sqrt{\phi}y) \\ &+ l_5 y\cos(\sqrt{\phi}y) + l_6 y\sin(\sqrt{\phi}y) + l_7 y^4 + l_8 y^3 + l_9 y^2 \end{aligned} \tag{40}$$

for the case of heat generation and

$$\begin{aligned} u_1 &= C_5 + C_6 y + C_7\cosh(\sqrt{\phi}y) + C_8\sinh(\sqrt{\phi}y) \\ &+ l_1\cosh(2\sqrt{\phi}y) + l_2\sinh(2\sqrt{\phi}y) + l_3 y^2\cosh(\sqrt{\phi}y) \\ &+ l_4 y^2\sinh(\sqrt{\phi}y) + l_5 y\cosh(\sqrt{\phi}y) + l_6 y\sinh(\sqrt{\phi}y) \\ &+ l_7 y^4 + l_8 y^3 + l_9 y^2 \end{aligned} \tag{41}$$

for the case of heat absorption.

Evaluation of exact solution for $n=2$ becomes complicated and hence neglecting the terms for $n=2$ the solution is obtained up to $0(\varepsilon^1)$ as

$$u = u_0 + \varepsilon\, u_1 \tag{42}$$

The dimensionless temperature field is obtained from Eq. (17) considering velocity field defined as in Eq. (41) become

$$\begin{aligned} \theta &= \frac{1}{\lambda}\Big((C_3 + \varepsilon C_7)\phi\cos(\sqrt{\phi}y) + (C_4 + \varepsilon C_8)\phi\sin(\sqrt{\phi}y) \\ &- \varepsilon(4l_1\phi\cos(2\sqrt{\phi}y) - 4l_2\phi\sin(2\sqrt{\phi}y) - l_3\phi y^2\cos(\sqrt{\phi}y) \\ &- l_4\phi y^2\sin(\sqrt{\phi}y) + l_{10} y\cos(\sqrt{\phi}y) + l_{11} y\sin(\sqrt{\phi}y) \\ &+ l_{12}\cos(\sqrt{\phi}y) + l_{13}\sin(\sqrt{\phi}y) + 12l_7 y^2 + 6l_8 y + 2l_9)\Big) \end{aligned} \tag{43}$$

for the case of heat generation and

$$\begin{aligned} \theta &= -\frac{1}{\lambda}\Big((C_3 + \varepsilon C_7)\phi\cosh(\sqrt{\phi}y) + (C_4 + \varepsilon C_8)\phi\sinh(\sqrt{\phi}y) \\ &+ \varepsilon(4l_1\phi\cosh(2\sqrt{\phi}y) + 4l_2\phi\sinh(2\sqrt{\phi}y) + l_3\phi y^2\cosh(\sqrt{\phi}y) \\ &+ l_4\phi y^2\sinh(\sqrt{\phi}y) + l_{10} y\cosh(\sqrt{\phi}y) + l_{11} y\sinh(\sqrt{\phi}y) \\ &+ l_{12}\cosh(\sqrt{\phi}y) + l_{13}\sinh(\sqrt{\phi}y) + 12l_7 y^2 + 6l_8 y + 2l_9)\Big) \end{aligned} \tag{44}$$

for the case of heat absorption.

Isoflux-isothermal $(q_1 - T_2)$ walls:

For this case, the thermal boundary conditions for the channel walls can be written in the dimensional form as

$$q_1 = -K\left.\frac{dT}{dY}\right|_{Y=-\frac{L}{2}}; \qquad T\left(\frac{L}{2}\right) = T_2 \tag{45}$$

The dimensionless form of the above equations can be obtained by using the Eq. (11) with $\Delta T = q_1 D / K$ to give

$$\left.\frac{d\theta}{dy}\right|_{y=-\frac{1}{4}} = -1; \quad \theta\left(\frac{1}{4}\right) = R_{qt} \tag{46}$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter for isoflux-isothermal case.

Other than the no-slip conditions at the channel walls, two more boundary conditions in terms of *U* are required to solve the Eq. (7). These are the conditions given in Eq. (45) and are obtained from Eq. (2) as follows.

Differentiating Eq. (2) with respect to Y with $dP/dX = A$ gives

$$\frac{d^3U}{dY^3} + \frac{\beta g}{\nu}\frac{dT}{dY} = 0 \tag{47}$$

Equation (45) is non-dimensionalzed by using Eq. (11) to give

$$\frac{d^3u}{dy^3} + \lambda\frac{d\theta}{dy} = 0 \tag{48}$$

Evaluating Eq. (46) at the left wall $(y = -1/4)$ yields

$$\frac{d^3u}{dy^3}\left(-\frac{1}{4}\right) = \lambda \tag{49}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal wall with R_T replaced by R_{qt} such that

$$\frac{d^2u}{dy^2}\left(\frac{1}{4}\right) = -48 - \frac{\lambda R_{qt}}{2} \tag{50}$$

The solutions of velocity field and temperature field can be obtained from Eqs. (40) – (42) by using the boundary conditions (34) - (37) along with (49) and (50).

Isothermal-isoflux $(T_1 - q_2)$ walls:

For this case, the thermal boundary conditions for the channel walls can be written in the dimensional form as

$$q_2 = -K\left.\frac{dT}{dY}\right|_{Y=\frac{L}{2}};\ T\left(-\frac{L}{2}\right) = T_1 \tag{51}$$

The dimensionless form of above equations can be obtained by using the Eq. (11) with $\Delta T = q_2 D/K$ to give

$$\left.\frac{d\theta}{dy}\right|_{\frac{1}{4}} = -1;\ \theta\left(-\frac{1}{4}\right) = -R_{tq} \tag{52}$$

where $R_{tq} = (T_1 - T_0)/\Delta T$ is the thermal ratio parameter for isothermal-isoflux case.

Similar to the procedure done in the previous section on isoflux-isothermal walls, the dimensionless form of the boundary conditions obtained from Eq. (2) and by using Eq. (52) is

$$\frac{d^3u}{dy^3}\left(\frac{1}{4}\right) = \lambda \tag{53}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal wall with R_T replaced by R_{tq} such that

$$\frac{d^2u}{dy^2}\left(-\frac{1}{4}\right) = -48 + \frac{\lambda R_{tq}}{2} \tag{54}$$

The solutions of velocity field and temperature field can be obtained from Eqs. (40) – (42) by using the boundary conditions (34) - (37) along with (53) and (54).

4. NUMERICAL SOLUTION

The analytical solutions obtained in the preceding section include only two terms of the series, which are not applicable for large values of λ i.e., for large values of buoyancy force. In many practical problems, the values of λ are usually large. Therefore, a numerical scheme is used to solve non-linear boundary value problem using finite-difference method, the details of which can be found in Umavathi [29]. Replacing the derivatives with corresponding central difference approximations, lead to m linear algebraic equations, where m is the number of divisions from $y = -1/4$ to $y = 1/4$. The solutions of reduced algebraic equations are solved by Successive-over relaxation method. The relaxation parameter is fixed by comparing the numerical values with the analytical results for the case $\lambda = 0$. The convergence criterion is based on the step size and the previous iterations for the iterative difference to the order 10^{-6}. Validity of the numerical scheme is justified by comparing the numerical solution with the analytical solution and it is found that this agreement is good enough.

4. RESULTS AND DISCUSSION

Analytical and numerical solutions for the steady fully developed mixed convective flow and heat transfer in a vertical channel containing purely viscous fluid are obtained using regular perturbation technique and finite difference method respectively. The product of λ and *Br*, where λ is the mixed convective parameter and *Br* is the Brinkman number is used as the perturbation parameter. The flow filed in the case of heat generation and heat absorption are obtained and depicted in Figs. 2-10. Equation (18) for the velocity u is evaluated for different values of λ and is shown in Fig. 2 when $Br = 0$ and $\phi = 2$. It is seen that the velocity profile for $\lambda = 400$ shows a flow reversal near the cold wall at $y = -1/4$. With $U_0 > 0$ i.e. for upward flow it is expected that for sufficiently large value of Grashof number a flow reversal induced by the buoyancy forces occurs at the cold wall. By performing a reflection of y-axis of Fig.2, plots of u for λ =-200 and -400 can be obtained. Equation (29) for the non-dimensional temperature field θ is evaluated for different values of Brinkman number with $\phi = 2$, and is shown in Fig. 3.

We notice that the temperature field is linear indicating that the heat transfer is purely by conduction in the absence of dissipation $(Br = 0)$. This figure shows that, although the conductive regime holds only for $Br = 0$, the temperature is almost linear in the middle of the channel for $Br \neq 0$ also indicating that the convection dominates in the boundary layer region. Also, the temperature filed increases with increasing the value of Br .

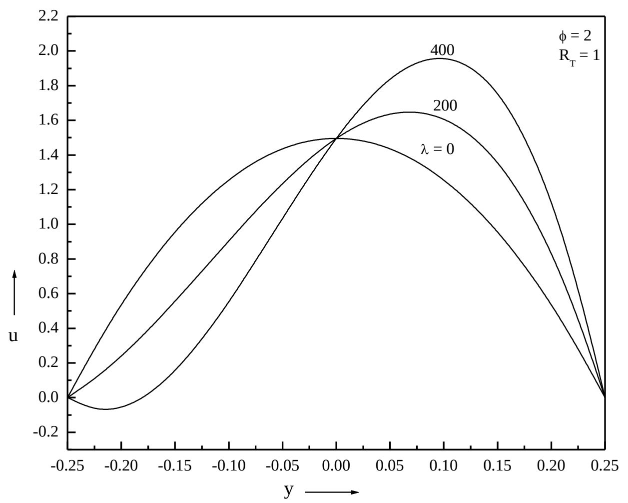

Figure 2: Plots of u versus y in the case of asymmetric heating, for different values of λ and $Br = 0$.

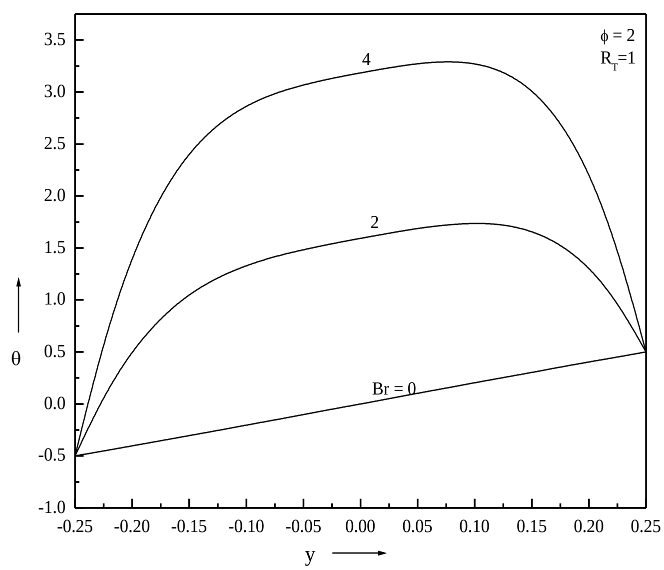

Figure 3: Plots of θ versus y in the case of asymmetric heating, for different values of Br and $\lambda = 0$.

Figures 4 compare the analytical with the numerical solution of the non-dimensional velocity and temperature profiles for $\lambda = \pm 500$ when $\varepsilon = 8$ and $\phi = 5$. We notice that there is a very good agreement between the analytical and numerical solutions. It is also observed from Fig. 4 that for large value of λ, flow reversal occurs at both the cold and hot walls.

Variation of non-dimensional velocity and temperature profiles u and θ for $\lambda = \pm 500$, $R_T = 1$ for different values of heat generation coefficient ϕ is shown in Figs. 5 and 6 respectively. For values of ε and λ positive, the flow is upward, and contrarily the flow is downward when ε and λ are negative. Flow reversal is noticed for both upward and downward flow near the channel walls for the values of ϕ and ε as shown in Fig. 5. It is seen from Figs. 5 and 6 that for $\lambda > 0$, as the heat generation co-efficient ϕ increases, velocity and temperature increases whereas for $\lambda < 0$, velocity and temperature decreases as ϕ increases. One can also observe from these figures that the heat generation coefficient is operative as ε increases.

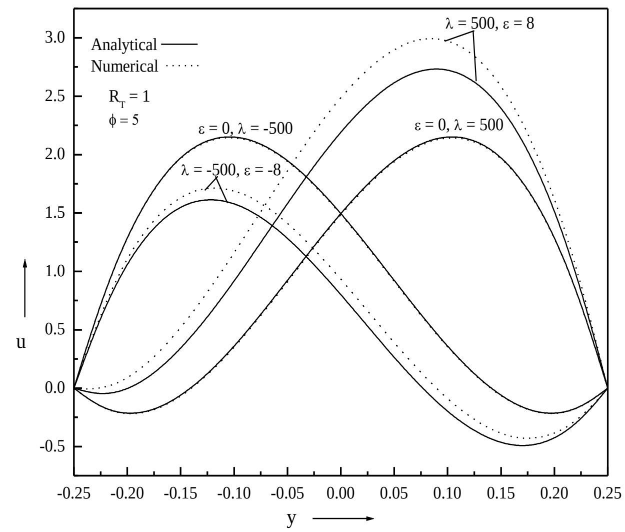

Figure 4: Effects of ε and λ on velocity profiles for $(T_1 - T_2)$ case.

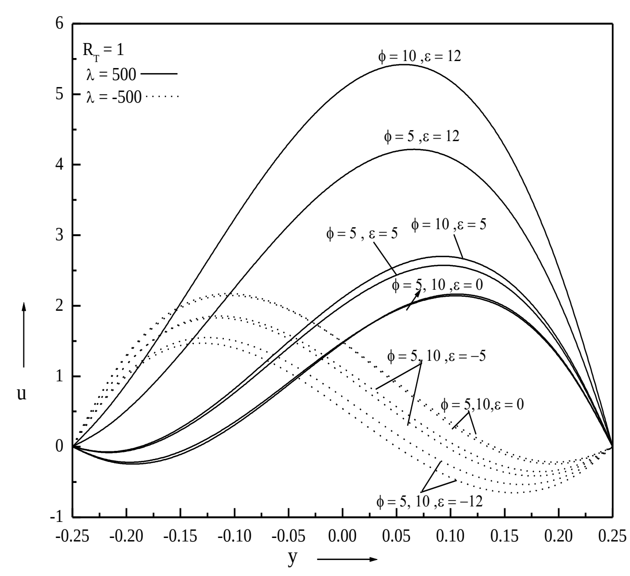

Figure 5: Effects of ε and ϕ on velocity profiles for $(T_1 - T_2)$ case.

The effect of heat generation coefficient on the velocity u and temperature θ fields for isoflux-isothermal and isothermal-isoflux wall conditions are shown in Figs. 7-10. It is seen that as ϕ increases flow is upward for $\lambda > 0$ and is downward for $\lambda < 0$ as seen in Fig. 7. As ϕ increases, temperature increases for both $\lambda > 0$ and $\lambda < 0$ as seen in Fig. 8. However the magnitude is large for positive λ compared to negative value of λ. The effect of ϕ on velocity and temperature for isothermal-isoflux wall condition is similar to that for isoflux-isothermal wall condition as seen in Figs. 9 and 10. However the flow direction for isothermal-isoflux is in contrary for isoflux-isothermal wall condition.

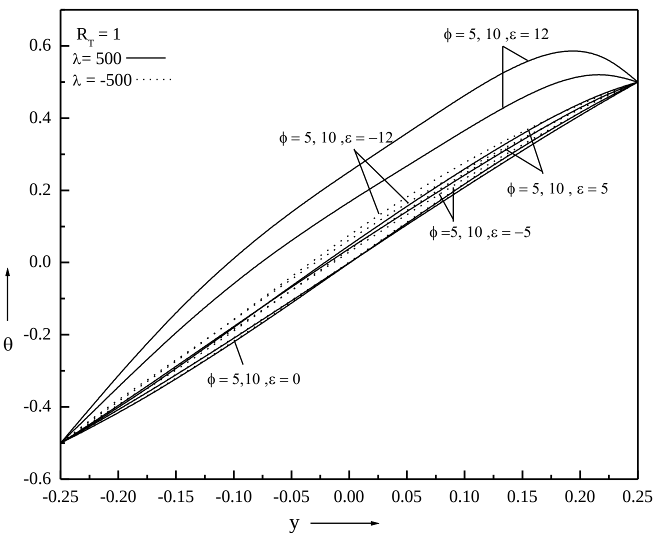

Figure 6: Effects of ε and ϕ on temperature profiles for $(T_1 - T_2)$ case.

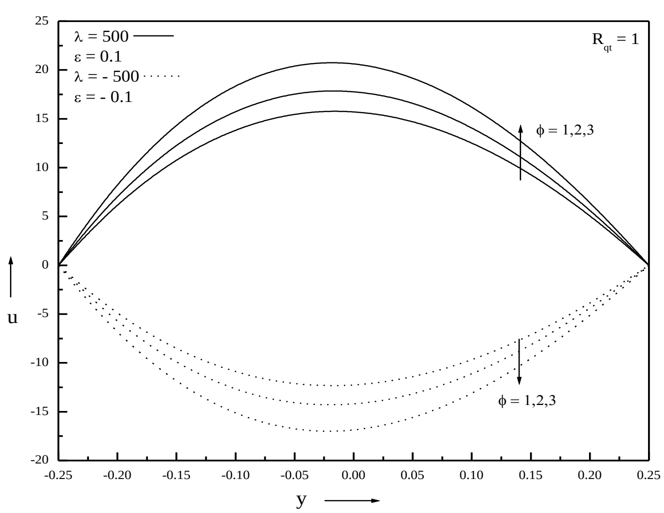

Figure 7: Effects of ϕ and λ on velocity profiles for $(q_1 - T_2)$ case

Results were also observed for variations of heat absorption coefficient ϕ on the flow. Similar results were obtained when compared to the effects of heat generation coefficient ϕ except that the effects of ϕ for heat absorption is in contrary with heat generation coefficient ϕ, and hence not shown graphically.

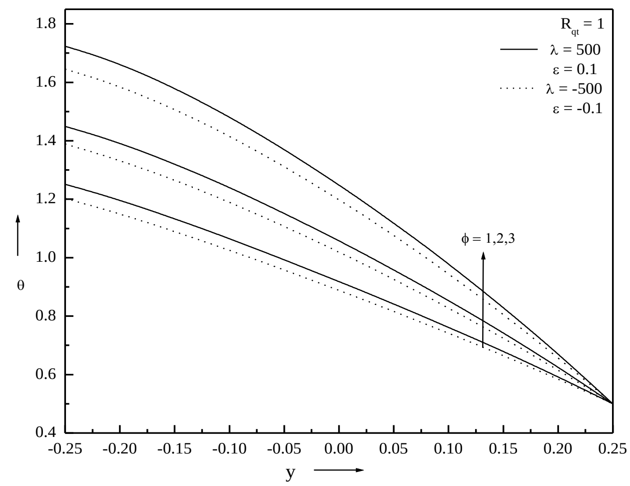

Figure 8: Effects of ϕ and λ on temperature profiles for $(q_1 - T_2)$ case.

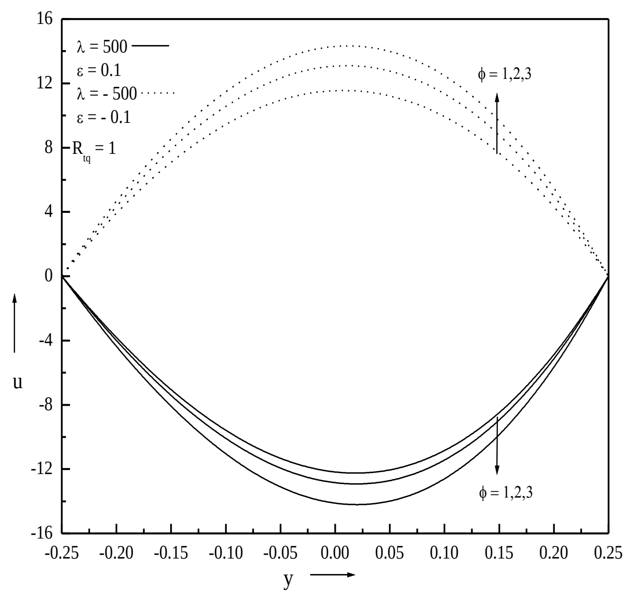

Figure 9: Effects of ϕ and λ on velocity profiles for $(T_1 - q_2)$ case.

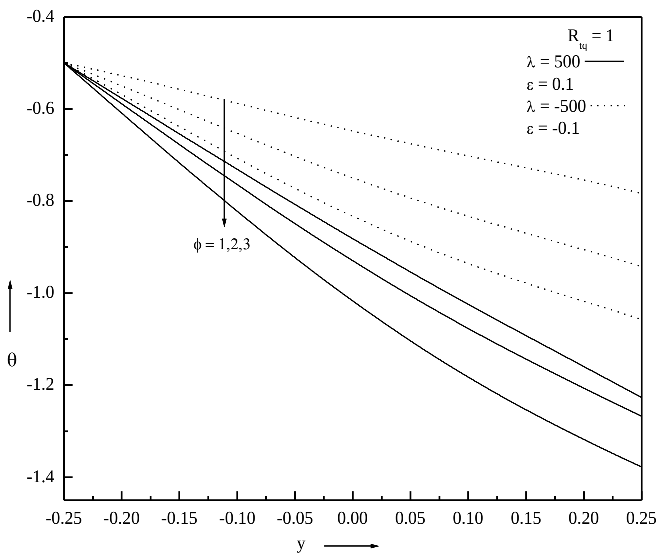

Figure 10: Effects of ϕ and λ on temperature profiles for $\left(T_1 - q_2\right)$ case.

5. CONCLUSION

The work presented focused on the problem of steady laminar, mixed convective flow in a vertical channel filled with purely viscous fluid in the presence of viscous dissipation and heat generation or absorption effects. Three different combination of thermal left-right wall condition were presented. Various analytical solutions on the flow for special cases were obtained. The general mixed convection problem which includes the effects of viscous dissipation was solved numerically using finite difference technique with Successive-Over-Relaxation method. Obtained numerical solutions were found to be in good agreement with analytical solution in the absence of viscous dissipation. Graphical results were displayed for different parameters governing the flow and heat transfer. It is found that fixing heat generation or heat absorption coefficient we obtain the results reported by Barletta [2]. It is also found that heat generation coefficient aids the flow whereas heat absorption opposes the flow.

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Nomenclature

A	constant
Br	Brinkman number
C_p	specific heat at constant pressure [$\mathrm{kJ\,kg^{-1}\,K^{-1}}$]
D	$= 2L$, hydraulic diameter [m]
g	acceleration due to gravity [$\mathrm{m\,s^{-2}}$]
Gr	Grashof number
K	thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$]
L	channel width [m]
p	pressure [$\mathrm{N\,m^{-2}}$]
P	$= p + \rho_o g X$ difference between the pressure and the hydrostatic pressure [$\mathrm{N\,m^{-2}}$]
Pr	Prandtl number
Re	Reynolds number
R_T	temperature difference ratio [K]
T	temperature [K]
T_1, T_2	prescribed boundary temperatures [K]
T_0	reference temperature [K]
u	dimensionless velocity component in the x-direction
U	velocity component in the X-direction [$\mathrm{m\,s^{-1}}$]
U_0	reference velocity [$\mathrm{m\,s^{-1}}$]
V	velocity component in the Y-direction [$\mathrm{m\,s^{-1}}$]
X, Y	space coordinates [m]
y	dimensionless transverse coordinate

Greek symbols

λ	dimensionless parameter
α	$= K / \left(\rho_0 C_p\right)$ thermal diffusivity
β	thermal expansion coefficient
ε	dimensionless parameter
θ	dimensionless temperature defined in
μ	dynamic viscosity [$\mathrm{kg\,m^{-1}\,s^{-1}}$]
ρ_0	density [$\mathrm{kg\,m^{-3}}$]
ν	$= \mu / \rho_0$, kinematics viscosity

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