Thermo-Solutal Convection in an Inclined Porous Cavity with Various Aspect Ratios Under Mixed Thermal and Species Boundary Conditions

Ali J. Chamkha, Ali Al-Mudhaf, and Eisa Al-Meshaieie
Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh, Kuwait

The problem of laminar thermo-solutal convective flow of a binary fluid mixture in an inclined rectangular cavity filled with a uniform porous medium is considered. Mixed heat and mass fluxes and uniform temperature and concentration conditions are applied on two opposing walls of the cavity while the other two walls are kept adiabatic and impermeable to mass transfer. The problem is put in terms of the stream function-vorticity formulation. A numerical solution based on the finite-difference methodology is obtained. Representative results illustrating the effects of the inclination angle of the cavity, buoyancy ratio, Darcy number, and the cavity aspect ratio on the contour maps of the streamline, temperature, and concentration as well as the profiles of velocity, temperature, and concentration at mid-section of the cavity are reported. In addition, numerical results for the average Nusselt and Sherwood numbers as well as some useful correlations are presented for various parametric conditions and discussed. © 2011 Wiley Periodicals, Inc. Heat Trans Asian Res, 40(8): 693–720, 2011; Published online 11 August 2011 in Wiley Online Library (wileyonlinelibrary.com/journal/htj). DOI 10.1002/htj.20369

Key words: porous media, double-diffusive convection, inclined cavity, mixed boundary conditions

1. Introduction

Thermo-solutal or double-diffusive convection is referred to as buoyancy-driven flows induced by combined temperature and concentration gradients. Double diffusion occurs in a wide range of scientific fields such as oceanography, astrophysics, geology, biology, and chemical processes (see, for instance, Beghein et al. [3]). Ostrach [18] and Viskanta et al. [26] have reported complete reviews on the subject. Lee and Hyun [9] and Hyun and Lee [8] have reported numerical solutions for double-diffusive convection in a rectangular enclosure with aiding and opposing temperature and concentration gradients. Their solutions compared favorably with reported experimental results. Mamou et al. [11] have reported an analytical and numerical study of double-diffusive convection in a vertical enclosure.

© 2011 Wiley Periodicals, Inc.
In the past decade or so, interest in studying double-diffusive convective flows induced by the combined action of both temperature and concentration gradients in porous media has surged in view of its importance in many engineering problems such as migration of moisture contained in fibrous insulation, grain storage, transport of contaminants in saturated soil, underground disposal of nuclear wastes, and drying processes (Mamou et al. [12]). Trevisan and Bejan [24] have studied heat and mass transfer by natural convection in a vertical slot filled with a porous medium. Chen and Chen [5] have considered double-diffusive fingering convection in a porous medium. Lin [10] has studied unsteady natural convection heat and mass transfer in a saturated porous medium. Mamou et al. [13] have analyzed double-diffusion convection in an inclined slot filled with a porous medium.

Natural convection in enclosures with uniform heat and mass fluxes and mixed thermal and species boundary conditions imposed on some of their walls has also been considered in the literature. For example, Trevisan and Bejan [25] studied combined heat and mass transfer by natural convection in a vertical enclosure having two adiabatic and impermeable horizontal walls and two heat and mass isoflux vertical walls. Alavyoon [1] reported on natural convection in vertical porous enclosures due to prescribed fluxes of heat and mass at the vertical boundaries. Alavyoon et al. [2] investigated natural convection in vertical enclosures filled with porous media due to opposing fluxes of heat and mass prescribed at the vertical walls. Prasad and Kulacki [20] studied natural convection in a rectangular porous cavity with constant heat flux in one vertical wall. Nithiarasu et al. [16] reported a numerical study on buoyancy driven flow in a non-Darcian porous medium enclosure subjected to uniform heat flux. Masuda et al. [14] considered double-diffusive natural convection in porous medium under constant heat and mass fluxes.

In a variety of engineering applications, enclosures are inclined in the direction of gravity. Hence, the buoyancy forces have components relative to the directions of the walls of the enclosure. This modifies strongly the flow structure and the heat transfer characteristics within the enclosure. The effect of inclination on natural convection in an enclosure has been discussed by several investigators (see, for instance, Raithby and Hollands [21] and Yang [27]). As mentioned by Rasoul and Prinos [22], the first studies on inclined enclosures were devoted to the stability problem of such a flow by Hart [7]. Ozoe et al. [19] predicted experimentally that the Nusselt number achieved first a minimum and then a maximum with increasing enclosure inclination angle, but their work was limited to Rayleigh numbers of up to $10^4$. Hamady et al. [6] measured local and mean Nusselt numbers at various inclination angles and Rayleigh numbers from $10^4$ to $10^6$. They found a strong dependence of the heat transfer rate on the enclosure inclination angle and Rayleigh number. Ravi et al. [23] studied the structure of steady, laminar, natural convection in a square enclosure for high Ra numbers (up to $10^5$) and an angle of inclination 90°. They examined the recirculating pocket appearing near the corners downstream of the vertical walls and they found that it was caused by the thermal effects. They also studied the effect of the Prandtl number on the flow structure and they concluded that, for water ($Pr = 7.0$), the recirculation region in the corner does not appear even for high Rayleigh numbers. Rasoul and Prinos [22] studied the effect of the inclination angle on steady natural convection in a square enclosure for Rayleigh numbers ranging from $10^3$ to $10^6$ and Prandtl numbers from 0.02 to 4,000. They found that the Nusselt number depended strongly on the inclination angle and Rayleigh and Prandtl numbers. Chamkha and Al-Mudhaf [4] studied double-diffusive natural convection in inclined porous cavities with various aspect ratios and temperature-dependent heat source or sink. In their work, Chamkha and Al-Mudhaf considered inclined cavities with constant temperatures and concentrations on two opposing walls while the other two walls are insulated.
The objective of this work is to consider various aspects of laminar, double-diffusive natural convection flow inside an inclined rectangular enclosure with various aspect ratios filled with a uniform porous medium under mixed constant temperature and concentration and heat and mass flux conditions. This problem can be validated by the works of Nishimura et al. [17] who considered opposing temperature and concentration gradients.

**Nomenclature**

AR: enclosure aspect ratio = H/W  
C: concentration of species  
c_l: low species concentration (sink)  
c_p: specific heat of the fluid  
c_s: specific heat of porous medium material  
C: dimensionless species concentration = (c – c_l)/Δc  
D: species diffusivity  
Da: Darcy number = κ/W²  
G: gravitational acceleration  
H: enclosure height  
k: thermal conductivity  
Le: Lewis number = α_e/D  
mo: wall mass flux  
N: buoyancy ratio = (β_cm o/D)/(β_T q’/k)  
Nu: average Nusselt number at heated vertical wall  
P: fluid pressure  
Pr: Prandtl number = ν/α_e  
q’: wall heat flux  
Ra: thermal Rayleigh number = gβ_T q’W⁴/(kα_e ν)  
Sh: average Sherwood number at heated vertical wall  
T: time  
T: temperature  
T_c: cold wall temperature (sink)  
U: horizontal velocity component  
U: dimensionless horizontal velocity component = uW/α_e  
V: vertical velocity component  
V: dimensionless vertical velocity component = vW/α_e  
W: enclosure width  
X: horizontal coordinate  
X: dimensionless horizontal coordinate = x/W  
Y: vertical coordinate  
Y: dimensionless vertical coordinate = y/W  

**Greek Symbols**

a: inclination angle of the cavity  
a_e: effective thermal diffusivity of the porous medium  
b_T: thermal expansion coefficient
2. Mathematical Model

Consider unsteady, laminar, two-dimensional double-diffusive convective flow inside a rectangular porous enclosure. The temperature \( T_c \) and concentration \( c_1 \) are uniformly imposed on the right (vertical when non-inclined) wall of the enclosure while constant heat and mass fluxes are imposed on the left wall while the two horizontal walls are assumed to be adiabatic and impermeable to mass transfer. Figure 1 shows the schematics of the problem under consideration. The fluid is assumed to be incompressible, Newtonian, and viscous. The porous medium is assumed to be uniform and in local thermal and compositional equilibrium with the fluid. The effects due to viscous dissipation and porous medium inertia are assumed to be negligible.

The governing equations for this problem are based on the balance laws of mass, linear momentum, thermal energy, and concentration. Taking into account the assumptions mentioned above, and applying the Boussinesq approximation for the body force terms in the momentum equations, the governing equations can be written in dimensional form as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{1}{\varepsilon} \frac{\partial u}{\partial t} + \frac{1}{\varepsilon^2} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\nu}{\varepsilon} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g \beta \left( T - T_c \right) \sin(\alpha) + g \beta_c (c - c_1) \sin(\alpha) - \frac{\mu}{\rho k} u
\]
where x, y, and t are the horizontal and vertical distances and time, respectively. u, v, p, T, and c are the velocity components in the x and y directions, pressure, temperature, and concentration, respectively. $\beta_T$ and $\beta_c$ are the thermal and compositional expansion coefficients, respectively. The parameters $\varepsilon$, $\kappa$, $\alpha_e$, $\nu$, $\mu$, $c_p$, and $\rho$ are the porosity, permeability, and effective thermal diffusivity of the porous medium, the fluid kinematic and dynamic viscosities, specific heat at constant pressure and the fluid density, respectively. $D$ is the species diffusivity, $\sigma = \varepsilon \rho c_p + (1 - \varepsilon) \rho_s c_1)/(\rho c_p)$ is the specific heat ratio, $c_s$ and $\rho_s$ are the specific heat and density of the porous medium material, respectively, $T_c$ is the cold wall temperature, $c_1$ is the concentration at the cold wall, and $g$ is the gravitational acceleration.

The initial and boundary conditions for the problem can be written as
where \( W \) and \( H \) are the width and height of the enclosure, respectively.

The dimensional stream function and vorticity can be defined in the usual way as

\[
\begin{align*}
\psi &= \frac{\partial \Psi}{\partial y}, \\
v &= -\frac{\partial \Psi}{\partial x} , \\
\Omega &= \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} 
\end{align*}
\]

It is convenient to nondimensionalize Eqs. (1) through (7) by using the following dimensionless variables:

\[
\begin{align*}
X &= \frac{x}{W} , \\
Y &= \frac{y}{W} , \\
\tau &= \frac{\alpha_e t}{W^2} , \\
\zeta &= \frac{\Omega W^2}{\alpha_e} , \\
Pr &= \frac{\nu}{\alpha_e} , \\
Da &= \frac{k}{W^3} , \\
Le &= \frac{\alpha_e}{D} , \\
\theta &= \frac{T - T_e}{\Delta T} , \\
C &= \frac{c - c_i}{\Delta c} , \\
\Delta T &= \frac{q W}{k} , \\
\Delta c &= \frac{m^2 W}{D} , \\
N &= \frac{\beta_s m^2}{D} , \\
Ra &= \frac{\beta_f q W^4}{k \alpha v} 
\end{align*}
\]

where the dimensionless parameters appearing in the above equations are given in the Nomenclature list.

By employing Eqs. (8) and combining Eqs. (2) and (3) by eliminating the pressure gradient terms, the resulting dimensionless equations can be written as

\[
\begin{align*}
\zeta &= \frac{\partial \psi}{\partial x} - \frac{\partial U}{\partial y} = -V^2 \psi \\
\frac{1}{\varepsilon} \frac{\partial \zeta}{\partial \tau} + \frac{1}{\varepsilon^2} \left( U \frac{\partial \xi}{\partial X} + V \frac{\partial \xi}{\partial Y} \right) &= \frac{Pr}{\varepsilon} V^2 \xi + \frac{\varepsilon}{\varepsilon} \xi + \frac{Ra Pr}{\varepsilon} \left( \frac{\partial \theta}{\partial X} + N \frac{\partial C}{\partial Y} \right) \cos(\alpha) \\
- \frac{Ra Pr}{\varepsilon} \left( \frac{\partial \theta}{\partial Y} + N \frac{\partial C}{\partial Y} \right) \sin(\alpha) - \frac{Pr}{\varepsilon} \zeta \\
\sigma \frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} &= \nabla^2 \theta \\
\varepsilon \frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} &= \nabla^2 C / Le
\end{align*}
\]

The initial and boundary conditions in dimensionless form become

\[
\begin{align*}
\tau &= 0 : \\
U &= V = \psi = 0 , \\
\theta &= 0 , \\
C &= 0
\end{align*}
\]

(13a)
The average Nusselt and Sherwood numbers at the left heated wall of the enclosure are given by

\[ Y = 0 : \quad U = V = \psi = 0, \quad \zeta = -\left( \frac{\partial^2 \psi}{\partial Y^2} \right), \quad \frac{\partial \theta}{\partial Y} = 0, \quad \frac{\partial C}{\partial Y} = 0 \quad (13b) \]

\[ Y = \frac{H}{W} : \quad U = V = \psi = 0, \quad \zeta = -\left( \frac{\partial^2 \psi}{\partial Y^2} \right), \quad \frac{\partial \theta}{\partial Y} = 0, \quad \frac{\partial C}{\partial Y} = 0 \quad (13c) \]

\[ X = 0 : \quad U = V = \psi = 0, \quad \zeta = -\left( \frac{\partial^2 \psi}{\partial X^2} \right), \quad \frac{\partial \theta}{\partial X} = -1, \quad \frac{\partial C}{\partial X} = -1 \quad (13d) \]

\[ X = 1 : \quad U = V = \psi = 0, \quad \zeta = -\left( \frac{\partial^2 \psi}{\partial X^2} \right), \quad \theta = 0, \quad C = 0 \quad (14) \]

The average Nusselt and Sherwood numbers at the left heated wall of the enclosure are given by

\[ \overline{Nu} = \frac{1}{\text{AR}} \int_0^{\text{AR}} \frac{1}{\Delta \theta_w} \, dY, \quad \overline{Sh} = \frac{1}{\text{AR}} \int_0^{\text{AR}} \frac{1}{\Delta C_w} \, dY \quad (15) \]

where AR = H/W is the aspect ratio of the cavity.

### 3. Numerical Algorithm

The numerical algorithm used to solve Eqs. (9) through (13) is based on the finite-difference methodology. Central difference quotients were used to approximate the second derivatives in both the X- and Y-directions. The governing equations are then transformed into tri-diagonal algebraic equations which were solved in the X- and Y-directions for the concentration, temperature, vorticity, and the stream function. This method was found to be stable and gave results that are very close to the numerical results obtained by Nishimura et al. [17] using the finite-element method.

The numerical computations were carried out for (61 × 61), (61 × 81), (61 × 121), and (61 × 161) grid nodal points with a time step of 10^{-5} and horizontal and vertical step sizes (\(\Delta X = 1/60\) and \(\Delta Y = 1/60\)), (\(\Delta X = 1/60\) and \(\Delta Y = 1/40\)), (\(\Delta X = 1/60\) and \(\Delta Y = 1/40\)), and (\(\Delta X = 1/60\) and \(\Delta Y = 1/40\)) for AR = 1, 2, 3, 4, respectively. The grid mesh points for the different AR values were arrived at after making various grid independence tests (for example, see Table 1). The convergence criterion required that the difference between the current and previous iterations for all of the dependent variables be 10^{-4}. A smaller error value than 10^{-4} did not change the results significantly. In all the results obtained, Ra = 10^5 and \(\varepsilon = 0.6\).

### 4. Numerical Validation Tests

In order to check on the accuracy of the numerical method employed for the solution of the problem under consideration, it was validated (after making the necessary modifications) with the problem of double-diffusive convective flow in a vertical rectangular enclosure with opposing...
combined temperature and concentration gradients reported earlier by Nishimura et al. [17]. Figures 2(a) and 2(b) present comparisons for the streamlines, isotherms, concentration contours, and density contours of the present work at N = 0.8 (thermal-dominated flow) and N = 1.3 (compositional-dominated flow) with those reported by Nishimura et al. [17]. These comparisons show good agreement between the results. Moreover, Table 1 shows a favorable quantitative comparison between numerical results for a period of oscillation and stream function extrema $|\psi_{\text{max}}|$ and $|\psi_{\text{min}}|$ at N = 1.0 obtained by three different numerical schemes, the finite-element method (reported by Nishimura et al. [17]), spectral method (reported by Morega and Nishimura [15]), and the finite-difference method of the present work. These various comparisons lend confidence in the numerical results to be reported subsequently. Results for two different mesh points (31 × 41) and (61 × 81) are present in Table 1. Because of the accuracy considerations, 61 × 81 mesh points are used for the cases of AR = 2.0 for the following study.

Fig. 2a. Comparison of thermal-dominated regime with Nishimura et al. [17] (dotted lines) for AR = 2.0, Da = $\infty$, Le = 2, N = 0.8, Pr = 1.0, and Ra = $10^5$. 
Fig. 2b. Comparison of compositional-dominated regime with Nishimura et al. [17] (dotted lines) for AR = 2.0, Da = ∞, Le = 2, N = 1.3, Pr = 1.0, and Ra = 10^5.

Table 1. Comparison between the Present Method and Two Numerical Methods for AR = 2.0, Da = ∞, Le = 2, N = 1.0, Pr = 1.0, and Ra = 10^5

<table>
<thead>
<tr>
<th></th>
<th>Finite element method (31×41 points)</th>
<th>Spectral method (40×80 points)</th>
<th>Finite difference method (31×41 points)</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_0 )</td>
<td>0.0497</td>
<td>0.0494</td>
<td>0.05091 (0.0499)</td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>(</td>
<td>\psi_{\text{max}}</td>
<td>)</td>
</tr>
<tr>
<td>Min</td>
<td></td>
<td>(</td>
<td>\psi_{\text{max}}</td>
<td>)</td>
</tr>
<tr>
<td>Max</td>
<td></td>
<td>(</td>
<td>\psi_{\text{max}}</td>
<td>)</td>
</tr>
<tr>
<td>Min</td>
<td></td>
<td>(</td>
<td>\psi_{\text{max}}</td>
<td>)</td>
</tr>
</tbody>
</table>

Quantities in ( ) are for a mesh size 61×81 points.
5. Results and Discussion

In this section, numerical results for the streamline, temperature, and concentration contours as well as selected velocity, temperature, and concentration profiles at mid-section of the cavity for various values of the cavity inclination angle $\alpha$, the buoyancy ratio $N$, and the Darcy number $Da$ will be reported for various cavity aspect ratios ($AR = 1.0\text{--}4.0$). In addition, representative results for the average Nusselt and Sherwood numbers $Nu$ and $Sh$ for various conditions will be presented in tabulated form and discussed. In all of these results, $Ra$ and $\varepsilon$ were fixed at the values of $10^5$ and 0.6, respectively.

Figure 3 shows steady-state contours for the streamline, temperature, and concentration for a square cavity ($AR = 1.0$) and various values of the cavity inclination angle $\alpha$. When $\alpha = 0^\circ$ (non-inclined cavity), a single stretched recirculating cell or vortex in the whole enclosure exists. Tilting the cavity by up to $60^\circ$ increases the strength of clockwise vortex movement and twisting and decreases cell stretching. However, when $\alpha = 90^\circ$ the flow dampens down significantly and two separated recirculating vortices are predicted within the cavity. It is also observed that the streamlines tend to move away from the cavity walls as $\alpha$ is increased from $0^\circ$ to $60^\circ$ before they split, forming

Fig. 3. Effects of cavity inclination angle on (a) streamlines, (b) isotherms, and (c) iso-concentration contours, $AR = 1$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, and $\varepsilon = 0.6$. 

702
two distinctive cells. The isotherms are uniformly distributed within the cavity. Slight changes in these contours are observed as $\alpha$ is increased from $0^\circ$ to $60^\circ$. However, for $\alpha = 90^\circ$ the isotherms show a more non-uniform behavior in the core region. As the inclination angle increases, the iso-concentration contours change from being more parallel to the insulated walls of the cavity in the core region for $\alpha = 0^\circ$ to becoming less parallel to the insulated walls and more non-uniform for $\alpha = 90^\circ$. By comparison with the same problem considered by Chamkha and Al-Mudhaf [4] but under constant temperatures and concentrations on two opposing walls, it is observed that the streamlines’ patterns and isotherms and iso-concentration contours show a very similar behavior.

Representative profiles for the X- and Y-components of velocity U and V, temperature $\theta$, and concentration C at the cavity mid-section for various values of cavity inclination angle $\alpha$ and AR = 1.0 are presented in Figs. 4 through 7, respectively. As mentioned before, tilting the cavity by up to $60^\circ$ increases the strength of the recirculating cell within the cavity. However, further increase in the inclination angle has the tendency to slow down the flow movement. It can be seen from Figs. 4 and 5 that the X-component of velocity U decreases close to the heated wall as $\alpha$ is increased from $0^\circ$ to $60^\circ$ while it increases significantly for $\alpha = 90^\circ$ (not shown here because it falls outside the scale of the figure). On the other hand, the Y-component of velocity V increases initially as $\alpha$ is increased from $0^\circ$ to $60^\circ$ and then decreases for $\alpha = 90^\circ$. The temperature and concentration profiles show decreases in temperature $\theta$ and solute concentration C close to the heated wall as the cavity is tilted by up to $60^\circ$ followed by significant increases for $\alpha = 90^\circ$. All of these behaviors are clear from Figs. 4 though 7.

Table 2 illustrates the influence of the cavity inclination angle $\alpha$ on the average Nusselt number $\bar{Nu}$ and the average Sherwood number $\bar{Sh}$ for a square cavity (AR = 1.0). It is observed that as the values of $\bar{Nu}$ at the heated wall of the cavity increase as $\alpha$ is increased from $\alpha = 0^\circ$ to $\alpha = 60^\circ$, the values of $\bar{Sh}$ increase as $\alpha$ is increased from $\alpha = 0^\circ$ to $\alpha = 30^\circ$ and then decrease as the cavity is tilted further. However, as $\alpha$ is increased beyond $60^\circ$, the average Nusselt number decreases. These behaviors are clear from Table 2.

![Fig. 4. Effects of $\alpha$ on X-component of velocity at cavity mid-section.](image-url)
Fig. 5. Effects of $\alpha$ on Y-component of velocity at cavity mid-section.

Fig. 6. Effects of $\alpha$ on temperature profiles at cavity mid-section.

Fig. 7. Effects of $\alpha$ on concentration profiles at cavity mid-section.
Table 2. Effects of \( \alpha \) on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for AR = 1.0, \( Da = 10^{-4}, Le = 10, Pr = 7.6, Ra = 10^5 \), and \( \varepsilon = 0.6 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \bar{Nu} )</th>
<th>( Sh )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0^\circ )</td>
<td>1.21451</td>
<td>6.76275</td>
</tr>
<tr>
<td>( \alpha = 30^\circ )</td>
<td>1.33359</td>
<td>7.21276</td>
</tr>
<tr>
<td>( \alpha = 45^\circ )</td>
<td>1.36732</td>
<td>7.04194</td>
</tr>
<tr>
<td>( \alpha = 60^\circ )</td>
<td>1.37579</td>
<td>6.64124</td>
</tr>
<tr>
<td>( \alpha = 90^\circ )</td>
<td>1.13600</td>
<td>4.72267</td>
</tr>
</tbody>
</table>

Figure 8 depicts the effects of the buoyancy ratio \( N \) on the contour maps of the streamlines, temperature, and concentration for AR = 1.0. The streamlines show a behavior where a single vortex close to the isothermal wall of the enclosure exists for \( N = -10 \) and \( N = -5 \) (opposing flow). For \( N = 0 \) (thermally-buoyant flow) a single vortex centered in the cavity is predicted while this vortex becomes twisted in the clockwise direction for \( N = 10 \) (aiding flow). It is observed from these contours plots that the flow becomes less intense with increasing values of \( N \). As \( N \) increases, the eye of the recirculating cell widens and the flow pattern becomes more aligned to the diagonal joining the right lower and left upper corners of the cavity. The isotherms and the iso-concentration contours are more twisted towards the heated wall and almost parallel to the heated wall for \( N = -10 \) and twisted towards
the cooled isothermal wall for \( N = 10 \). Furthermore, unlike the isotherms, the iso-concentration contours show relatively significant changes in the boundary-layer regions close to the walls of the cavity.

Table 3 illustrates the influence of the buoyancy ratio \( N \) on the values of \( \text{Nu} \) and \( \text{Sh} \) for a square cavity (\( \text{AR} = 1.0 \)). It is predicted that increases in the buoyancy ratio \( N \) produce higher values of \( \text{Nu} \) and \( \text{Sh} \) for aiding flow situations (\( N > 0 \)). However, for opposing flow situations (\( N < 0 \)), it is predicted that \( \text{Nu} \) and \( \text{Sh} \) decrease for \( N = -5.0 \) and then increases for \( N = -10 \) as \( N \) increases. Plots of \( \text{Nu} \) and \( \text{Sh} \) versus \( N \) show that their values are minimum at \( N = -5.0 \).

Table 3. Effects of \( N \) on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for \( \text{AR} = 1.0, \text{Da} = 10^{-4}, \text{Le} = 10, \text{Pr} = 7.6, \text{Ra} = 10^5, \alpha = 45^\circ, \) and \( \varepsilon = 0.6 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \text{Nu} )</th>
<th>( \text{Sh} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = -10 )</td>
<td>1.02721</td>
<td>1.88448</td>
</tr>
<tr>
<td>( N = -5.0 )</td>
<td>1.02301</td>
<td>1.54630</td>
</tr>
<tr>
<td>( N = 0.0 )</td>
<td>1.09393</td>
<td>3.56453</td>
</tr>
<tr>
<td>( N = 5.0 )</td>
<td>1.25747</td>
<td>5.87557</td>
</tr>
<tr>
<td>( N = 10 )</td>
<td>1.36732</td>
<td>7.04194</td>
</tr>
</tbody>
</table>

Figure 9 presents steady-state contour maps for the streamline, temperature, and concentration for various values of the Darcy number \( \text{Da} \) and \( \text{AR} = 1.0 \). It is seen that as \( \text{Da} \) decreases, the streamline contours show the existence of a single vortex which becomes distorted and less stretched and the flow moves slower. This is evident from the decreases in the extreme values of the stream function as \( \text{Da} \) decreases. The distortion effect which occurs in the clock-wise direction continues causing the recirculating vortex to become centered in the core of the cavity at \( \text{Da} = 10^{-7} \). It can be said that the flow, isotherms, and iso-concentration contour patterns at \( \text{Da} = 10^{-7} \) are similar to the patterns observed using the Darcy flow models (see Prasad and Kulacki [20]). However, the flow, isotherms, and iso-concentration contours at a Darcy number \( 10^{-4} \) show an entirely different structure. Unlike in the Darcy flow regime, the results presented for \( \text{Da} = 10^{-4} \) are very strongly in the boundary layer flow regime with relatively strong convective channelling near the hot and cold walls of the cavity. This is expected because the medium considered at \( \text{Da} = 10^{-4} \) is highly permeable and thus has strong convective motion. Also, for large values of \( \text{Da} \) the temperature and concentration contours show significant convection-diffusion effects. However, the isotherms and iso-concentration contours become more parallel to the heated wall of the cavity as \( \text{Da} \) decreases indicating the approach to a quasi-conduction regime.

Table 4 displays the effects of increasing the Darcy number \( \text{Da} \) on the values of the average Nusselt and Sherwood numbers \( \text{Nu} \) and \( \text{Sh} \) for a buoyancy ratio \( N = 10 \) and \( \text{AR} = 1.0 \). Again, it is predicted that both \( \text{Nu} \) and \( \text{Sh} \) exhibit an increasing trend with increases in the values of \( \text{Da} \). As is seen from Fig. 9, for \( \text{Da} = 10^{-4} \), the temperature and concentration gradients exist in the middle of the cavity to transfer the heat and mass by diffusion in the horizontal direction and convective motion of the fluid with a thick boundary layer aids the diffusion in transferring more energy and mass. However, for \( \text{Da} = 10^{-7} \), the transfer of heat is due to conduction and therefore less energy and mass are transferred as evident from Table 4.
Figure 10 presents steady-state contours for the streamline, temperature, and concentration for various values of the cavity inclination angle $\alpha$ and $AR = 2.0$. Again, for $\alpha = 0^\circ$ a single stretched recirculating cell or vortex in the whole enclosure exists. Tilting the cavity by up to $45^\circ$ increases the strength of clockwise flow movement. However, further tilting of the cavity decreases the flow movement. For $\alpha = 90^\circ$ six separated recirculating cells are predicted within the cavity. The cells are stretched and aligned parallel to the insulated walls. This is unlike the predictions reported by Chamkha and Al-Mudhaf [4] on the same problem with uniform temperatures and concentrations on two opposing walls under the same parametric conditions in which the six cells were not stretched.

**Table 4. Effects of Da on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for AR = 1.0, Le = 10, N = 10, Pr = 7.6, Ra = $10^5$, $\alpha = 45^\circ$, and $\varepsilon = 0.6$.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\overline{\text{Nu}}$</th>
<th>$\overline{\text{Sh}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Da = 10^{-4}$</td>
<td>1.34471</td>
<td>6.91969</td>
</tr>
<tr>
<td>$Da = 10^{-5}$</td>
<td>1.05380</td>
<td>2.89446</td>
</tr>
<tr>
<td>$Da = 10^{-6}$</td>
<td>1.01882</td>
<td>1.14654</td>
</tr>
<tr>
<td>$Da = 10^{-7}$</td>
<td>1.01736</td>
<td>1.18513</td>
</tr>
</tbody>
</table>
parallel to the insulated walls but rather stretched in a “zig-zag” pattern. The isotherms are uniformly distributed, satisfying the boundary conditions. Slight changes in these contours are observed as $\alpha$ is increased from $0^\circ$ to $60^\circ$. However, for $\alpha = 90^\circ$ the isotherms show a wiggly behavior related to the formation of the six-cell structure mentioned before. As for the iso-concentration contours, they are more non-uniform than the isotherms in the core region, and for $\alpha = 90^\circ$ the non-uniformities become more significant.

Representative profiles for the X- and Y-components of velocity $U$ and $V$, temperature $\theta$, and concentration $C$ at the cavity mid-section for various values of cavity inclination angle $\alpha$ and $AR = 2.0$ are presented in Figs. 11 through 14, respectively. As mentioned before, tilting the cavity by up to $45^\circ$ increases the strength of the recirculating cell within the cavity. However, further increase in
Fig. 11. Effects of $\alpha$ on X-component of velocity at cavity mid-section.

Fig. 12. Effects of $\alpha$ on Y-component of velocity at cavity mid-section.

Fig. 13. Effects of $\alpha$ on temperature profiles at cavity mid-section.
the inclination angle has the tendency to slow down the flow movement. It can be seen from Figs. 11 and 12 that the X-component of velocity $U$ close to the heated wall decreases as $\alpha$ is increased from $0^\circ$ to $60^\circ$ while it increases significantly for $\alpha = 90^\circ$ (not shown here because it falls outside the scale of the figure). On the other hand, the Y-component of velocity $V$ increases initially as $\alpha$ is increased from $0^\circ$ to $45^\circ$ and then decreases as $\alpha$ increases further. Again, it should be remembered that the net flow behavior is determined based on the magnitude of the net velocity defined by $\sqrt{U^2 + V^2}$. The temperature and concentration profiles show an initial decrease in temperature and solute concentration near the heated wall as the cavity is tilted by $30^\circ$ followed by slight increases as $\alpha$ is increased from $30^\circ$ to $60^\circ$ (but the temperature remains lower than that corresponding $\alpha = 0^\circ$) and then followed by significant increases for $\alpha = 90^\circ$.

Table 5 illustrates the influence of the cavity inclination angle $\alpha$ on the average Nusselt number $\overline{Nu}$ and the average Sherwood number $\overline{Sh}$ for $AR = 2.0$. It is seen from this figure that $\overline{Nu}$ (Sh) increases as $\alpha$ is increased from $0^\circ$ to $45^\circ$ ($30^\circ$). However, as $\alpha$ is increased beyond $45^\circ$ ($30^\circ$), both $\overline{Nu}$ and $\overline{Sh}$ decrease as $\alpha$ is increased further. It should be noted that by comparison with the work of Chamkha and Al-Mudhaf [4], the values of the average Nusselt and Sherwood numbers for this problem are predicted to be higher than those reported by Chamkha and Al-Mudhaf [4] for the constant temperatures and concentrations on two opposing walls under the same parametric conditions.

Table 5. Effects of $\alpha$ on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for $AR = 2.0$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, and $\varepsilon = 0.6$

<table>
<thead>
<tr>
<th>Parameter $\alpha$</th>
<th>$\overline{Nu}$</th>
<th>$\overline{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>1.32162</td>
<td>6.46558</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>1.40642</td>
<td>6.61157</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>1.41573</td>
<td>6.41339</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>1.40015</td>
<td>6.03535</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>1.06950</td>
<td>4.36872</td>
</tr>
</tbody>
</table>
Figure 15 shows the influence of the buoyancy ratio \( N \) on the contour maps of the streamlines, temperature, and concentration for \( AR = 2.0 \). The streamlines show a behavior where a single vortex close to the isothermal wall of the cavity exists and that this vortex is stretched for \( N = -10 \) (opposing flow). However, for \( N = 0 \) (thermally-buoyant flow) and \( N = 10 \) (aiding flow) the vortex becomes less stretched and twists in the clockwise direction. As in the case of \( AR = 1.0 \), the isotherms and the iso-concentration contours are more twisted towards the heated wall and almost parallel to the heated wall for \( N = -10 \) and twisted towards the cooled isothermal wall for \( N = 10 \). Furthermore, unlike the isotherms, the iso-concentration contours show relatively significant changes in the boundary-layer regions close to the walls of the cavity.

Table 6 depicts the influence of the buoyancy ratio \( N \) on the values of \( \overline{Nu} \) and \( \overline{Sh} \) for \( AR = 2.0 \). As in the case of \( AR = 1.0 \), plotting of \( \overline{Nu} \) and \( \overline{Sh} \) versus \( N \) show V-shaped figures, their values
Table 6. Effects of N on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for AR = 2.0, Da = 10^{-4}, Le = 10, Pr = 7.6, Ra = 10^5, α = 45°, and ε = 0.6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nu</th>
<th>Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = -10</td>
<td>1.03384</td>
<td>2.61541</td>
</tr>
<tr>
<td>N = -5.0</td>
<td>1.02091</td>
<td>1.91822</td>
</tr>
<tr>
<td>N = 0.0</td>
<td>1.12171</td>
<td>3.64243</td>
</tr>
<tr>
<td>N = 5.0</td>
<td>1.29972</td>
<td>5.45157</td>
</tr>
<tr>
<td>N = 10</td>
<td>1.41573</td>
<td>6.41339</td>
</tr>
</tbody>
</table>

being minimum at N = -5.0. That is, increases in the buoyancy ratio N produce higher values of Nu and Sh for aiding flow situations (N > 0). However, for opposing flow situations (N < 0), it is predicted that Nu and Sh increase as N decreases below N = -5.0.

Fig. 16. Effects of Darcy number on (a) streamlines, (b) isotherms, and (c) iso-concentration contours, AR = 2.0, Le = 10, N = 10, Pr = 7.6, Ra = 10^5, α = 45°, and ε = 0.6.
Figure 16 shows steady-state contour maps for the streamline, temperature, and concentration for various values of the Darcy number Da and AR = 2.0. It is seen that as Da decreases, the streamline contours show the existence of a single vortex which becomes distorted and less stretched and the flow moves slower. Also, for large values of Da the temperature and concentration contours show significant convection-diffusion effects. However, the isotherms and iso-concentration contours become more parallel to the heated wall of the cavity as Da decreases, indicating the approach to a quasi-conduction regime.

Finally, a representative set of results for the streamlines, temperature, and concentration and for the X- and Y-components of velocity U and V, temperature θ, and concentration C profiles at the cavity mid-section for various values of α and N for both AR = 3.0 and AR = 4.0 are presented in Figs. 17 through 23. Essentially, the same streamline, isotherm, and iso-concentration contour qualitative features are predicted for AR = 3.0 and AR = 4.0 as those observed for AR = 2.0 in the

Fig. 17. Effects of cavity inclination angle on (a) streamlines, (b) isotherms, and (c) iso-concentration contours, AR = 3, Da = 10^{-4}, Le = 10, Pr = 7.6, Ra = 10^5, and ε = 0.6.
Fig. 18. Effects of $\alpha$ on X-component of velocity at cavity mid-section.

Fig. 19. Effects of $\alpha$ on Y-component of velocity at cavity mid-section.

Fig. 20. Effects of $\alpha$ on temperature profiles at cavity mid-section.
Fig. 21. Effects of $\alpha$ on concentration profiles at cavity mid-section.

Fig. 22. Effects of buoyancy ratio on (a) streamlines, (b) isotherms, and (c) iso-concentration contours, $AR = 3, Da = 10^{-4}, Le = 10, Pr = 7.6, Ra = 10^5, \alpha = 45^\circ$, and $\varepsilon = 0.6$. 

715
previously discussed figures. By comparison with the previous work of Chamkha and Al-Mudhaf [4], it is noted that for the case of AR = 3.0 and $\alpha = 90^\circ$ under the same parametric conditions, the number of predicted recirculating cells is 11 and are stretched in a “zig-zag” pattern between the insulated walls. However, in this present problem, only 10 recirculating cells stretched uniformly parallel to the insulated walls are predicted for AR = 3.0 and $\alpha = 90^\circ$. The same behavior is predicted for the case of AR = 4.0 and $\alpha = 90^\circ$ in which only five wider non-uniformly aligned cells are formed for the problem of Chamkha and Al-Mudhaf [4] while in the present problem, 10 uniformly aligned cells are predicted under the same parametric conditions. This type of behavior has a clear impact on the isotherm and iso-concentration contours and therefore, on the average Nusslet and Sherwood numbers.

Fig. 23. Effects of cavity inclination angle on (a) streamlines, (b) isotherms, and (c) iso-concentration contours, AR = 4, Da = $10^{-4}$, Le = 10, Pr = 7.6, Ra = $10^5$, and $\varepsilon = 0.6$.

Tables 7 and 9 present the same physical parameters as in Table 5 except for AR = 3.0 and 4.0, respectively. The same behaviors in Nu and Sh as for AR = 2.0 are observed as $\alpha$ is increased.
Table 7. Effects of $\alpha$ on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for $AR = 3.0$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, and $\varepsilon = 0.6$

<table>
<thead>
<tr>
<th>Parameter $\alpha$</th>
<th>$\bar{Nu}$</th>
<th>$\bar{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0^\circ$</td>
<td>1.32573</td>
<td>6.00961</td>
</tr>
<tr>
<td>$-30^\circ$</td>
<td>1.37103</td>
<td>6.06613</td>
</tr>
<tr>
<td>$-45^\circ$</td>
<td>1.36234</td>
<td>5.86025</td>
</tr>
<tr>
<td>$-60^\circ$</td>
<td>1.33117</td>
<td>5.48881</td>
</tr>
<tr>
<td>$-90^\circ$</td>
<td>1.05643</td>
<td>4.11441</td>
</tr>
</tbody>
</table>

Table 8. Effects of $N$ on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for $AR = 3.0$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, $\alpha = 45^\circ$, and $\varepsilon = 0.6$

<table>
<thead>
<tr>
<th>Parameter $N$</th>
<th>$\bar{Nu}$</th>
<th>$\bar{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10$</td>
<td>1.05222</td>
<td>2.96168</td>
</tr>
<tr>
<td>$-5.0$</td>
<td>1.02808</td>
<td>2.10921</td>
</tr>
<tr>
<td>$0.0$</td>
<td>1.10113</td>
<td>3.34103</td>
</tr>
<tr>
<td>$5.0$</td>
<td>1.25905</td>
<td>4.99608</td>
</tr>
<tr>
<td>$10$</td>
<td>1.36234</td>
<td>5.86025</td>
</tr>
</tbody>
</table>

Table 9. Effects of $\alpha$ on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for $AR = 4.0$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, and $\varepsilon = 0.6$

<table>
<thead>
<tr>
<th>Parameter $\alpha$</th>
<th>$\bar{Nu}$</th>
<th>$\bar{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0^\circ$</td>
<td>1.30126</td>
<td>5.62960</td>
</tr>
<tr>
<td>$-30^\circ$</td>
<td>1.32586</td>
<td>5.63320</td>
</tr>
<tr>
<td>$-45^\circ$</td>
<td>1.30955</td>
<td>5.41905</td>
</tr>
<tr>
<td>$-60^\circ$</td>
<td>1.27331</td>
<td>5.04363</td>
</tr>
<tr>
<td>$-90^\circ$</td>
<td>1.09690</td>
<td>4.64236</td>
</tr>
</tbody>
</table>

Table 10. Effects of $N$ on the Average Nusselt and Sherwood Numbers at the Heated Wall of the Cavity for $AR = 4.0$, $Da = 10^{-4}$, $Le = 10$, $N = 10$, $Pr = 7.6$, $Ra = 10^5$, $\alpha = 45^\circ$, and $\varepsilon = 0.6$

<table>
<thead>
<tr>
<th>Parameter $N$</th>
<th>$\bar{Nu}$</th>
<th>$\bar{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10$</td>
<td>1.06033</td>
<td>3.03632</td>
</tr>
<tr>
<td>$-5.0$</td>
<td>1.03003</td>
<td>2.15453</td>
</tr>
<tr>
<td>$0.0$</td>
<td>1.07996</td>
<td>3.04256</td>
</tr>
<tr>
<td>$5.0$</td>
<td>1.21904</td>
<td>4.61545</td>
</tr>
<tr>
<td>$10$</td>
<td>1.30955</td>
<td>5.41905</td>
</tr>
</tbody>
</table>
from $\alpha = 0^\circ$ to $\alpha = 90^\circ$, with the exception that both $\overline{\text{Nu}}$ and $\overline{\text{Sh}}$ increase as $\alpha$ is increased from $0^\circ$ to $30^\circ$. However, as $\alpha$ is increased beyond $30^\circ$, both $\overline{\text{Nu}}$ and $\overline{\text{Sh}}$ decrease as $\alpha$ is increased further.

Tables 8 and 10 show the same physical parameters as in Table 6 except for $\text{AR} = 3.0$ and 4.0, respectively. The same behaviors in $\overline{\text{Nu}}$ and $\overline{\text{Sh}}$ as for $\text{AR} = 2.0$ are observed as $N$ is increased from $N = -10$ to $N = 10$.

By inspection of the above figures and tables, one can easily observe that increasing the cavity aspect ratio beyond 2 decreases both the average Nusselt and Sherwood numbers and causes increased multi-cellular structure within the cavity for $\alpha = 90^\circ$.

### 6. Useful Correlations

Table 11 summarizes some useful correlations for $\overline{\text{Nu}}$ and $\overline{\text{Sh}}$ based on the tables above. The correlations obtained are for $\overline{\text{Nu}} = \text{Nu}(\alpha, \text{AR})$ and $\overline{\text{Sh}} = \text{Sh}(\alpha, \text{AR})$ and for $\overline{\text{Nu}} = \text{Nu}(N, \text{AR})$ and $\overline{\text{Sh}} = \text{Sh}(N, \text{AR})$. All correlations follow the simple polynomial form $F = y_0 + ax + by + cx^2 + dy^2$ as explained in Table 11. In this table, $R$ is the coefficient of correlation and $R^2$ is the coefficient of determination.

### 7. Conclusions

The problem of thermo-solutal convective flow of a binary fluid mixture inside an inclined rectangular porous enclosure due to mixed temperature and concentration and heat and mass fluxes conditions was studied numerically using the finite-difference method. Comparisons with previously published work on modified cases of the problem were performed and found to be in good agreement. Graphical results for the streamline, temperature, and concentration contours and representative velocity, temperature, and concentration profiles at the cavity mid-section for various parametric conditions were presented and discussed. It was found that the heat and mass transfer and the flow characteristics inside the enclosure depended strongly on the buoyancy ratio, cavity inclination angle, Darcy number, and cavity aspect ratio. In general, increasing the cavity tilting angle produced reductions in the average Nusselt and Sherwood numbers with the exception of a critical angle for which they become maximum. The critical inclination angle depended on the values of the parameters involved in the problem. Decreasing the Darcy number was found to reduce the average Nusselt and
Sherwood numbers and the fluid circulation within the enclosure. In addition, increasing the buoyancy ratio was predicted to decrease both the average Nusselt and Sherwood numbers for opposing flow situations reaching a minimum at the thermal convection condition (or close to it depending on the walls’ heating conditions) while they increased for aiding flow situations regardless of the cavity aspect ratio. Furthermore, increasing the cavity aspect ratio beyond 2 decreased both the average Nusselt and Sherwood numbers and caused increased multi-cellular structure within the cavity for $\alpha = 90^\circ$.

**Literature Cited**