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Combined Convection in Micropolar Fluids from a Vertical Surface with Slip

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Abstract

Boundary layer solutions are presented to study the combined convection from a slotted vertical plate to a micropolar fluid. Numerical results are obtained for the velocity, angular velocity and temperature distribution. Asymptotic solutions are presented for the region close to the leading edge and also the region far away from leading edge. The missing wall values of the velocity, angular velocity and thermal functions are tabulated. The Prandtl number was taken as 10, while the dimensionless grouping of the material properties was allowed to vary over a wide range.

INTRODUCTION

Micropolar fluids are fluids containing micro-constituents which can undergo micro rotation and micro inertia, the presence of which can affect the hydrodynamics of the flow so that these fluids can distinctively be non-Newtonian. These micropolar fluids support couple stresses and body couples and are physically represented as fluids consisting of bar like elements. Some of the examples of micropolar fluids are certain anisotropic fluids, polymeric fluids, fluids with some additives, colloidal suspensions and animal blood.

It is a well known fact that the classical Navier-Stokes theory failed to describe adequately the flow properties of these micropolar fluids, but Eringen [1] successfully proposed the theory of these micropolar fluids. The theory of thermo micropolar fluids was also developed by Eringen [2] by extending the theory of micropolar fluids.

Lukaszewic [3] wrote a book on the theory of micropolar fluids. The stagnation point heat transfer in micropolar fluids was studied by Gorla [4]. The forced convective heat transfer in a micropolar fluid over a flat plate was

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NOMENCLATURE

G_r	Grashof Number. $\frac{g\beta\chi^3(T_w - T_\infty)}{\upsilon^2}$
f, F	dimensionless velocity
g, G	dimensionless micro rotation
h	heat transfer coefficient. (W/m^{2}k)
j	micro-inertia per unit mass. (m^2)
L	characteristic length. (m)
N	angular velocity.(rad/s)
N_u	Nusselt Number.(hx/k$_f$)
P	pressure
P_r	Prandtl Number. (C$_p\mu$/k$_f$)
q	surface heat flux.
R_e	Reynolds number (U$_\infty$x/υ)
T	Temperature. (K)
u	velocity in x-direction. (m/sec)
v	velocity in y-direction. (m/sec)
U_∞	velocity at boundary layer edge (m/sec)
x	distance along the surface (m)
y	distance normal to surface (m)

Greek symbols

ψ	Stream function $(2\nu U_\infty x)^{1/2} f(\xi,\eta)$
μ	dynamic viscosity. (kg m/sec)
ρ	density of fluid. (kg/m^3)
υ	kinematic viscosity. (kg m/sec)
χ, t	dimensionless distance parallel to plate
η	dimensionless distance normal to plate
θ	dimensionless temperature
Δ, Λ	dimensionless material properties
λ	slip velocity coefficient
γ	spin gradient viscosity or microrotation viscosity (kg m /sec)
κ	vortex viscosity (kg/m sec)
β	coefficient of thermal expansion (1/K)
k_f	thermal conductivity of fluid (W/m-K)
τ	shear stress (Pa)

Subscripts

w	conditions at the surface
∞	conditions far away from the surface
f	fluid properties

investigated by Gorla [5]. A similarity solution of the free convective flow of a thermo-micropolar fluid along a vertical plate was presented by Jena and Mathur [6]. Papers by Gorla[7-15] over a period of time on Micropolar fluids suggest the development of the theory of micropolar fluids.

Lukaszewic [3] wrote a book on the theory of micropolar fluids. The stagnation point heat transfer in micropolar fluids was studied by Gorla [4]. The forced convective heat transfer in a micropolar fluid over a flat plate was investigated by Gorla [5]. A similarity solution of the free convective flow of a thermo-micropolar fluid along a vertical plate was presented by Jena and Mathur [6]. Papers by Gorla[7-15] over a period of time on Micropolar fluids suggest the development of the theory of micropolar fluids.

In recent years the study of fluid flow and heat transfer through a porous medium has received considerable interest because of numerous applications as geothermal energy technology, filtration process in metallurgical industries, underground disposal of chemical waste, petroleum recovery etc.

There exist several other practical applications in which temperature differences between the surface of the body and the free stream are present. The density gradients are caused in the fluid medium as a result of the imposed temperature differences and in the presence of gravitational body force, natural convection effects have to be taken into account. Combined forced and free convection effects become important when both of them are of comparable order. Gorla [16] et.al. have presented asymptotic boundary layer solutions to study the combined convection from a vertical isothermal plate placed in a micropolar fluid stream.

Recently, there has been some interest in the boundary layer flows involving slip at the surface. Wang [17, 18] analyzed the stagnation point problem in the presence of slip at the surface. Fang et. al. [19] applied the slip flow model to the boundary layer flow on a continuous moving shrinking sheet.

In this paper, we propose to study the problem of combined convection boundary layer flow of a viscous incompressible micropolar fluid along a slotted vertical plate or a vertical plate with slip. The dimensionless equations that govern the flow have been solved numerically employing an implicit finite difference method. Solutions are presented in terms of the skin friction factor and local heat transfer rate showing the effects of different pertinent physical parameters.

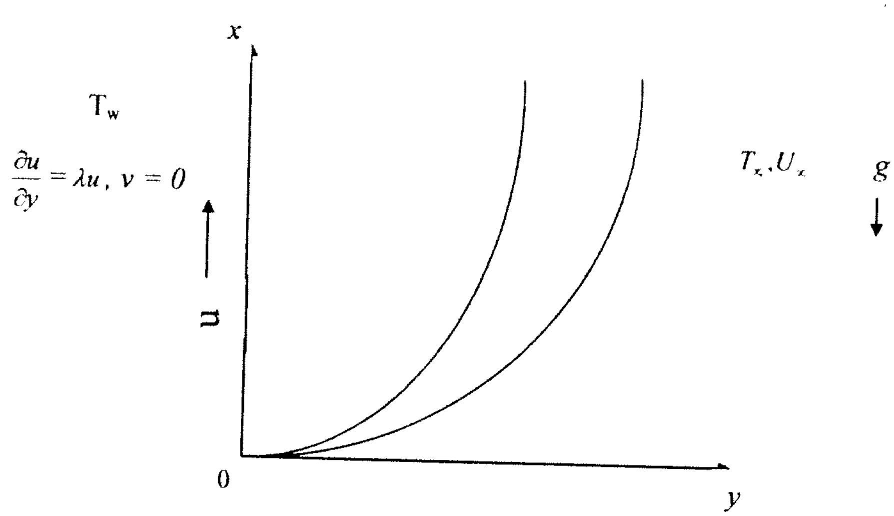

Figure 1. Flow model and coordinate system.

We consider the combined convection flow of a uniform laminar, incompressible, steady micropolar fluid stream over a vertical slotted plate. As shown in Figure 1, the x and y coordinates of the plate are selected such that the x-direction is along the leading edge of the plate and the y-direction is measured normal to the surface of the plate. The temperature at the wall is assumed to be T_w. T_∞ is the free stream fluid temperature and U_∞ is the velocity of the free stream at far away distance from the wall. The gravitational force acts in negative x-direction. When $T_w > T_\infty$ the buoyancy force acts in the same direction as the forced flow and acts in opposite direction when $T_w < T_\infty$. The former is called assisting flow and latter is referred as opposite flow.

The governing equations for a steady, laminar, incompressible, micro polar fluid over a semi-infinite vertical plate with variable micro-inertia may be written within boundary layer approximation as:
Mass equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial N}{\partial y}+\rho g\beta(T-T_\infty) \tag{2}$$

Angular Momentum equation

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-k\left(2N+\frac{\partial u}{\partial y}\right)+\gamma\frac{\partial^2 N}{\partial y^2} \tag{3}$$

Energy equation

$$\rho C_p\left[u\frac{\partial T}{\partial y}+v\frac{\partial T}{\partial y}\right]=k_f\frac{\partial^2 T}{\partial y^2} \tag{4}$$

In the above equations, T is the temperature of the fluid; N the microrotation component; u and v the velocity components in the x and y directions; ρ the fluid density; g acceleration due to gravity and β the volumetric coefficient of thermal expansion; α the thermal diffusivity; N the microrotation component normal to (x,y)-plane j the micro-inertia density; k the vortex viscosity; k_f the thermal conductivity of the fluid and γ the spin-gradient viscosity given by $\gamma=(\mu+k/2)j$

The boundary condition for the velocity and angular velocity fields may be written as

$$y=0 \ : \quad \frac{\partial u}{\partial y}=\lambda u\ ,\ v=0\ ,\ N=-n\frac{\partial u}{\partial y} \tag{5}$$

$$y\to\infty : \quad u\to U_\infty, \quad N\to 0$$

For the temperature field, we have

$$y = 0: \qquad T = T_w$$
$$y \to \infty: \qquad T \to T_\infty$$

A variable relation between microrotation and the surface skin frictic
[12], where n is a constant and varies from 0 to 1. The value n=
concentrated particle flows in which the particle density is sufficier
microelements close to the wall are unable to rotate. This condition
as the strong interaction. When n=0.5 the particle rotation is e
viscosity at the boundary for fine particle suspension. When n=1, w
which are representative to turbulent boundary layers.
Proceeding with the analysis, we define a stream function ψ such th:

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

We now define the following dimensionless variables:

$$\eta = \left(\frac{U_\infty}{2vx}\right)^{1/2} y$$

$$\chi = \lambda x\left(\frac{2}{\mathrm{Re}_x}\right)^{1/2}$$

$$N = \left(\frac{U_\infty}{2vx}\right)^{1/2} U_\infty g(\chi,\eta)$$

$$\psi = (2vU_\infty x)^{1/2} f(\chi,\eta)$$

$$\theta = \frac{T - T_\infty}{T_W - T_\infty} \quad \text{(Isothermal surface)}$$

$$\theta = \frac{T - T_\infty}{\frac{q_w}{k_f}\left(\frac{\upsilon x}{U_\infty}\right)} \text{ (Uniform Heat Flux)}$$

Case (a): Isothermal Surface

$$u = \frac{\partial \psi}{\partial y}$$

$$\psi = (2\nu U_\infty x)^{1/2} f(\chi, \eta)$$

$$u = (2\nu U_\infty x)^{1/2} f'\left(\frac{U_\infty}{2\nu x}\right)^{1/2} = U_\infty f'$$

$$c = \frac{g\beta\Delta T}{U_\infty{}^2}$$

$$c_1 = \lambda\left(\frac{2\nu}{U_\infty}\right)^{1/2}$$

Substituting (8) in (2)-(4) we get the following Governing Equations:

$$(1+\Delta)f''' + ff'' + \Delta g' \pm 2\left(\frac{c}{c_1^2}\right)\chi^2\theta = \chi\left[f'\frac{\partial f'}{\partial \chi} - f''\frac{\partial f}{\partial \chi}\right] \tag{9}$$

$$\Lambda G g'' - 2\Lambda(2g + f'') + G(f'g + fg') = \chi\left[g'\frac{\partial f}{\partial \chi} - f'\frac{\partial g}{\partial \chi}\right] \tag{10}$$

$$\frac{\theta''}{\mathrm{Pr}} + f\theta' = \chi\left[f'\frac{\partial \theta}{\partial \chi} - \theta'\frac{\partial f}{\partial \chi}\right] \tag{11}$$

Boundary Conditions

$$f(\chi,0) = 0,\ f''(\chi,0) = \chi f'(\chi,0),\ \mathrm{f}'(\chi,\infty) = 1 \tag{12}$$

$$g(\chi,0) = -nf''(\chi,0),\ g(\chi,\infty) = 0 \tag{13}$$

$$\theta(\chi,0) = 1,\ \theta(\chi,\infty) = 0 \tag{14}$$

Case (b) Uniform Surface Heat Flux

$$\psi = (2vU_\infty x)^{1/2} \cdot f(\chi,\eta)$$

$$u = \frac{\partial \psi}{\partial y} = (2vU_\infty x)^{1/2} \cdot f' \left(\frac{U_\infty}{2vx}\right)^{1/2} = U_\infty \cdot f'$$

$$\chi = \lambda x \left(\frac{2}{\mathrm{Re}_x}\right)^{1/2}$$

$$c = \frac{8g^2 \beta^2 q w^2 v}{25 k_f^{\,2} U_\infty}$$

$$c_1 = \lambda \left(\frac{2v}{U_\infty}\right)^{1/2}$$

Substituting (8) in (2)-(4) we get following Governing Equations:

$$(1+\Delta) f''' + ff'' + \Delta g' + 5\left(\frac{c}{c_1^2}\right)^{3/2} \cdot \chi^3 \cdot \theta = \chi \left[f' \frac{\partial f'}{\partial \chi} - f'' \frac{\partial f}{\partial \chi} \right] \tag{15}$$

$$\Lambda G g'' - 2\Lambda(2g + f'') + G(f'g + fg') = \chi \left[g' \frac{\partial f}{\partial \chi} - f' \frac{\partial g}{\partial \chi} \right] \tag{16}$$

$$\frac{\theta''}{P_r} + f\theta' - f'\theta = \chi \cdot \left[f' \frac{\partial \theta}{\partial \chi} - \theta' \frac{\partial f}{\partial \chi} \right] \tag{17}$$

Boundary Conditions

$$f(\chi,0)=0, f''(\chi,0)=\chi\ f'(\chi,0), f'(\chi,\infty)=1 \tag{18}$$

$$g(\chi,0)=-n\ f''(\chi,0), g(\chi,\infty)=0 \tag{19}$$

$$\theta'(\chi,0)=-1, \theta(\chi,\infty)=0 \tag{20}$$

SOLUTION

Numerical Solution by Keller-Box Method

The governing equations of the problem under investigation were solved by the Keller-Box method which is an implicit scheme with unconditional stability. The method allows for non-uniform grid discretion and converts the differential equations into algebraic ones which are then solved by Thomas algorithm. We have used 301 grid points in the χ direction and 196 grid points in the η direction. Variable step sizes in the η direction with an initial step size of 0.001 and a growth factor of 1.03 and constant step sizes of 0.01 in the χ direction are employed. A mesh sensitivity exercise has been performed to ensure grid independence. The solution convergence criterion required that the difference between the current and previous iterations be less than 10^{-5}.

Asymptotic Solution with χ as a small parameter

Here we use a series expansion to solve equations (9)-(11) for the isothermal wall case and equations (15)-(17) for the uniform surface heat flux case. We treat χ as a small parameter so that we get the combined effects of forced and free convection on the flow near the leading edge. The Asymptotic expansion of f, g and θ in χ may be assumed to be

$$f(\chi,\eta)=\sum_{j=0}^{\infty}\chi^j f_j(\eta)=f_0(\eta)+\chi.f_1(\eta)+\chi^2 f_2(\eta)+\ldots\ldots$$

$$g(\chi,\eta)=\sum_{j=0}^{\infty}\chi^j g_j(\eta)=g_0(\eta)+\chi.g_1(\eta)+\chi^2 g_2(\eta)+\ldots\ldots$$

$$\theta(\chi,\eta)=\sum_{j=0}^{\infty}\chi^j \theta_j(\eta)=\theta_0(\eta)+\chi.\theta_1(\eta)+\chi^2\theta_2(\eta)+\ldots\ldots \tag{21}$$

Case(a): Isothermal Surface

By substituting expressions in equation (21) into equations (9), (10) and (11) and equating the like powers of χ, we obtain series solution valid for small values of χ.

Equating like powers of χ to zero we have:

Coefficient of χ^0:

$$(1+\Delta).f_0'''+\Delta.g_0'+f_0.f_0''=0 \tag{22}$$

$$\Lambda(g_0''-4g_0-2f_0'')+f_0'g_0+f_0g_0'=0 \tag{23}$$

$$\frac{\theta_0''}{P_r}+f_0\theta_0'=0 \tag{24}$$

Coefficient of χ

$$(1+\Delta).f_1'''+\Delta.g_1'+2.f_0''.f_1+f_0.f_1''-f_0'.f_1'=0 \tag{25}$$

$$\Lambda(g_1''-4g_1-2f_1'')+2f_0'g_1+f_0g_1'+f_1'g_0=0 \tag{26}$$

$$\frac{\theta_1''}{P_r}+f_0\theta_1'-f_0'\theta_1+2f_1\theta_0'=0 \tag{27}$$

Coefficient of χ^2

$$(1+\Delta).f_2'''+\Delta.g_2'+2\theta_0+3.f_2.f_0''+2.f_1.f_1''+f_0.f_2''-2.f_0'.f_2'-(f_1')^2=0 \tag{28}$$

$$\Lambda(g_2''-4g_2-2f_2'')-f_2g_0'+3f_0'g_2+f_2'g_0+f_0g_2'+2f_1'g_1=0 \tag{29}$$

$$\frac{\theta_2''}{P_r}+f_0\theta_2'+2f_1\theta_1'+3f_2\theta_0'-2.f_0'\theta_2-f_1'\theta_1=0 \tag{30}$$

Equations (22)-(30) were solved numerically using boundary conditions (12-14) by the fourth order Runge Kutta method. The missing wall values for the velocity, angular velocity and thermal functions are tabulated in Table 1.

Table 1. Wall values for. $f_0', \theta_0', g_0', f_1', \theta_1', g_1', f_2', \theta_2', g_2'$ for Pr=10.

Δ	Λ	n	$f_0'(0)$	$\theta_0'(0)$	$g_0'(0)$	$f_0'(0)$	$\theta_1'(0)$	$g_1'(0)$	$f_2'(0)$	$\theta_2'(0)$	$g_2'(0)$
0.5	0.5	0	0.5725	-1.9231	-2.3210	-0.9774	2.5471	-1.5734	-1.6512	9.6046	-0.5137
5	0.5	0	0.3678	-1.5981	-1.3410	-1.0241	2.9187	-2.1340	-1.7413	11.2750	-0.9271
50	0.5	0	0.1573	-1.2153	-1.4922	-1.1169	3.6378	-2.6730	-2.3491	12.1240	-1.0219
0.5	0.5	0.5	0.6412	-2.0201	-1.9827	-0.8941	2.2146	-1.4634	-1.4914	7.8964	-0.4871
5	0.5	0.5	0.4292	-1.6288	-1.2314	-1.0674	2.8871	-2.3491	-1.6091	8.2419	-0.6993
50	0.5	0.5	0.1814	-1.2934	-1.4256	-1.3269	3.0280	-2.9741	-2.1389	9.3292	-0.9781
0.5	0.5	1	0.7198	-2.1320	-1.7491	-0.7293	1.9573	-1.3314	-1.4109	6.7182	-0.3981
5	0.5	1	0.5391	-1.8988	-1.0637	-1.1894	3.2211	-3.0891	-1.5192	7.3412	-0.4781
50	0.5	1	0.2173	-1.3634	-1.3795	-1.3932	4.5728	-3.2149	-1.9341	8.7182	-0.8169
0.5	0.5	0.5	0.6412	-2.0201	-1.9827	-0.8941	2.2146	-1.4634	-1.4914	7.8964	-0.4871
0.5	5	0.5	0.5435	-1.8970	-1.2110	-1.2740	3.6746	-1.7634	-1.6231	8.3198	-0.6418
0.5	50	0.5	0.5035	-1.5989	-0.9751	-1.4312	3.7731	-1.9841	-2.0947	9.1941	-0.8947

Asymptotic Solution with χ as a large parameter

Here we investigate the boundary layer behavior at large distances away from leading edge. We now define

$$h = g + \frac{f''}{2}$$

We now set for the main layer equation (9)-(11) the following expansion

$$f(\chi,\eta) = F_0(\eta) + \chi^{\frac{1}{2}} F_1(\eta) + \ldots\ldots$$

$$h(\chi,\eta)=\chi^{-1}H_0(\eta)+\chi^{\frac{-3}{2}}H_1(\eta)+....$$

$$\theta(\chi,\eta)=\theta_0(\eta)+\chi^{\frac{1}{2}}\theta_1(\eta)+....... \tag{31}$$

By substituting expressions in equation (31) into equations (9) - (11) and equating the like powers of χ to zero , we obtain the following set of ordinary differential equations governing the momentum, angular momentum and thermal fields:

$$(1+\Delta)F_0'''+F_0F_0''=0 \tag{32}$$

$$(1+\Delta)F_1'''+F_0F_1''+\frac{1}{2}F_1F_0''+\frac{1}{2}F_0'F_1'=0 \tag{33}$$

$$\Lambda H_0''-4\Lambda H_0+F_0H_0'-\frac{1}{4}F_1'F_1''-\frac{3}{4}F_1F_1'''=0 \tag{34}$$

$$\Lambda H_1''-4\Lambda H_1+F_0H_1'-\frac{1}{2}F_0'H_1+\frac{3}{2}F_1H_0'=0 \tag{35}$$

$$\frac{\theta_0''}{\mathrm{Pr}}+F_0\theta_0'=0 \tag{36}$$

$$\frac{\theta_1''}{\mathrm{Pr}}+F_0\theta_1'+\frac{1}{2}F_1\theta_0'+\frac{1}{2}F_0'\theta_1=0 \tag{37}$$

Main Layer Boundary conditions are

$$f(\chi,0)=0,\ f''(\chi,0)=\chi f'(\chi,0),\ f'(\chi,\infty)=1 \tag{38}$$

$$h(\chi,0)=(\frac{1}{2}-n)f''(\chi,0),\quad h(\chi,\infty)=\frac{f''(\chi,\infty)}{2} \tag{39}$$

$$\theta(\chi,0)=1,\ \theta(\chi,\infty)=0 \tag{40}$$

Equations (32)-(37) were solved numerically by means of the fourth order Runge Kutta procedure to satisfy the boundary conditions (38-40).

Table 2. Wall values for $F_0', \theta_0', H_0', F_1', \theta_1', H_1'$ are obtained for Pr=10.

Δ	Λ	n	$F_0'(0)$	$\theta_0'(0)$	$H_0'(0)$	$F_1'(0)$	$\theta_1'(0)$	$H_1'(0)$
0.5	0.5	0	0.9999	-2.4712	0.7126	-0.9774	2.5471	-0.0847
5	0.5	0	0.9999	-2.4893	0.6934	-1.0241	2.9187	-0.0789
50	0.5	0	0.9999	-2.4914	0.5813	-1.1169	3.6378	-0.0675
0.5	0.5	0.5	0.9999	-2.4712	0.7126	-0.8941	2.2146	-0.0847
0.5	5	0.5	0.9999	-2.4712	0.0873	-1.0674	2.8871	0.3412
0.5	50	0.5	0.9999	-2.4712	0.06832	-1.3269	3.0280	0.4091

Inner Layer

We now define

$$h = g + \frac{f''}{2}$$

$$\varsigma = \eta\chi^{\frac{1}{2}}$$

We set $\Lambda = 1 + \frac{\Delta}{2}$ following Gorla et. al. [11].

We now set the inner layer equations (9)-(11) in the following expansion:

$$f(\chi,\varsigma) = \chi^{-1}F_0(\varsigma) + \chi^{\frac{-3}{2}}F_1(\varsigma) + \ldots\ldots$$

$$h(\chi,\varsigma) = H_0(\varsigma) + \chi^{\frac{-1}{2}}H_1(\varsigma) + \ldots.$$

$$\theta(\chi,\varsigma) = \chi^{\frac{-3}{2}}\theta_0(\varsigma) + \chi^{-2}\theta_1(\varsigma) + \ldots\ldots. \tag{41}$$

By substituting expressions in equation (41) into equations (9)-(11) and equating the like powers of χ to zero , we obtain the following set of ordinary differential equations governing the momentum, angular momentum and thermal fields:

$$(1+\frac{\Delta}{2})F_0'''+\Delta H_0'+2\theta_0=0 \tag{42}$$

$$(1+\frac{\Delta}{2})F_1'''+\Delta H_1'+2\theta_1=0 \tag{43}$$

$$(1+\Delta)H_0''+\theta_0'=0 \tag{44}$$

$$(1+\Delta)H_1''+\theta_1'=0 \tag{45}$$

$$\frac{\theta_0''}{\Pr}=0 \tag{46}$$

$$\frac{\theta_1''}{\Pr}=0 \tag{47}$$

The boundary conditions are

$$F_0=F_1=F_0''=F_1''=0,$$

$$H_0=\left(\frac{1}{2}-n\right)F''_0\ ,\quad H_1=\left(\frac{1}{2}-n\right)F''_1$$

$$\theta_0=1\quad ,\quad \theta_1=0 \tag{48}$$

The Solutions are given by

$$\theta_0(\varsigma)=\theta_0'(0)\varsigma+1 \tag{49}$$

$$H_0(\varsigma)=-[\varsigma+\frac{\theta_0'(0)}{(1+\Delta)}\frac{\varsigma^2}{2}+H_0'(0)\varsigma] \tag{50}$$

$$F_0(\varsigma)=F_0'(0)\varsigma-\frac{\theta_0'(0)}{(1+\Delta)}\frac{\varsigma^4}{12}-(1+\Delta)H_0'(0)\frac{\varsigma^3}{6} \tag{51}$$

$$\theta_1(\varsigma)=\theta_1'(0)\varsigma \tag{52}$$

$$H_1(\varsigma) = -\frac{\theta_1'(0)}{(1+\Delta)}\frac{\varsigma^2}{2} + H_1'(0)\varsigma \tag{53}$$

$$F_1(\varsigma) = F_1'(0)\varsigma - \frac{\theta_1'(0)}{(1+\Delta)}\frac{\varsigma^4}{12} - \frac{\varsigma^3}{3(2+\Delta)}H_1'[0] \tag{54}$$

Case(b): Uniform Heat Flux

By substituting expressions in equation (21) into equations (15), (16) and (17) and equating the like powers of χ, we obtain series solution valid for small values of χ.

Equating like powers of χ to zero we have:

Coefficient of χ^0

$$(1+\Delta)f_0''' + \Delta g_0' + f_0 f_0'' = 0 \tag{55}$$

$$\Lambda(g_0'' - 4g_0 - 2f_0'') + f_0' g_0 + f_0 g_0' = 0 \tag{56}$$

$$\frac{\theta_0''}{P_r} + f_0\theta_0' - f_0'\theta_0 = 0 \tag{57}$$

Coefficient of χ

$$(1+\Delta)f_1''' + \Delta g_1' + f_0 f_1'' + 2f_1 f_0'' - f_1 f_0' = 0 \tag{58}$$

$$\Lambda(g_1'' - 4g_1 - 2f_1'') + 2f_0' g_1 + f_1' g_0 + f_0 g_1' = 0 \tag{59}$$

$$\frac{\theta_1''}{P_r} + f_0\theta_1' + 2f_1\theta_0' - f_1'\theta_0 - 2f_0'\theta_1 = 0 \tag{60}$$

Coefficient of χ^2

$$(1+\Delta)f_2''' + \Delta g_2' + 3f_2 f_0'' + 2f_1 f_1'' + f_0 f_2'' - 2f_2' f_0' - f_1 f_1' = 0 \tag{61}$$

$$\Lambda(g_2'' - 4g_2 - 2f_2'') + 3f_0' g_2 + 2f_1' g_1 + f_2' g_0 + f_0 g_2' - f_2 g_0' = 0$$

$$\frac{\theta_2''}{P_r} + f_0\theta_2' + 2f_1\theta_1' + 3f_2\theta_0' - 2f_1'\theta_1 - 3f_0'\theta_2 - f_2'\theta_0 = 0 \qquad (62)$$

Equations (55)-(62) were solved numerically to satisfy the boundary conditions (18, 19, 20). The missing wall values of the velocity, angular velocity and thermal functions are tabulated in Table 3.

Table 3. Wall Values for $f_0', \theta_0', g_0', f_1', \theta_1', g_1', f_2', \theta_2', g_2'$ for Pr=10.

Δ	Λ	N	f_0'	θ_0	g_0'	f_1'	θ_1	g_1'	f_2'	θ_2	g_2'
0.5	0.5	0	0.5725	0.5312	-2.3210	-0.9774	0.6821	-1.5734	-0.2812	0.0346	-0.5137
5	0.5	0	0.3678	0.6223	-1.3410	-1.0241	1.1397	-2.1340	-0.3413	0.0791	-0.9271
50	0.5	0	0.1573	0.7984	-1.4922	-1.1169	2.2718	-2.6730	-0.5941	0.1174	-1.0219
0.5	0.5	0.5	0.6412	0.5014	-1.9827	-0.8941	0.5971	-1.4634	-0.2013	0.0276	-0.4871
5	0.5	0.5	0.4292	0.5908	-1.2314	-1.0674	1.0167	-2.3491	-0.5123	0.0614	-0.6993
50	0.5	0.5	0.1814	0.7794	-1.4256	-1.3269	2.5582	-2.9741	-0.9341	0.0973	-0.9781
0.5	0.5	1	0.7198	0.4738	-1.7491	-0.7293	0.3872	-1.3314	0.01778	0.0157	-0.3981
5	0.5	1	0.5391	0.5373	-1.0637	-1.1894	0.8827	-3.0891	-0.0214	0.0321	-0.4781
50	0.5	1	0.2173	0.7424	-1.3795	-1.3932	2.6316	-3.2149	-0.0419	0.0779	-0.8169
0.5	0.5	0.5	0.6412	0.5014	-1.9827	-0.8941	0.5971	-1.4634	-0.2013	0.0276	-0.4871
0.5	5	0.5	0.5435	0.5341	-1.2110	-1.2740	1.0181	-1.7634	-0.4141	0.0717	-0.6418
0.5	50	0.5	0.5035	0.5537	-0.9751	-1.4312	1.3202	-1.9841	-0.6193	0.0894	-0.8947

Asymptotic Solution with χ as a Large Parameter

By substituting expressions in equation (40) into equations (15),(16),(17) and equating the like powers of χ to zero , we obtain the following set of ordinary differential equations governing the momentum, angular momentum and thermal fields:

$$(1+\Delta)F_0''' + F_0F_0'' = 0 \qquad (63)$$

$$(1+\Delta)F_1''' + F_0F_1'' + \frac{1}{2}F_1F_0'' + \frac{1}{2}F_0'F_1' + = 0 \qquad (64)$$

$$\Lambda H_0'' - 4\Lambda H_0 + F_0H_0' - \frac{1}{4}F_1'F_1'' - \frac{3}{4}F_1F_1''' = 0 \qquad (65)$$

$$\Lambda H_1'' - 4\Lambda H_1 + F_0H_1' - \frac{1}{2}F_0'H_1 + \frac{3}{2}F_1H_0' = 0 \qquad (66)$$

2

$$\frac{\theta_0''}{\Pr} + F_0\theta_0' - F_0\theta_0' = 0 \tag{67}$$

$$\frac{\theta_1''}{\Pr} + F_0\theta_1' - F_1'\theta_0 + \frac{1}{2}F_1\theta_0' - \frac{1}{2}F_0'\theta_1 = 0 \tag{68}$$

Main Layer Boundary conditions are

$$f(\chi,0) = 0,\ f''(\chi,0) = \chi f'(\chi,0),\ f'(\chi,\infty) = 1 \tag{69}$$

$$h(\chi,0) = (\frac{1}{2} - n) f''(\chi,0),\quad h(\chi,\infty) = \frac{f''(\chi,\infty)}{2} \tag{70}$$

$$\theta'(\chi,0) = -1, \theta(\chi,\infty) = 0 \tag{71}$$

Equations (63)-(68) were solved numerically to satisfy the boundary conditions (69)-(71). The fourth order Runge Kutta method was used. The missing wall values of the velocity, angular velocity and thermal functions are tabulated in Table 4.

Table 4. Wall values for $F_0', \theta_0', H_0', F_1', \theta_1', H_1'$ for Pr=10.

Δ	Λ	n	F_0'	θ_0	H_0'	F_1'	θ_1	H_1'
0.5	0.5	0	0.9999	0.4712	0.7126	0.2740	0.3732	-0.0847
5	0.5	0	0.9999	0.4241	0.6813	0.1971	0.2919	-0.0789
50	0.5	0	0.9999	0.4016	0.5917	0.0813	0.2162	-0.0692
0.5	0.5	0.5	0.9999	0.4712	0.7126	0.2740	0.3732	-0.0847
0.5	5	0.5	0.9999	0.4712	0.0902	0.2740	0.3732	-0.3187
0.5	50	0.5	0.9999	0.4712	0.0813	0.2740	0.3732	-0.4271

Inner Layer

We now set the inner layer equations (15)-(17) in the following expansion:

$$f(\chi,\varsigma) = \chi^{-1}F_0(\varsigma) + \chi^{\frac{-3}{2}}F_1(\varsigma) + \ldots\ldots$$

$$h(\chi,\varsigma)=H_0(\varsigma)+\chi^{\frac{-1}{2}}H_1(\varsigma)+....$$

$$\theta(\chi,\varsigma)=\chi^{\frac{-5}{2}}\theta_0(\varsigma)+\chi^{-3}\theta_1(\varsigma)+....... \tag{72}$$

By substituting expressions in equation (72) into equations (15)-(17) and equating the like powers of χ to zero , we obtain the following set of ordinary differential equations governing the momentum, angular momentum and thermal fields:

$$(1+\frac{\Delta}{2})F_0'''+\Delta H_0'+5\theta_0=0 \tag{74}$$

$$(1+\frac{\Delta}{2})F_1'''+\Delta H_1'+5\theta_1=0 \tag{75}$$

$$(1+\Delta)H_0''+\frac{5}{2}\theta_0'=0 \tag{76}$$

$$(1+\Delta)H_1''+\frac{5}{2}\theta_1'=0 \tag{77}$$

$$\frac{\theta_0''}{\Pr}=0 \tag{78}$$

$$\frac{\theta_1''}{\Pr}=0 \tag{79}$$

The boundary conditions are

$$F_0=F_1=F_0''=F_1''=0,$$

$$H_0=\left(\frac{1}{2}-n\right)F''_0\ ,\quad H_1=\left(\frac{1}{2}-n\right)F''_1$$

$$\theta_0'=-1,\quad \theta_1'=0 \tag{80}$$

The solutions are given by

$$\theta_0(\varsigma) = \theta_0(0) - \varsigma \tag{81}$$

$$H_0(\varsigma) = -\frac{5}{(1+\Delta)}\frac{\varsigma}{2} - H_0'(0) \tag{82}$$

$$F_0(\varsigma) = F_0'(0)\varsigma - \frac{2}{2+\Delta}\left[(5\theta_0(0) - H_0'(0))\frac{\varsigma^3}{6} + \frac{5(3+2\Delta)}{(1+\Delta)}\frac{\varsigma^4}{48}\right] \tag{83}$$

$$\theta_1(t) = \theta_1(0) \tag{84}$$

$$H_1(t) = H_1'(0)\varsigma \tag{85}$$

$$F_1(\varsigma) = F_1'(0)\varsigma + \frac{5\varsigma^3.\theta_1(0).\Delta}{3(1+\Delta)(2+\Delta)}\left[\Delta - \frac{1}{2}\right]\varsigma^4 + H_1'(0)\frac{\varsigma^3}{3(2+\Delta)} \tag{86}$$

RESULTS AND DISCUSSION

The numerical results of the present analysis are obtained by Keller box method for the boundary conditions of Isothermal surface and uniform heat flux cases. The velocity and temperature profiles for the cases of Isothermal surface are plotted in the figures varying n and Δ.

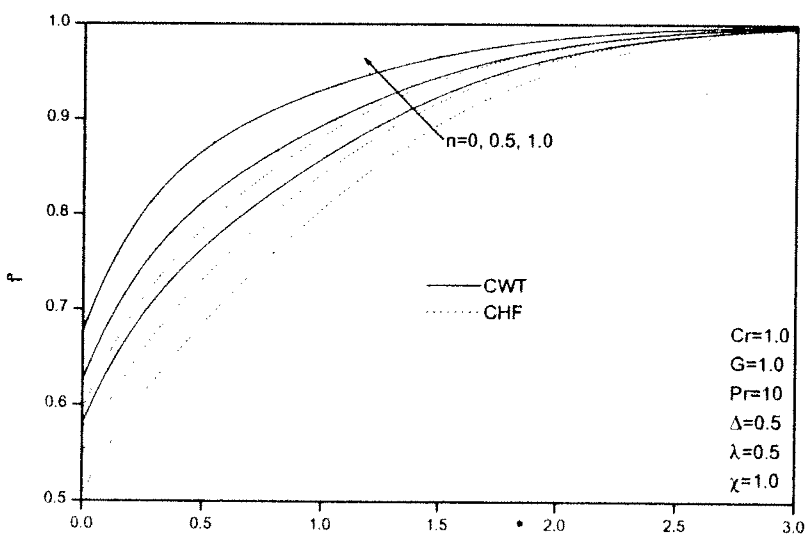

Figure 2. Velocity profiles for different values of n.

Figure 2 shows the velocity distribution within the boundary layer. As n increases from 0 to 1 the velocity increases. Figure 3 shows the microrotation profiles. Here microrotation decreases as the n value increases. Figure 4 shows the temperature profiles where temperature θ decreases when n increases.

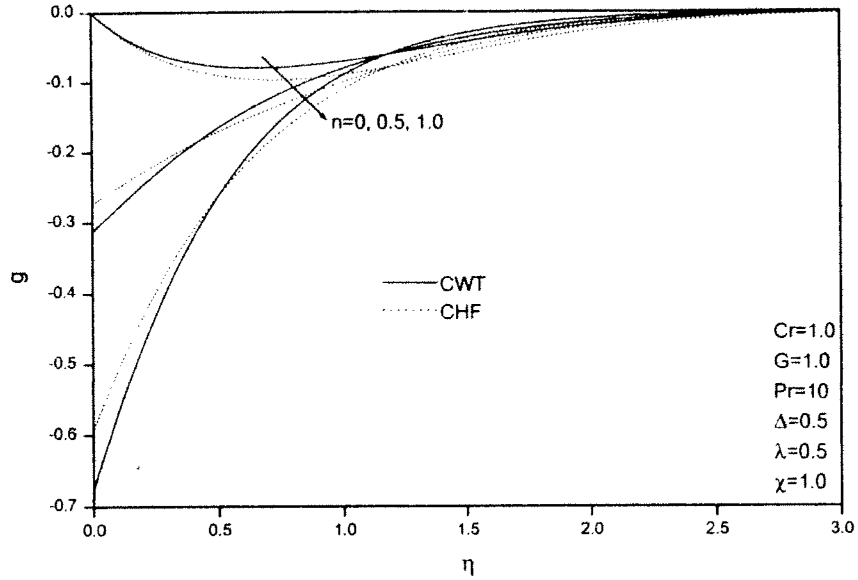

Figure 3. Microrotation profiles for different values of n.

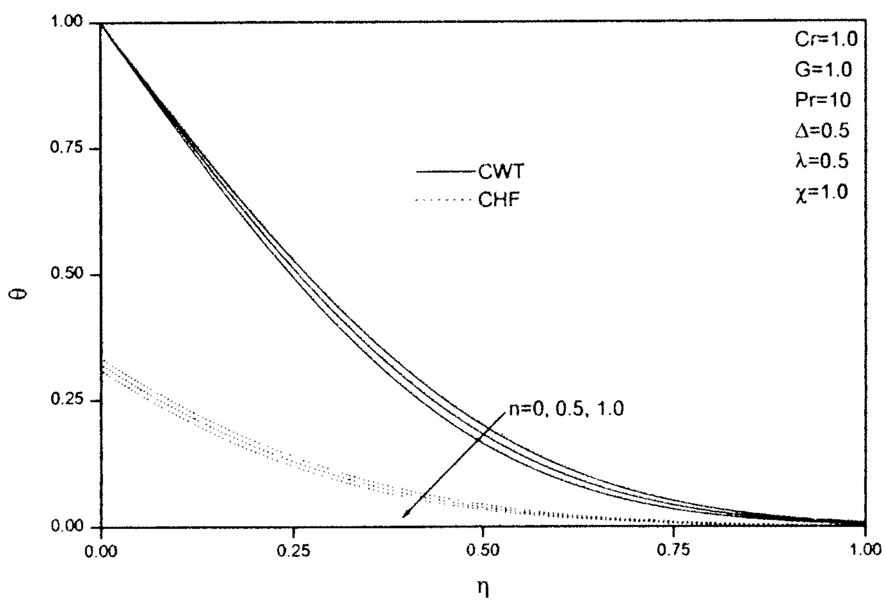

Figure 4. Tempeature profiles for different values of n.

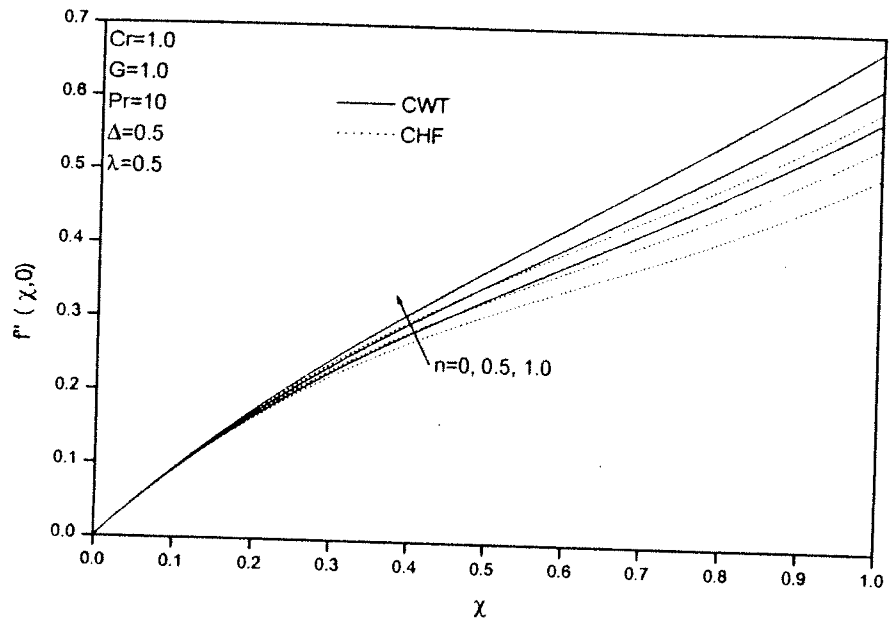

Figure 5. Effects of n of $f'(\chi,0)$ for different values of χ.

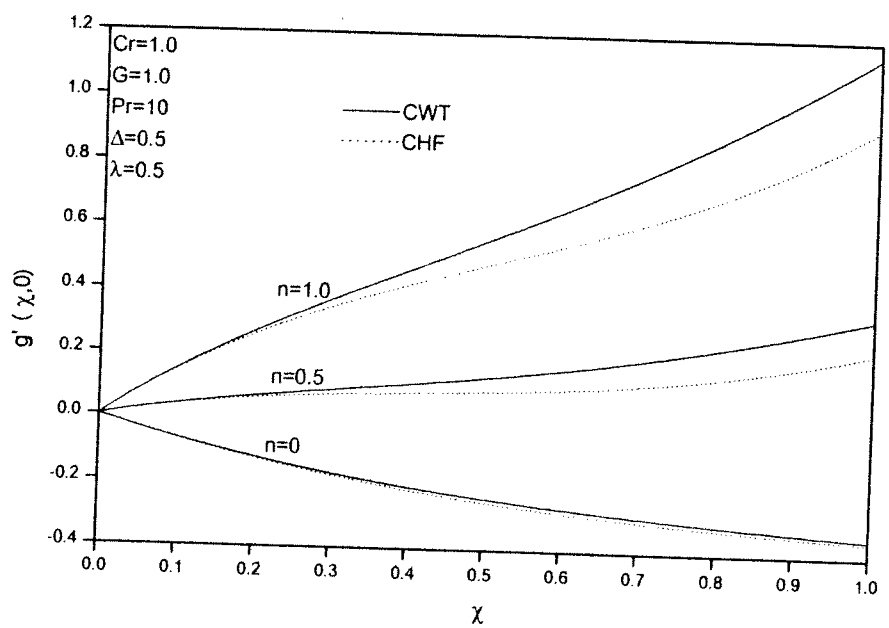

Figure 6. Effects of n of $g'(\chi,0)$ for different values of χ.

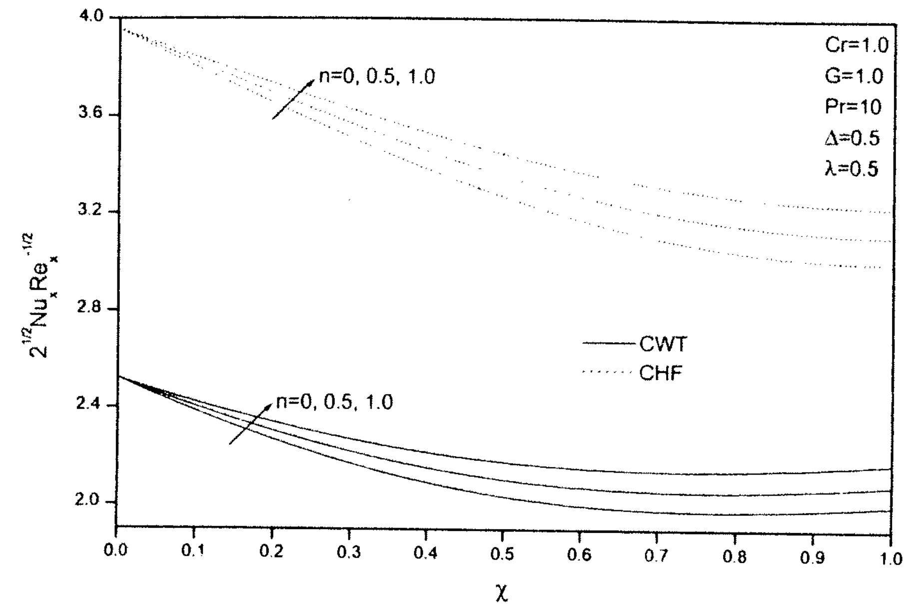

Figure 7. Effects of n on $2^{1/2}$ Nu_x $Re_x^{-1/2}$ for different values of χ.

Figure 5 shows the results of $f''(\chi,0)$. As n increases from 0 to 1 $f''(\chi,0)$ also increases. Figure 6 clearly shows that $g'(\chi,0)$ increases with the increasing values of n. Figure 7 shows the variation of dimensionless heat transfer rate $(2^{\frac{1}{2}} Nu \mathrm{Re}^{\frac{-1}{2}})$ both at constant wall temperature (CWT) and constant heat flux (CHF). Here dimensionless heat transfer rate increases with increasing values of n.

Figures 8 to 10 show the distributions of velocity, microrotation and temperature within the boundary layer for several values of the material parameter, Δ. Figure 8 shows that the velocity decreases with increasing values of Δ. Figure 9 shows that as Δ increases, the microrotation decreases. Figure 10 shows the temperature distribution. As Δ increases, temperature increases.

Figure 11 shows the results for the friction factor, $f''(\chi,0)$. As Δ increases from 0.5 to 50, the friction factor decreases. This indicates that micropolar fluids display drag reducing properties. Figure 12 clearly shows the $g'(\chi,0)$ decreases as Δ increases. Figure 13 shows the variation of dimensionless heat transfer rate $(2^{\frac{1}{2}} Nu \mathrm{Re}^{\frac{-1}{2}})$. Here dimensionless heat transfer rate decreases with the increase of Δ. Therefore, micropolar fluids display heat transfer rate reducing properties.

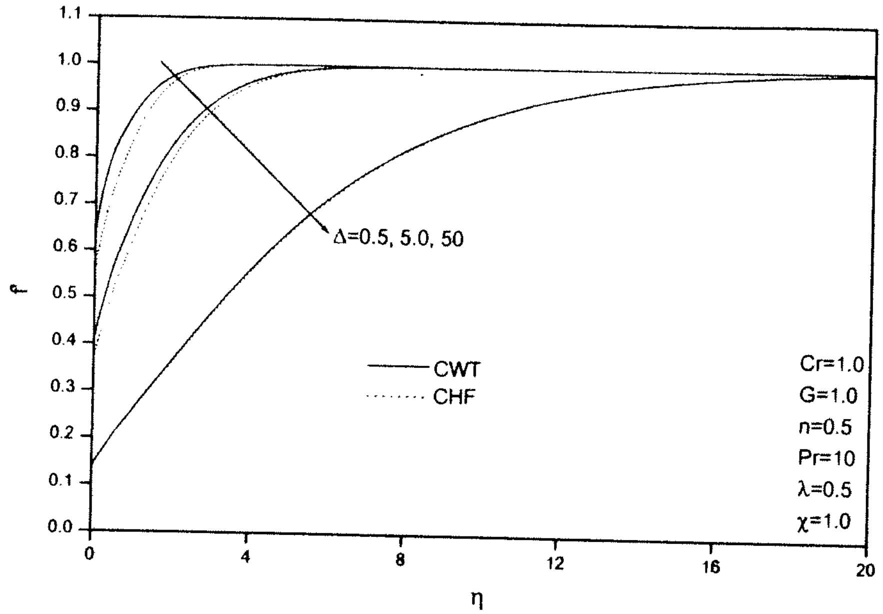

Figure 8. Velocity profiles for different values of Δ.

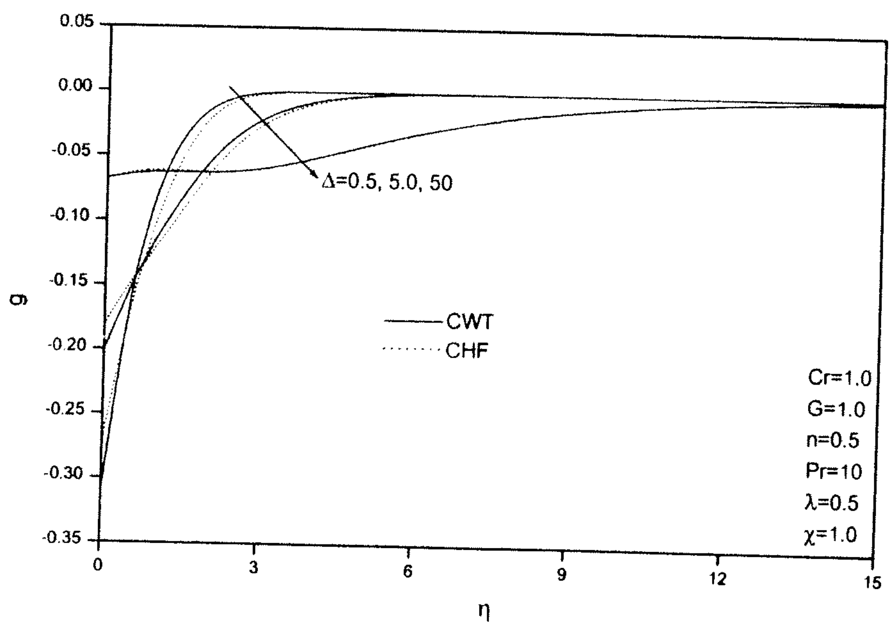

Figure 9. Microrotation profiles for different values of Δ.

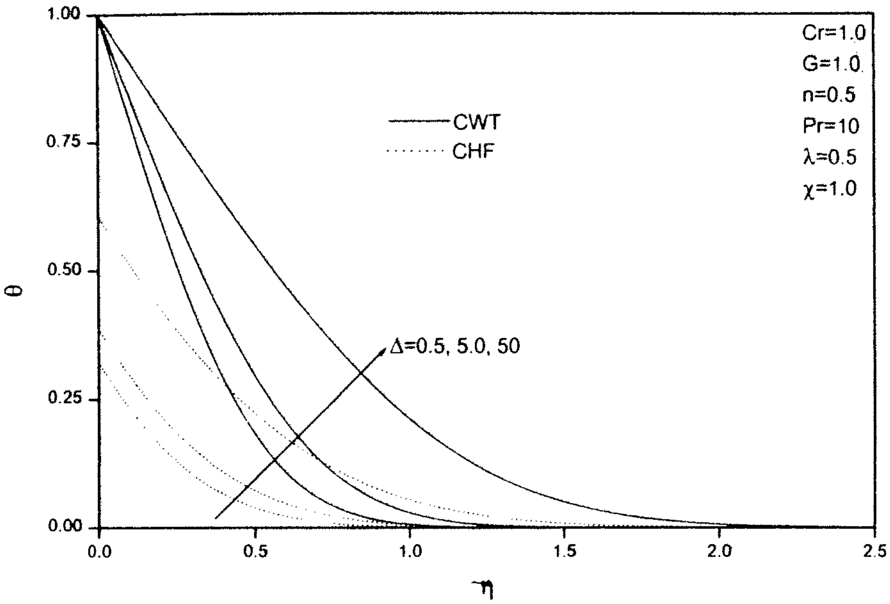

Figure 10. Temperature profiles for different values of Δ.

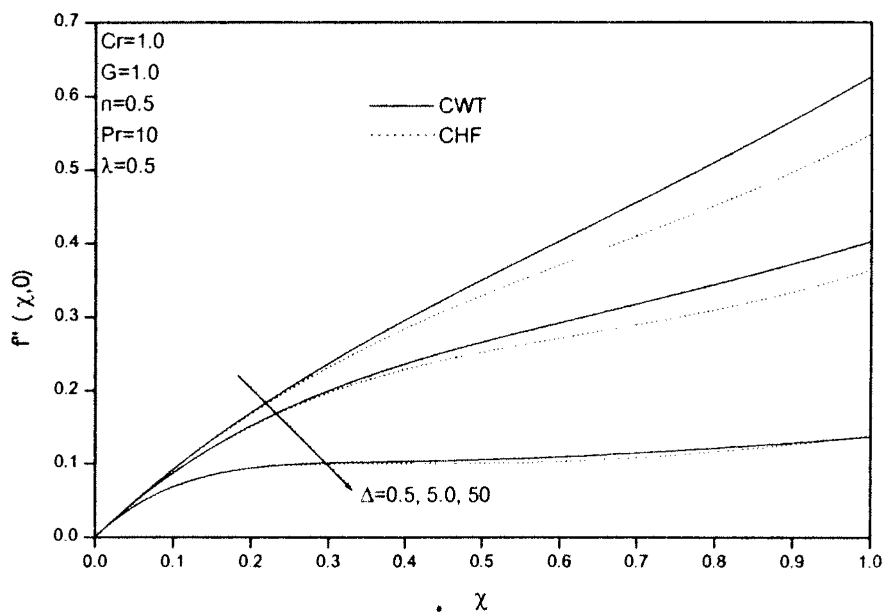

Figure 11. Effects of Δ on $f'(\chi,0)$ for different values of χ.

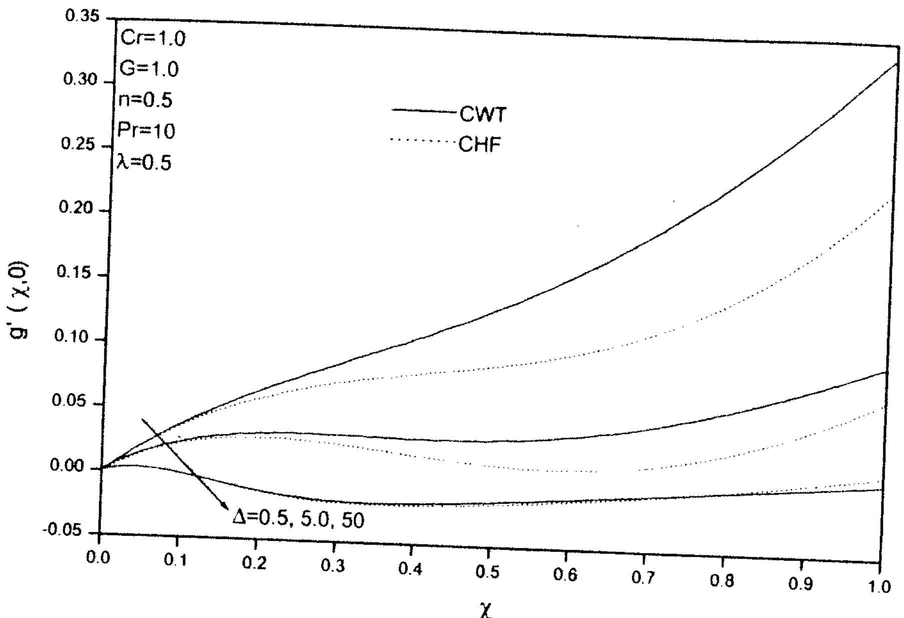

Figure 12. Effects of Δ on $g'(\chi,0)$ for different values of χ.

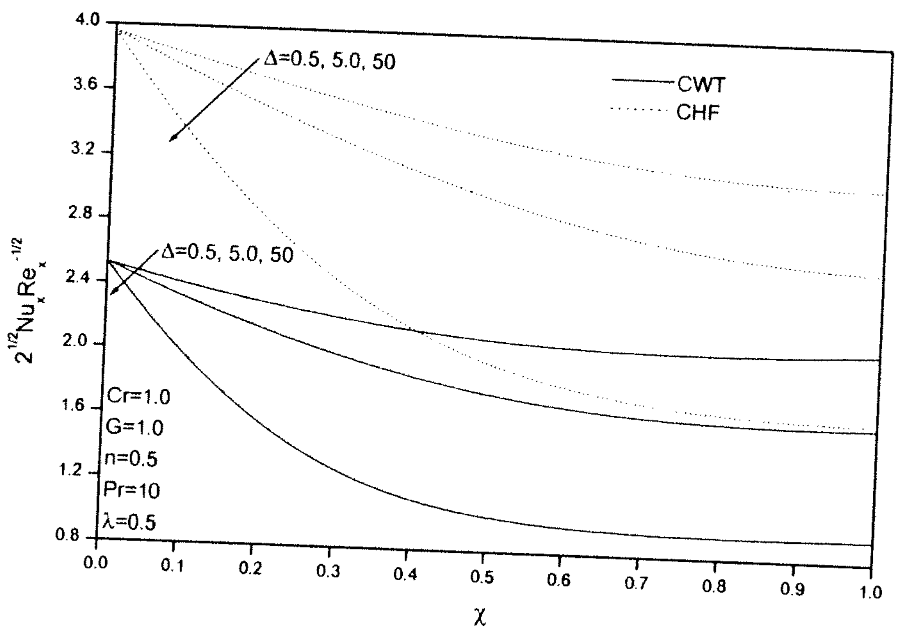

Figure 13. Effects of Δ on $2^{1/2} Nu_x \operatorname{Re}_x^{-1/2}$ for different values of χ.

CONCLUSION

In this paper, we have presented a boundary layer analysis for flow of a micropolar fluid over a vertical plate with slotted surface. The governing equations are transformed into a set of nonlinear ordinary differential equations, where a numerical solution has been presented for a wide range of parameters. Asymptotic boundary layer solutions have been presented for the two regions, both near as well as far away from the leading edge. Numerical results indicate that as dimensionless parameter n increases, the surface heat transfer rate increases. As dimensionless material property Δ increases the friction factor and heat transfer rate decreases.

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