NATURAL CONVECTIVE BOUNDARY LAYER FLOW OVER A HORIZONTAL PLATE EMBEDDED IN A POROUS MEDIUM SATURATED WITH A NON-NEWTONIAN NANOFLUID

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ABSTRACT

A boundary layer analysis is presented for the natural convection past a horizontal plate in a porous medium saturated with a non-Newtonian nano fluid. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the buoyancy ratio parameter Nr, Brownian motion parameter Nb, thermophoresis parameter Nt and Lewis number Le. The dependency of the friction factor, surface heat transfer rate (Nusselt number) and mass transfer rate on these parameters has been discussed.

Keywords: natural convection, porous medium, non-Newtonian flow, nanofluid.

NOMENCLATURE

\( D_B \) Brownian diffusion coefficient \([\text{m}^2 \text{s}^{-1}]\)
\( D_T \) thermophoretic diffusion coefficient \([\text{m}^2 \text{s}^{-1}]\)
\( f \) rescaled nano-particle volume fraction
\( g \) gravitational acceleration vector \([\text{m} \text{s}^{-2}]\)
\( k_m \) effective thermal conductivity of the porous medium \([\text{W} \text{m}^{-1} \text{K}^{-1}]\)
\( K \) permeability of porous medium
\( Le \) Lewis number (equation 11)
\( Nr \) Buoyancy Ratio (equation 11)
\( Nb \) Brownian motion parameter (equation 11)

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The study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many applications in the industry since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. For example, a small amount (<1% volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil is reported to increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively [1,2]. Conventional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such
enhancement. However, problems of rheology and stability are amplified at high concentrations, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Other perplexing results in this rapidly evolving field include a surprisingly strong temperature dependence of the thermal conductivity [3] and a three-fold higher critical heat flux compared with the base fluids [4,5]. These enhanced thermal properties are not merely of academic interest. If confirmed and found consistent, they would make nanofluids promising for applications in thermal management. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology.

Porous media heat transfer problems have several engineering applications such as geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage and flow through filtering media. Cheng and Minkowycz [6] presented similarity solutions for free convective heat transfer from a vertical plate in a fluid-saturated porous medium. Gorla and co-workers [7,8] solved the nonsimilar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. Chen and Chen [9] and Mehta and Rao [10] presented similarity solutions for free convection of non-Newtonian fluids over horizontal surfaces in porous media. Nakayama and Koyama [11] studied the natural convection over a non-isothermal body of arbitrary geometry placed in a porous medium. All these studies were concerned with Newtonian fluid flows. The boundary layer flows in nano fluids have been analyzed recently by Nield and Kuznetsov and Kuznetsov [12] and Nield and Kuznetsov [13]. A clear picture about the nanofluid boundary layer flows is still to emerge.

The present work has been undertaken in order to analyze the natural convection past an isothermal horizontal plate in a porous medium saturated by a non-Newtonian nanofluid. The effects of Brownian motion and thermophoresis are included for the nanofluid. Numerical solutions of the boundary layer equations are obtained and discussion is provided for several values of the nanofluid parameters governing the problem.

We consider the steady free convection boundary layer flow past a horizontal plate placed in a power-law type non-Newtonian nano-fluid saturated porous medium. The coordinate system is selected such that x-axis is in the horizontal direction. We consider the two-dimensional problem. Figure 1 shows the coordinate system and flow model. At the surface, the temperature T and the nano-particle fraction \( \phi \) take constant values \( T_W \) and \( \phi_W \), respectively. The ambient values, attained as y tends to infinity, of T and \( \phi \) are denoted by \( T_\infty \) and \( \phi_\infty \), respectively.

The Oberbeck-Boussinesq approximation is employed and the homogeneity and local thermal equilibrium in the porous medium are assumed. We consider the porous medium whose porosity is denoted by \( \varepsilon \) and permeability by K. The Darcy velocity is denoted by \( \nabla \). The following four field equations embody the conservation of total mass, momentum, thermal energy, and nano-particles, respectively.
Analysis

The field variables are the Darcy velocity $\hat{v}$, the temperature $T$ and the nano-particle volume fraction $\phi$.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\frac{\mu}{k} \frac{\partial u^n}{\partial y} = - (1 - \Phi_{\infty}) \rho_f \beta g \frac{\partial T}{\partial x} + (\rho_p - \rho_f) \frac{g}{\partial x} \tag{2}
\]

\[
u \frac{\partial \tau}{\partial x} + \nu \frac{\partial \tau}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[ B_T \frac{\partial \phi}{\partial y} + \frac{\rho_f}{T_w} \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{3}
\]

\[
\frac{1}{\varepsilon} \left[ u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right] = B_T \frac{\partial^2 \phi}{\partial y^2} + \frac{\rho_f}{T_w} \frac{\partial^2 T}{\partial y^2} \tag{4}
\]

where

\[
\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \tau = \frac{\varepsilon (\rho c)_p}{(\rho c)_f}
\]

The power-law model is used for the non-Newtonian fluid, according to which the relationship between the shear stress and the strain rate is given as follows:

\[
\tau_{xy} = \mu \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \tag{5}
\]

when $n = 1$, Eq. (5) represents a Newtonian fluid with dynamic coefficient of viscosity $\mu$ and when $n \neq 1$, Eq. (5) represent dilatant or shear-thickening ($n > 0$) and a pseudoplastic or shear-thinning ($n < 0$) fluids with $\mu$ as the fluid consistency.

The boundary conditions are given by:

\[
y = 0: v = 0, T = T_w, \phi = \phi_w
\]

\[
y \to \infty: u \to 0, T \to T_\infty, T \to T_\infty
\]
We define the following transformations:

\[ \eta = \frac{y}{x} (R_{ax})^{2n+1} \]
\[ \Psi = \alpha_m (R_{ax})^{2n+1} S(\eta) \]
\[ R_{ax} = \frac{x}{\alpha_m} \left( \frac{(1-\phi_w) \rho k g \beta (T_w - T_x)}{\mu} \right)^{\frac{1}{n}} \]
\[ \theta = \frac{T - T_x}{T_w - T_x} \]
\[ f = \frac{\phi - \phi_x}{\phi_w - \phi_x} \]

(7)

The continuity equation is automatically satisfied by defining a stream function \( \psi \) such that \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \).

Substituting the transformations in equation (7) into the governing equations (1-4) we have:

\[ (S')^{n-1} S'' = \left[ \frac{n+1}{n (2n+1)} \right] \eta [\theta' - N_r f'] \]
\[ \theta'' + \left( \frac{n}{2n+1} \right) S \theta' + N_b f' \theta' + N_t (\theta')^2 = 0 \]
\[ f'' + \text{Le} \left( \frac{n}{2n+1} \right) S f' + \frac{N_t}{N_b} \theta'' = 0 \]

(8) (9) (10)

The transformed boundary conditions are:

\[ \eta = 0: \quad S = 0, \quad \theta = 1, \quad f = 1 \]
\[ \eta \to \infty: \quad S' = 0, \quad \theta = 0, \quad f = 0 \]

(11)

where the four parameters are defined as:

\[ N_r = \frac{(\rho_p - \rho_{\infty})(\phi_w - \phi_x)}{\rho_{\infty} \beta (T_w - T_x)(1 - \phi_x)} \]
\[ N_b = \frac{\varepsilon (\rho c_p) D_B (\phi_w - \phi_x)}{(\rho c_p) \alpha_m} \]
\[ N_t = \frac{\varepsilon (\rho c_p) D_T (T_w - T_x)}{(\rho c_p) \alpha_m T_{\infty}} \]
The local friction factor may be written as

\[ Cf_x = \frac{\frac{\partial u}{\partial y}}{\frac{\rho u^2}{2}} = \frac{2R_{ax}^{3n+1}}{\nu R e_x} S''(0)^n \tag{13} \]

The heat transfer rate at the surface is given by:

\[ q_w = -k_j \frac{\partial T}{\partial y} \bigg|_{y=0} \]

The heat transfer coefficient is given by:

\[ h = \frac{q_w}{(T_w - T_x)} \]

Local Nusselt number is given by:

\[ Nu_x = \frac{h \cdot x}{k_j} = -Ra_x^{(2n+1)} \cdot \theta'(\xi, 0) \tag{14} \]

The mass transfer rate at the surface is given by:

\[ N_w = -D \frac{\partial \phi}{\partial y} \bigg|_{y=0} = h_m (\phi_w - \phi_\infty) \]

where \( h_m = \) mass transfer coefficient,

The local Sherwood number is given by:

\[ Sh = \frac{h_m \cdot x}{D} = -Ra_x^{(2n+1)} \cdot f'(\xi, 0) \tag{15} \]
RESULTS AND DISCUSSION

Equations (8-10) were solved numerically to satisfy the boundary conditions (11) for parametric values of Le, Nr (buoyancy ratio number), Nb (Brownian motion parameter) and Nt (thermophoresis parameter) and the viscosity index n using finite difference method.

Tables 1-5 indicate results for wall values for the gradients of velocity, temperature and concentration functions which are proportional to the friction factor, Nusselt number and Sherwood number, respectively.

Table 1. Effects of Nr on \(S''(0)^n\), \(-\theta'(0)\) and \(-f'(0)\) for \(n=0.7\), \(N_b=0.3\), \(N_t=0.1\) and \(Le=10\)

<table>
<thead>
<tr>
<th>Nr</th>
<th>(S''(0)^n)</th>
<th>(-\theta'(0))</th>
<th>(-f'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.725660E-03</td>
<td>3.770110E-01</td>
<td>1.764546</td>
</tr>
<tr>
<td>0.1</td>
<td>2.956277E-03</td>
<td>3.737452E-01</td>
<td>1.742344</td>
</tr>
<tr>
<td>0.2</td>
<td>3.895301E-03</td>
<td>3.712463E-01</td>
<td>1.718035</td>
</tr>
<tr>
<td>0.3</td>
<td>1.111151E-03</td>
<td>3.683985E-01</td>
<td>1.694436</td>
</tr>
<tr>
<td>0.4</td>
<td>3.670370E-03</td>
<td>3.658519E-01</td>
<td>1.669248</td>
</tr>
<tr>
<td>0.5</td>
<td>5.211735E-04</td>
<td>3.625295E-01</td>
<td>1.643667</td>
</tr>
</tbody>
</table>

Table 2. Effects of Nt on \(S''(0)^n\), \(-\theta'(0)\) and \(-f'(0)\) for \(n=0.7\), \(N_b=0.3\), \(N_r=0.5\) and \(Le=10\)

<table>
<thead>
<tr>
<th>Nt</th>
<th>(S''(0)^n)</th>
<th>(-\theta'(0))</th>
<th>(-f'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>5.211735E-04</td>
<td>3.625295E-01</td>
<td>1.643667</td>
</tr>
<tr>
<td>0.2</td>
<td>3.549956E-03</td>
<td>3.477103E-01</td>
<td>1.640142</td>
</tr>
<tr>
<td>0.3</td>
<td>1.041389E-03</td>
<td>3.344773E-01</td>
<td>1.640064</td>
</tr>
<tr>
<td>0.4</td>
<td>1.875751E-03</td>
<td>3.213755E-01</td>
<td>1.645531</td>
</tr>
</tbody>
</table>

Table 3. Effects of Nb on \(S''(0)^n\), \(-\theta'(0)\) and \(-f'(0)\) for \(n=1.5\), \(N_r=0.5\), \(N_t=0.1\) and \(Le=10\)

<table>
<thead>
<tr>
<th>Nb</th>
<th>(S''(0)^n)</th>
<th>(-\theta'(0))</th>
<th>(-f'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.708242E-07</td>
<td>3.476051E-01</td>
<td>1.234733</td>
</tr>
<tr>
<td>0.2</td>
<td>3.862558E-07</td>
<td>3.230105E-01</td>
<td>1.292362</td>
</tr>
<tr>
<td>0.3</td>
<td>7.619389E-07</td>
<td>3.000357E-01</td>
<td>1.313371</td>
</tr>
<tr>
<td>0.4</td>
<td>1.213397E-06</td>
<td>2.768814E-01</td>
<td>1.326720</td>
</tr>
<tr>
<td>0.5</td>
<td>3.750701E-08</td>
<td>2.547894E-01</td>
<td>1.335746</td>
</tr>
</tbody>
</table>

From Table 1-3, we notice that as Nr and Nt increase, the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decrease. As Nb increases, surface mass transfer rate increases whereas the surface heat transfer rate decreases. Results from Table 4 indicate that as Le increases, the heat and mass transfer rates increase. From Table 5, we observe that the nano fluids display drag reducing and heat and mass transfer rate reducing characteristics. Table 6 displays the effect of the viscosity index n on the heat and mass transfer rates. As n increases, the friction factor as well as the heat and mass transfer rates decrease.
Table 4. Effects of $\text{Le}$ on $S''(0)^n$, $-\theta'(0)$, and $-\overline{f}'(0)$ for $n=1.5$, $N_b=0.3$, $N_r=0.5$ and $N_t=0.1$

<table>
<thead>
<tr>
<th>$\text{Le}$</th>
<th>$S''(0)^n$</th>
<th>$-\theta'(0)$</th>
<th>$-\overline{f}'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.545870E-06</td>
<td>3.013375E-01</td>
<td>8.952270E-01</td>
</tr>
<tr>
<td>10</td>
<td>7.619389E-07</td>
<td>3.000557E-01</td>
<td>1.313371</td>
</tr>
<tr>
<td>100</td>
<td>1.707007E-07</td>
<td>2.917377E-01</td>
<td>4.312719</td>
</tr>
<tr>
<td>1000</td>
<td>4.237932E-06</td>
<td>2.882833E-01</td>
<td>13.711130</td>
</tr>
</tbody>
</table>

Table 5. Effects of $\text{Le}$ on $S''(0)^n$, $-\theta'(0)$, and $-\overline{f}'(0)$ for $n=1.5$, $N_b=0$, $N_r=0$, and $N_t=0$

<table>
<thead>
<tr>
<th>$\text{Le}$</th>
<th>$S''(0)^n$</th>
<th>$-\theta'(0)$</th>
<th>$-\overline{f}'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.535335E-07</td>
<td>4.037194E-01</td>
<td>4.037194E-01</td>
</tr>
<tr>
<td>10</td>
<td>3.921939E-07</td>
<td>4.037666E-01</td>
<td>1.357157</td>
</tr>
<tr>
<td>100</td>
<td>1.312309E-06</td>
<td>4.037666E-01</td>
<td>4.318549</td>
</tr>
<tr>
<td>1000</td>
<td>1.019299E-06</td>
<td>4.037666E-01</td>
<td>13.665670</td>
</tr>
</tbody>
</table>

Table 6. Effects of $N_r$ on $S''(0)^n$, $-\theta'(0)$, and $-\overline{f}'(0)$ for $N_b=0.3$, $N_r=0.5$, $N_t=0.1$ and $\text{Le}=10$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$S''(0)^n$</th>
<th>$-\theta'(0)$</th>
<th>$-\overline{f}'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.056422E-02</td>
<td>4.428760E-01</td>
<td>2.050320</td>
</tr>
<tr>
<td>0.8</td>
<td>1.717160E-03</td>
<td>3.420633E-01</td>
<td>1.540997</td>
</tr>
<tr>
<td>1.0</td>
<td>1.645268E-04</td>
<td>3.185953E-01</td>
<td>1.419499</td>
</tr>
<tr>
<td>1.2</td>
<td>1.329801E-05</td>
<td>3.065256E-01</td>
<td>1.356616</td>
</tr>
<tr>
<td>1.5</td>
<td>6.867181E-07</td>
<td>2.996677E-01</td>
<td>1.313580</td>
</tr>
</tbody>
</table>

Figures 2-4 indicate that as $N_r$ increases, the velocity decreases and the temperature and concentration increase. Similar effects are observed from Figures 5-10 as $N_t$ and $N_b$ vary.

![Figure 2](image-url)
Figure 3. Effects of $N_r$ on temperature profiles.

Figure 4. Effects of $N_r$ on volume fraction profiles.
Figure 5. Effects of $N_t$ on velocity profiles.

Figure 6. Effects of $N_t$ on temperature profiles.
Figure 7. Effects of $N_t$ on volume fraction profiles.

Figure 8. Effects of $N_b$ on velocity profiles.
Figure 9. Effects of \( N_b \) on temperature profile.

Figure 10. Effects of \( N_b \) on volume fraction profiles.

Figure 11 illustrates the variation of velocity within the boundary layer as \( \text{Le} \) increases. The velocity increases as \( \text{Le} \) increases. From Figures 12 and 13, we observe that as \( \text{Le} \)
increases, the temperature and concentration within the boundary layer decrease and the thermal and concentration boundary layer thicknesses decrease. As the viscosity index $n$ increases, the velocity within the boundary layer decreases, temperature increases and concentration increases.

Figure 11. Effects of $Le$ on velocity profiles.

Figure 12. Effects of $Le$ on temperature profiles.
Figure 13. Effects of Le on volume fraction profiles.

Figure 14. Effects of n on velocity profiles.
When \( n = 1 \), Eq. (5) represents a Newtonian fluid with dynamic coefficient of viscosity \( \mu \) and when \( n \neq 1 \), Eq. (5) represent dilatant or shear-thickening (\( n > 1 \)) and a pseudoplastic or shear-thinning (\( n < 1 \)) fluids. The present results for \( n = 1 \) as shown in Table 6 agree with the published results of Gorla and Chamkha [14].

![Figure 15. Effects of n on temperature profiles.](image1)

![Figure 16. Effects of n on volume fraction profiles.](image2)
The effective values of viscosity and thermal conductivity for the nanofluid may be taken as recommended by Oztop and Abu Nada [15] as:

\[
\begin{align*}
\mu_m &= \frac{\mu_f}{(1-\phi)^{2.5}} \\
\frac{k_m}{k_f} &= \frac{(k_f+2k_f)-2\phi(k_f-k_p)}{(k_f+2k_f)+\phi(k_f-k_p)}
\end{align*}
\]

(16)

**CONCLUDING REMARKS**

In this paper, we presented a boundary layer analysis for the natural convection past a non-isothermal vertical plate in a porous medium saturated with a power-law type non-Newtonian nano fluid. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the buoyancy ratio parameter \(N_r\), Brownian motion parameter \(N_b\), thermophoresis parameter \(N_t\) and Lewis number \(L e\). The results indicate that as \(N_r\) and \(N_t\) increase, the friction factor increases whereas the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decrease. As \(N_b\) increases, the friction factor and surface mass transfer rates increase whereas the surface heat transfer rate decreases. As \(L e\) increases, the heat and mass transfer rates increase. Nano fluids display drag reducing and heat and mass transfer rate reducing characteristics.

**REFERENCES**


Natural Convective Boundary Layer Flow over a Horizontal Plate . . . .


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