MIXED CONVECTION FLOW OF A NANOFUID
IN A SQUARE LID-DRIVEN CAVITY WITH
A LOCALIZED HEAT SOURCE AT THE BOTTOM WALL

M. A. Mansour¹, Ali J. Chamkha ², and Sameh E. Ahmed³

¹Department of Mathematics, Faculty of Sciences, Assuit University, Assuit, Egypt
²Manufacturing Engineering Department, The Public Authority for Applied
   Education and Training, Shuweikh 70654, Kuwait
³Department of Mathematics, Faculty of Sciences, South Valley University,
   Qena, Egypt

ABSTRACT

A numerical analysis is performed to examine laminar mixed convection cooling of a
constant heat flux at the bottom wall of a square enclosure filled with water-base
nanofuid containing various volume fractions of Cu, Ag, Al₂O₃ and TiO₂. The finite
difference method is employed to solve the dimensionless governing equations of the
problem. The influences of the governing parameters, namely, Reynolds number, location
and geometry of the heat source, the type of nanofuid and solid volume fraction of
nanoparticles on the cooling performance are studied. The present results are validated by
favorable comparisons with previously published results. The results of the problem are
presented in graphical and tabular forms and discussed.

Keywords: Mixed convection, nanofuid, heat source, finite difference method.

NOMENCLATURE

B length of the heat source ( b / L )
Cₚ specific heat, J kg⁻¹ K⁻¹
D distance between the left wall and center of the heat source ( d / L )
Gr Grashof number ( gβT L³ΔT / ν² )
g gravitational acceleration, m s⁻²

Email: a chamkha@yahoo.com
\( k \) thermal conductivity, \( \text{Wm}^{-1}\text{K}^{-1} \)
\( L \) length of the cavity, m
\( Nu_s \) Nusselt number along the heat source
\( Nu_m \) average Nusselt number along the heat source
\( p \) pressure, pa
\( P \) dimensionless pressure \((p / p_{nf} U_0^2)\)
\( \text{Pr} \) Prandtl number \((\nu_f / \alpha_f)\)
\( q^\prime \) heat generation per area, \( \text{W} / \text{m}^2 \)
\( Ra \) Rayleigh number \((g \beta_f L^3 \Delta T / \nu_f \alpha_f)\)
\( \text{Re} \) Reynolds number \((\rho_f U_0 L / \mu_f)\)
\( T \) temperature, K
\( u, v \) velocity components in \( x, y \) directions, \( \text{ms}^{-1} \)
\( U, V \) dimensionless velocity components \((u / U_0, v / U_0)\)
\( x, y \) Cartesian coordinates, m
\( X, Y \) dimensionless coordinates \((x / L, y / L)\)

**Greek Symbols**

\( \alpha \) thermal diffusivity, \( \text{m}^2\text{s}^{-1}(k / \rho C_p) \)
\( \beta \) thermal expansion coefficient, \( \text{k}^{-1} \)
\( \Delta T \) reference temperature difference \((q^\prime L / \nu_f)\)
\( \phi \) solid volume fraction
\( \Omega \) dimensionless vorticity \((\omega L / U_0)\)
\( \theta \) dimensionless temperature \((T - T_c / \Delta T)\)
\( \mu \) dynamic viscosity, \( \text{kgm}^{-1}\text{s}^{-1} \)
\( \nu \) kinematic viscosity, \( \text{m}^2\text{s}^{-1} \( \mu / \rho \) \)
\( \rho \) density, \( \text{kg} / \text{m}^3 \)
\( \psi \) dimensionless stream function \((\Psi / U_0 L)\)

**Subscripts**

\( c \) cold wall
\( f \) pure fluid
\( m \) average
INTRODUCTION

The term “nanofluid” was coined by Choi [1]. This liquid is a suspension of nanoparticles in a base fluid. Nanofluids have attracted enormous interest from researchers due to their potential for high rate of heat exchange incurring either little or no penalty in pressure drop. The convective heat transfer characteristic of nanofluids depends on the thermo-physical properties of the base fluid and the ultrafine particles, the flow pattern and flow structure, the volume fraction of the suspended particles, and the dimensions and the shape of these particles. Xuan et al. [2] have examined the transport properties of a nanofluid and have expressed that thermal dispersion, which takes place due to the random movement of particles, takes a major role in increasing the heat transfer rate between the fluid and the wall. This requires a thermal dispersion coefficient, which is still unknown. Brownian motion of the particles, ballistic phonon transport through the particles and nanoparticles clustering can also be the possible reason for this enhancement [3]. Das et al. [4] has observed that the thermal conductivity for a nanofluid increases with increasing temperature. They have also observed the stability of $\text{Al}_2\text{O}_3$–water and $\text{Cu}$–water nanofluid. Experiments on heat transfer due to natural convection with a nanofluid have been studied by Putra et al. [5] and Wen and Ding [6]. They have observed that heat transfer decreases with increases in the concentration of nanoparticles. The viscosity of this nanofluid increases rapidly with inclusion of nanoparticles as the shear rate decreases. Recently, Kumar et al. [7] used a single phase thermal dispersion model to study the flow and thermal field in a nanofluid. Talebi et al. [8] investigated numerically the problem of mixed convection flow through a Cu-water nanofluid in a square lid-driven cavity. The problem of natural convection cooling of a localized heat source at the bottom of a nanofluid-filled enclosure was discussed by Aminossadati and Ghasemi [9]. Oztop and Abu-Nada [10] presented a numerical study for heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using nanofluids made with different types of nanoparticles. Putra et al. [11] and Wen and Ding [12] found a systematic and definite deterioration in the heat transfer for a particular range of Rayleigh numbers and density and concentration of nanoparticles. Similar results were also obtained by Santra et al. [13] who modeled the nanofluid as a non-Newtonian fluid. Ho et al. [14] argued that the heat transfer in a square enclosure filled nanofluids can be enhanced or mitigated depending on the formulas used for the estimated dynamic viscosity of the nanofluid. Hwang et al. [15] theoretically investigated thermal characteristics of natural convection in a rectangular cavity filled with a water-based nanofluid containing alumina. Abu-Nada et al. [16] investigated the heat transfer enhancement in a differentially heated enclosure using variable thermal conductivity and variable viscosity of $\text{Al}_2\text{O}_3$–water and Cu–water nanofluids.

On the other hand, fluid flow and heat transfer in a cavity filled by pure fluid which is driven by buoyancy and shear effects have been studied extensively in the literature [17–19].
The most usage of the mixed convection flow with lid-driven effect is to include the cooling of the electronic devices, lubrication technologies, drying technologies, etc.

The present work is concerned with laminar mixed convection flows of nanofluids in a square cavity with a moving lid that moves uniformly in the horizontal plane while all other walls of the cavity are fixed. The focus of the present study is focused on the analysis of several pertinent parameters such as Reynolds number, length and location of the heat source and the solid volume fraction.

**Problem Description**

The schematic diagram of the two-dimensional enclosure considered in this study is displayed in Figure 1.

![Figure 1. Physical model of the problem.](image)

The non-heated parts of the bottom wall and the left vertical wall are insulated. The right vertical wall and the horizontal top wall of the enclosure are maintained at a relatively low temperature \(T_c\). The top wall moves from left to right with a uniform velocity \(U_0\). The nanofluids used in the analysis are assumed to be Newtonian, incompressible and laminar. The base fluid (water) and the solid spherical nanoparticles (Cu, Ag, Al\(_2\)O\(_3\) and TiO\(_2\)) are in thermal equilibrium. The thermo-physical properties of the base fluid and the nanoparticles are given in Table 1. The thermo-physical properties of the nanofluid are assumed constant except for the density variation, which is determined based on the Boussinesq approximation.
Mixed Convection Flow of a Nanofluid in a Square Lid-Driven Cavity.

Table 1. Thermo-physical properties of water and nanoparticles [9]

<table>
<thead>
<tr>
<th></th>
<th>Pure water</th>
<th>Copper (Cu)</th>
<th>Silver (Ag)</th>
<th>Alumina Al₂O₃</th>
<th>Titanium Oxide (TiO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho (kgm^{-3}) )</td>
<td>997.1</td>
<td>8933</td>
<td>10500</td>
<td>3970</td>
<td>4250</td>
</tr>
<tr>
<td>( C_p (Jkg^{-1}K^{-1}) )</td>
<td>4179</td>
<td>385</td>
<td>235</td>
<td>765</td>
<td>686.2</td>
</tr>
<tr>
<td>( k (Wm^{-1}K^{-1}) )</td>
<td>0.613</td>
<td>401</td>
<td>429</td>
<td>40</td>
<td>8.9538</td>
</tr>
<tr>
<td>( \beta (K^{-1}) )</td>
<td>( 21\times10^{-5} )</td>
<td>( 1.67\times10^{-5} )</td>
<td>( 1.89\times10^{-5} )</td>
<td>( 0.85\times10^{-5} )</td>
<td>( 0.9\times10^{-5} )</td>
</tr>
</tbody>
</table>

**Mathematical Formulation**

The continuity, momentum and energy equations for laminar and steady-state mixed convection in the two-dimensional enclosure shown in Figure 1 can be written in the dimensional form as follows see [8]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right)
\]  

\[
u_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{g}{\rho_{nf}} (T - T_\infty) \left[ \phi (\rho \beta)_{p} + (1 - \phi) (\rho \beta)_{f} \right]
\]  

\[
u \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

where all parameters are defined in the Nomenclature section.

The effective density of the nanofluid is given as

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p
\]

where \( \phi \) is the solid volume fraction of nanoparticles.

The thermal diffusivity of the nanofluid is defined as:

\[
\alpha_{nf} = K_{nf} / (\rho C_p)_{nf}
\]
where the heat capacitance of the nanofluid given by
\[(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_p,\] (7)

The thermal expansion coefficient of the nanofluid can be determined from
\[(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_p\] (8)

whereas the effective dynamic viscosity of the nanofluid based on Brikman [20] is given by
\[\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{\frac{1}{3}}}\] (9)

In Eq. (6), \(K_{nf}\) is the thermal conductivity of the nanofluid for spherical nanoparticles. According to Maxwell [21], \(K_{nf}\) is given by
\[K_{nf} = K_f \left[\frac{(K_p + 2K_f) - 2\phi(K_f - K_p)}{(K_p + 2K_f) + \phi(K_f - K_p)}\right]\] (10)

where \(K_p\) is the thermal conductivity of dispersed nanoparticles and \(K_f\) is the thermal conductivity of the pure fluid.

Substituting the following dimensionless parameters
\[X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \theta = \frac{T - T_c}{\Delta T}, P = \frac{p}{\rho_{nf} U_0^2},\]
\[\Omega = \frac{\alpha L}{U_0}, \psi = \frac{\psi}{U_0 L}, \Delta T = \frac{q^* L}{k_f}, Ra = \frac{g \beta_f L^3 \Delta T}{\nu_f \alpha_f}, Pr = \frac{\nu_f}{\alpha_f},\] (11)

in Eqs.(1)-(4) and using the vorticity-stream function formulation yields the following set of dimensionless equations:
\[\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\Omega\] (12)
\[U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \rho_f \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} + \frac{Ra \rho_f}{Re \rho_{nf} (1 - \phi)^{\frac{1}{3}}} \left[\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2}\right] + \frac{Ra \rho_f}{Re^2 Pr \rho_{nf}} (1 - \phi + \phi \frac{\rho_p \beta_p}{\rho_f \beta_f}) \frac{\partial \theta}{\partial X}\] (13)
Mixed Convection Flow of a Nanofluid in a Square Lid-Driven Cavity.

The dimensionless form of the boundary conditions can be written as:

\[ \frac{U}{K_f} \frac{\partial \theta}{\partial X} + \frac{V}{K_f} \frac{\partial \theta}{\partial Y} = \frac{K_{nf}}{K_f} (\rho C_p)_{nf} \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \]  \hspace{1cm} (14)

The boundary condition imposed on the vorticity is written as

\[ \frac{\partial \psi_w}{\partial S} = -S \left( \frac{\partial^2 \psi_w}{\partial S^2} \right) \]  \hspace{1cm} (16)

where the subscript \( w \) refers to the wall condition and \( S \) indicates the direction normal to the wall surface.

The local Nusselt number on the heat source surface can be defined as

\[ Nu_s = \frac{hL}{K_f} \]  \hspace{1cm} (17)

where \( h \) is the convection heat transfer coefficient given by

\[ h = \frac{q^*}{T_s - T_c} \]  \hspace{1cm} (18)

The local Nusselt number by using the dimensionless parameters (Eq. 11) is given by:

\[ Nu_s(X) = \frac{1}{\theta_s(X)} \]  \hspace{1cm} (19)
where \( \theta_i \) is the dimensionless heat flux temperature. The average Nusselt number is determined by

\[
Nu_m = \frac{1}{B} \int_{-B/2}^{D+B/2} \frac{Nu_i(X)}{dX}
\] (20)

**NUMERICAL METHOD AND VALIDATION**

Equations (12)-(14) subject to the boundary conditions (15) are solved numerically by using the finite difference methodology [22-24]. Central difference quotients [25] were used to approximate the second derivatives in both the X- and Y-directions. Then, By following Grosan et al. [26] and Singh and Venkateshan [27], the obtained discretized equations are solved using a Gauss-Seidel iteration technique. For more details for this procedure, the numerical form of the equation (14) can be presented as follows:

\[
\theta_{i,j} = \frac{1}{D_2} \left[ U_{i,j} \left( \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X} \right) + V_{i,j} \left( \frac{\theta_{i,j+1} - \theta_{i,j-1}}{2\Delta Y} \right) - D_1 \left( \frac{\theta_{i+1,j} + \theta_{i-1,j}}{(\Delta X)^2} \right) \right] - D_1 \left( \frac{\theta_{i,j+1} + \theta_{i,j-1}}{(\Delta Y)^2} \right)
\]

where, \( D_1 = \frac{K_{nf} (\rho C_p)_f}{K_f (\rho C_p)_{nf} Pr Re} \) and \( D_2 = -2D_1 \left( \frac{(\Delta X)^2 + (\Delta Y)^2}{(\Delta X\Delta Y)^2} \right) \). In the same manner, Equations (12) and (13) can be analyzed then all these analyses are implemented in a FORTRAN program. The solution procedure is iterated until the following convergence criterion is satisfied:

\[
\sum_{i,j} \left| \chi_{i,j}^{new} - \chi_{i,j}^{old} \right| \leq 10^{-7}
\] (21)

where \( \chi \) is the general dependent variable. Table 2 shows an accuracy tests using the finite difference method with four sets of grid sizes: \( 31 \times 31, 75 \times 75, 81 \times 81, 101 \times 101 \). A good agreement was found between the numerical results based on the \( 75 \times 75 \) and \( 81 \times 81 \) grid sizes with reasonable accuracy. Therefore, all of the numerical computations were carried out for either \( 75 \times 75 \) or \( 81 \times 81 \) grid nodal points. This method was found to be suitable and gave results that are very close to the numerical results obtained previously by Hsu et al. [17], Rubin and Khosla [18], Ghia et al. [19] for classical pure fluids. As we can see from Table 3, the present results are in good agreement with those of Hsu et al. [17], Rubin and Khosla [18] and Ghia et al. [19]. These favorable comparisons lend confidence in the numerical results to be reported subsequently.
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Table 2. Grid independency results (Cu-water). Re = 10, B = 1/3, D = 0.5, $\phi = 0.1$ and $Ra = 1.47 \times 10^5$

<table>
<thead>
<tr>
<th>Grid</th>
<th>$-\theta_{min}$</th>
<th>$-Nu_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31×31</td>
<td>0.2029924</td>
<td>5.590473</td>
</tr>
<tr>
<td>75×75</td>
<td>0.1910532</td>
<td>5.928671</td>
</tr>
<tr>
<td>81×81</td>
<td>0.1910579</td>
<td>5.931643</td>
</tr>
<tr>
<td>101×101</td>
<td>0.1876571</td>
<td>6.020414</td>
</tr>
</tbody>
</table>

Table 3. Comparisons of $-\psi_{min}$ for classical fluids

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>$Gr = 0$, $Re = 100$</th>
<th>$Gr = 10^4$, $Re = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite difference [18]</td>
<td>0.1013</td>
<td>-</td>
</tr>
<tr>
<td>Multigrid [19]</td>
<td>0.1034</td>
<td>-</td>
</tr>
<tr>
<td>Spline [17]</td>
<td>0.1054</td>
<td>0.0934</td>
</tr>
<tr>
<td>Finite difference (Present study)</td>
<td>0.1042</td>
<td>0.0914</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

A two-dimensional mixed convection flow of a nanofluid in a square lid-driven cavity is studied for a range of values of Rayleigh number ($1.47 \times 10^3 \leq Ra \leq 1.47 \times 10^6$), Reynolds number ($1 \leq Re \leq 50$), solid volume fraction ($0 \leq \phi \leq 0.2$), heat source lengths ($0.2 \leq B \leq 0.8$), heat source locations ($0.2 \leq D \leq 0.5$) and a choice of nanoparticles (Cu, Ag, Al$_2$O$_3$ and TiO$_2$). For all simulations, pure water is considered as the base fluid with $Pr = 6.2$. Figure 2 shows the effects of the Reynolds number $Re$ on the contours of streamlines and isotherms for a cavity filled with Cu-water nanofluid.

It is found that, for small values of the Reynolds number ($Re = 1$) the effect of shear effect due to moving lid is less significant with two (clockwise and anti-clockwise) circular cells formed inside the cavity. However, as the Reynolds number increases, the shear effect of the lid-driven flow increases and hence, forced convection flow is realized. In addition, for the high values of Reynolds number ($Re = 50$), the presence of the heat source does not have a noticeable effect on the streamlines. The fluid motion is concentrated at the top of the cavity and the motion of the upper lid stretches the streamlines towards the left wall of the cavity. Moreover, Figure 3 is consistent with this observation.

This figure shows the effect of the Reynolds number $Re$ on the vertical velocity component at the mid-section of the cavity. It is clear that, increasing the value of the Reynolds number $Re$ leads to decreases in the vertical velocity component. The isotherms contours are distributed inside the whole cavity for a low value of the Reynolds number ($Re = 1$), but as the Reynolds number increases, the fluid temperature decreases and isotherms contours concentrate beside the left wall.
Figure 2. Streamlines (left) and isotherms (right) for an enclosures filled with Cu-water nanofluid at $Re = 1, 10, 50$. (Increasing from top towards bottom). The reference case is $Ra = 1.47 \times 10^3$, $B=1/3$, $D=0.5$ and $\phi = 0.1$.

Figure 3. Effect of the Reynolds number on the vertical velocity component at the cavity mid section.
The same behavior is seen from Figure 4 for the local Nusselt number along the heat source \( N \), which increases as \( Re \) increases. These behaviors are clearly shown in Figures 2-4.

Figure 5 shows the effects of the solid volume fraction \( \phi = 0.0, 0.05, 0.15, 0.2 \) on the streamlines and isotherms contours for a square cavity filled with Cu-water nanofluid. As seen for each case, a main recirculation flow cell is formed inside the enclosure. This cell is generated by the movement of the top lid. Increasing the volume fraction of nanoparticles results in decreasing the intensity of the buoyancy effect and hence, the flow intensity. For more understanding of this influence on the fluid flow, the vertical velocity component at the enclosure mid section is plotted in Figure 6.

As expected, it is found that increasing the solid volume fraction leads to decreases in the vertical velocity component. Regarding the isotherms contours, it is seen that when there is no concentration of nanoparticles in the base fluid (i.e. \( \phi = 0 \)), the isotherm lines distribute along the whole cavity and concentrate beside the left wall (adiabatic wall). Addition of the nanoparticles causes an increase in the local Nusselt number distribution along the heat source \( N \). This is clearly shown in Figure 7 which displays the variation of the local Nusselt number along the heat source with solid volume fraction.

Figure 8 shows the effects of the heat source length \( B = 0.2, 0.4, 0.6, 0.8 \) on the streamlines and isotherms contours for a cavity filled with Cu-water nanofluid. It is found that increasing the heat source length \( B \) causes the activity of the fluid motion in the cavity to increase resulting in increased clockwise speed of circular cell that is formed inside the cavity. This increase in the fluid motion in the cavity is related to the association between the buoyancy forces generated due to the fluid temperature differences and activity of the fluid motion. In addition, in order to understand the flow behavior in this situation, the vertical velocity profiles at the enclosure mid-section are presented in Figure 9.
Figure 5. Streamlines (left) and isotherms (right) for an enclosure filled with Cu-water nanofluid at $\phi = 0, 0.05, 0.15, 0.2$. (Increasing from top towards bottom). The reference case is Ra= $1.47 \times 10^4$, Re=10, B=1/3 and D=0.5.

It is clear that the vertical velocity component increases with increasing values of the heat source length. This is because of the stronger buoyant flow for higher heat flux rates caused by increasing the heat source length $B$. On the other hand, as the length of the heat source increases, the heat flux rates decrease. Therefore, the higher temperature patterns corresponds to the taller heat source and the opposite is valid. In addition, as we can see from Figure 10, increasing length of the heat source results in decreasing the corresponding local Nusselt number - $Nu_x$. 
Figure 6. Effect of the solid volume fraction on the vertical velocity component at the cavity mid section.

Figure 7. Profiles of the local Nusselt number along the heat source for various solid volume fractions.

Figure 11 displays the effects of varying the location of the heat source \( D = 0.2, 0.3, 0.4, 0.5 \) on the streamlines and isotherms contours for a square cavity filled with Cu-water nanofluid. It is observed that the fluid follows the geometry of the cavity by forming one clockwise circular cell. As the heat source moves away from the left wall of the cavity, the intensity of the fluid motion decreases. This can also be seen from Figure 12. It is clear that the vertical velocity component decreases with moving the heat source away from the left wall.
Figure 8. Streamlines (left) and isotherms (right) for an enclosure filled with Cu-water nanofluid at $B = 0.2, 0.4, 0.6, 0.8$. (Increasing from top towards bottom). The reference case is $Ra = 1.47 \times 10^3$, $Re = 10$, $D = 0.5$ and $\phi = 0.1$.

On the other hand, the fluid temperature is influenced by changing the heat source location. The fluid temperature follows the movement of the heat source. Moreover, the profiles of local Nusselt number along the heat source at different locations of the heat source are displayed in Figure 13. The results show that as the distance between the left wall and center of the heat source increases, the local Nusselt number increases.
Figure 14 depicts the profiles of local Nusselt number along the heat source $-\text{Nu}_x$ for various nanofluids made up of pure water and different nanoparticles (Cu, Ag, Al$_2$O$_3$ and TiO$_2$) at $B = 0.4$, $Re = 10$, $D = 0.5$, $\phi = 0.1$ and $Ra = 1.47 \times 10^3$.

![Diagram](image1)

Figure 9. Effect of varying the heat source length on the vertical velocity component at the cavity mid section.

![Diagram](image2)

Figure 10. Profiles of the local Nusselt number along the heat source for different heat source lengths.

It is found that the lowest rate of heat transfer can be obtained by adding titanium oxide (TiO$_2$) compared to other nanoparticles. On the contrary, copper (Cu) gives the highest rate of heat transfer. In addition, in order to clarify the effects of the addition of nanoparticles on increasing of the average Nusselt number, Figure 15 shows the percentages of the absolute average Nusselt number increasing along the heat source for different pure water-base nanofluids at various values of solid volume fraction of nanoparticles.
Figure 11. Streamlines (left) and isotherms (right) for an enclosure filled with Cu-water nanofluid at \( D = 0.2, 0.3, 0.4, 0.5 \). (Increasing from top towards bottom). The reference case is \( \text{Ra}=1.47\times10^5, \text{Re}=10, \text{B}=0.4 \) and \( \phi = 0.1 \).
It is clear that, in general, amongst all nanoparticles, Cu provides the highest average Nusselt number increase (13.23\% for $\phi = 0.05$) and TiO$_2$ provides the lowest average Nusselt number increase (10.98\% for $\phi = 0.05$). Also, Tables 4-6 display the effects of the governing parameters, namely, nanoparticles volume fraction $\phi$, variations of the heat source length $B$, different location of the heat source $D$ and the Rayleigh number $Ra$ on the absolute values of the average Nusselt number $Nu_m$ and minimum temperature $\theta_{\text{min}}$ for
It is found that increasing both of the solid volume fraction $\phi$ and the Rayleigh number $Ra$ leads to increases in the average Nusselt number $Nu_m$ and decreases in the corresponding minimum temperature $\theta_{\text{min}}$. However, the inverse behaviors are observed when the heat source length $B$ and the location parameter of the heat source $D$ increase.

Figure 14. Profiles of the local Nusselt number along the heat source for different nanoparticles.

Figure 15. The percentages of increase of the absolute average Nusselt number along the heat source for various water-based nanofluids and different values of solid volume fractions.
Table 4. Effect of solid volume fraction on the average Nusselt number and the minimum temperature for different nanoparticles at $B = 1/3$, $D = 0.5$, $Ra = 1.47 \times 10^5$ and $Re = 10$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Cu</th>
<th>Ag</th>
<th>Al$_2$O$_3$</th>
<th>TiO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-Nu_m$</td>
<td>$\theta_{\min}$</td>
<td>$-Nu_m$</td>
<td>$\theta_{\min}$</td>
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<tr>
<td>0</td>
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</tr>
<tr>
<td>0.05</td>
<td>5.174535</td>
<td>0.2189692</td>
<td>5.153874</td>
<td>0.2196564</td>
</tr>
<tr>
<td>0.1</td>
<td>5.802563</td>
<td>0.1956831</td>
<td>5.7591</td>
<td>0.1968672</td>
</tr>
<tr>
<td>0.15</td>
<td>6.465145</td>
<td>0.1755273</td>
<td>6.404451</td>
<td>0.1768156</td>
</tr>
<tr>
<td>0.2</td>
<td>7.178102</td>
<td>0.1578385</td>
<td>7.089272</td>
<td>0.1594323</td>
</tr>
</tbody>
</table>

Table 5. Effect of varying the heat source length on the average Nusselt number and the minimum temperature for different nanoparticles at $D = 0.5$, $Ra = 1.47 \times 10^5$, $\phi = 0.1$ and $Re = 10$

<table>
<thead>
<tr>
<th>$B$</th>
<th>Cu</th>
<th>Ag</th>
<th>Al$_2$O$_3$</th>
<th>TiO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-Nu_m$</td>
<td>$\theta_{\min}$</td>
<td>$-Nu_m$</td>
<td>$\theta_{\min}$</td>
</tr>
<tr>
<td>0.2</td>
<td>8.400028</td>
<td>0.1206035</td>
<td>8.368852</td>
<td>0.1210891</td>
</tr>
<tr>
<td>0.4</td>
<td>5.909739</td>
<td>0.1665051</td>
<td>5.883652</td>
<td>0.1673735</td>
</tr>
<tr>
<td>0.6</td>
<td>4.757862</td>
<td>0.2063496</td>
<td>4.738077</td>
<td>0.207409</td>
</tr>
<tr>
<td>0.8</td>
<td>4.118363</td>
<td>0.2396312</td>
<td>4.105006</td>
<td>0.2401205</td>
</tr>
</tbody>
</table>
Table 6. Effect of varying the location parameter of the heat source on the average Nusselt number and the minimum temperature for different nanoparticles at $B=0.4$, $Ra = 1.47 \times 10^6$, $\phi = 0.1$ and $Re = 10$

<table>
<thead>
<tr>
<th>$D$</th>
<th>Cu $- Nu_m$</th>
<th>Cu $- \theta_{min}$</th>
<th>Ag $- Nu_m$</th>
<th>Ag $- \theta_{min}$</th>
<th>Al$_2$O$_3$ $- Nu_m$</th>
<th>Al$_2$O$<em>3$ $- \theta</em>{min}$</th>
<th>TiO$_2$ $- Nu_m$</th>
<th>TiO$<em>2$ $- \theta</em>{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>14.01287</td>
<td>0.1035415</td>
<td>13.8662</td>
<td>0.1053652</td>
<td>14.25698</td>
<td>0.099977</td>
<td>13.80135</td>
<td>0.1028633</td>
</tr>
<tr>
<td>0.3</td>
<td>8.65544</td>
<td>0.1110631</td>
<td>8.53078</td>
<td>0.112901</td>
<td>8.710432</td>
<td>0.1076917</td>
<td>8.688111</td>
<td>0.1098748</td>
</tr>
<tr>
<td>0.4</td>
<td>8.627617</td>
<td>0.111963</td>
<td>8.518521</td>
<td>0.1133163</td>
<td>8.719746</td>
<td>0.1098265</td>
<td>8.445221</td>
<td>0.1137491</td>
</tr>
<tr>
<td>0.5</td>
<td>9.201649</td>
<td>0.1061889</td>
<td>9.497358</td>
<td>0.1074502</td>
<td>9.099399</td>
<td>0.1099926</td>
<td>9.014672</td>
<td>0.1084195</td>
</tr>
</tbody>
</table>
CONCLUSION

A numerical simulation of mixed convection flows in a square lid-driven cavity partially heated from below using Cu-water, Ag-water, Al₂O₃-water and TiO₂-water nanofluids was studied. The finite difference method was employed for the solution of the present problem. Comparisons with previously published work on special cases of the problem were performed and found to be in good agreement. Graphical and tabular results for various parametric conditions were presented and discussed. From this investigation, the following conclusions can be reported:

1. Increasing the nanoparticles volume fraction led to decreases in both the activity of the fluid motion and the fluid temperature. However, it increased the corresponding average Nusselt number.
2. As the length of the heat source increased, the flow intensity and the fluid temperature increased whereas the corresponding average Nusselt number decreased.
3. As the distance between the left wall and center of the heat source increased, the average Nusselt number increased.
4. The lowest rate of heat transfer was obtained by adding titanium oxide (TiO₂) nanoparticles compared to other nanoparticles whereas addition of copper (Cu) nanoparticles gave the highest rate of heat transfer.

REFERENCES


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