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SECOND LAW ANALYSIS FOR COMBINED CONVECTION IN NON-NEWTONIAN FLUIDS OVER A VERTICAL WEDGE EMBEDDED IN A POROUS MEDIUM

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The second law analysis for the combined convective heat transfer in a non-Newtonian boundary-layer flow over a wedge embedded in a porous medium is presented. Velocity and temperature fields are solved numerically under the similarity assumption. These results are used to compute the entropy generation number Ns, irreversibility ratio ø, and the Bejan number Be for a range of values of the viscosity index (n), streamwise distance (x), buoyancy parameter (λ), and viscous frictional parameter, G. The results show that the Bejan number increases with the viscosity index and the buoyancy parameter.

KEY WORDS: second law analysis, combined convection, non-Newtonian fluids, porous medium, wedge flow

1. INTRODUCTION

Exergy analysis is pure thermodynamics. It relies on the laws of thermodynamics to establish the theoretical limit of ideal or reversible operation and the extent to which the operation of the given system departs from the ideal. The departure is measured by the calculated quantity called destroyed exergy or irreversibility. This quantity is proportional to the generated entropy. Exergy is the thermodynamic property that describes the useful energy content or the work-producing potential of substances and streams. In real systems exergy is always destroyed partially or totally when components and streams interact. The minimization of entropy generation requires the use of more than thermodynamics; fluid mechanics, heat and mass transfer, materials, constraints, and geometry are also needed in order to establish the relationships between the physical configuration and the destruction of exergy. Reduction in exergy destruction is pursued through changes in configuration. In the field of heat transfer, the entropy generation minimization method brings out the inherent competition between heat transfer and fluid flow irreversibilities in the optimization of devices subjected to overall constraints.

A study of thermodynamic irreversibilities is an important factor in the design of thermal systems. Entropy is a measure of molecular disorder within a system, which translates into the amount of energy not available to be converted into work. Therefore the main objective is to determine possible ways of minimizing it. The method utilized to achieve this purpose is known as entropy generation minimization or thermodynamic optimization. This
method is a combination of the basic principles of thermodynamics, heat transfer, and fluid mechanics in order to analyze the irreversibilities present in real devices, concentrating special attention in finding the physical quantities that can be modified to optimize their specific performance.


The present work has been undertaken in order to study entropy generation for the combined convection heat transfer in non-Newtonian fluids over a vertical wedge embedded in a porous medium. We have considered the power law (Ostwald-de Waele) model for the non-Newtonian fluid. Using this model, the shear stress is given by $\tau = \mu^* (\partial u / \partial y)^n$, where $\mu^*$ is the effective viscosity and $n$ the viscosity index. Here, $n < 1$ denotes shear thinning fluids called pseudoplastic fluids, whereas $n > 1$ denotes dilatant fluids. Conditions for similarity are derived and the transformed momentum and energy equations are solved numerically. The entropy generation rate was evaluated and the entropy generation number, irreversibility ratio, and Bejan number were computed.

2. ANALYSIS

Consideration is given to the mixed convection from a permeable vertical plate embedded in a non-Newtonian fluid-saturated porous medium. The coordinate system and flow model are shown in Fig. 1. We consider the Darcy model assuming low velocity and porosity. The governing equations under the Boussinesq and boundary-layer approximations are given by Nakayama and Koyama (1991):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$\frac{\partial u^*}{\partial y} = \gamma \frac{\partial T}{\partial y}$$

(2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

(3)
The nondimensional quantities \( x, y, u, v, \) and \( T \) are related to their dimensional counterparts \( \bar{x}, \bar{y}, \bar{u}, \bar{v}, \) and \( \bar{T} \) by

\[
\begin{align*}
    x &= \frac{\bar{x}}{L} \\
    y &= \frac{\bar{y}Pe^{1/2}}{L} \\
    u &= \frac{\bar{u}L}{\alpha Pe} \\
    v &= \frac{\bar{v}L}{\alpha Pe^{1/2}} \\
    T &= \frac{k(\bar{T} - \bar{T}_\infty)}{q_0 L} \\
    Pe &= \frac{\bar{u}_\infty L}{\alpha} \\
\end{align*}
\]

\[\gamma = \frac{\rho k^* g_x \beta_0^* L}{\mu^* K u_\infty} \quad \text{Buoyancy parameter}\]

\[g_x = g \cos \phi \quad \text{x component of acceleration due to gravity}\]

The boundary conditions are given by

\[
\begin{align*}
    y &= 0 : v = 0, \quad \frac{\partial T}{\partial y} = -Q(x) = x^\lambda \\
    y \to \infty : u \to u_\infty, \quad T \to 0
\end{align*}
\]

where \( Q = q_w/(q_0 Pe^{1/2}) \) = Dimensionless surface heat flux.

We assume that \( u_\infty(x) = Cx^m \), where \( C \) is a constant.

Proceeding with the analysis, we define the following:

\[\eta = y \cdot x^{\frac{1-n}{2n+1}}\]

\[\Psi = f(\eta) \cdot x^{\frac{n+\lambda+1}{2n+1}}\]

\[T = \theta(\eta) \cdot x^{\frac{n(2\lambda+1)}{2n+1}}\]

We have

\[u = f'(\eta) \cdot x^{\frac{(2\lambda+1)}{2n+1}}\]

\[v = -x^{\frac{1-n}{2n+1}} \left\{ \left( \frac{n+\lambda+1}{2n+1} \right) \cdot f + \left( \frac{\lambda-n}{2n+1} \right) \cdot \eta \cdot f' \right\}\]

The equations of motion then become

\[\left( f' \right)^n = x^{2mn - \frac{n(2\lambda+1)}{2n+1}} + \gamma \theta \]

\[\theta'' + \left( \frac{\lambda+n+1}{2n+1} \right) f \theta' - \frac{n(2\lambda+1)}{2n+1} f' \theta = 0\]

The transformed boundary conditions are given by

\[f(0) = 0, \quad \theta'(0) = -1\]

\[f'(\infty) = x^{-\frac{2\lambda+1}{2n+1}}, \quad \theta(\infty) = 0\]

To make Eqs. (9)–(11) independent of \( x \), or to seek similarity solutions, we set \( m = (2\lambda+1)/(2n+1) \). Then we have

\[\left( f' \right)^n = 1 + \gamma \theta \]

\[\theta'' + \left( \frac{\lambda+n+1}{2n+1} \right) f \theta' - \frac{n(2\lambda+1)}{2n+1} f' \theta = 0\]

The transformed boundary conditions are given by

\[f(0) = 0, \quad \theta'(0) = -1\]

\[f'(\infty) = 1, \quad \theta(\infty) = 0\]

Equations (12) and (13) were solved using the fourth-order Runge-Kutta numerical procedure. More details of the numerical solutions are documented by Gorla (1999, 2000).

The entropy generation rate is given by

\[S_g'' = \frac{k}{T_0^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0} \left( \frac{\partial u}{\partial y} \right)^{n+1}\]
Therefore
\[
S_g^{m_kT_0^2} = N_C + N_Y + N_F
\]
(16)
where
\[
N_C = \left\{ \frac{n(2\lambda + 1)}{2n + 1} \cdot x^{\frac{\alpha(2\lambda-1)-1}{2n+1}} \cdot \theta + \frac{\lambda-n}{2n+1} \right. \\
\times \left. x^{\frac{\alpha(2\lambda-1)-1}{2n+1}} \cdot \eta \cdot \theta' \right\}^2
\]
\[
N_Y = Pe \cdot (x^\lambda \theta')^2
\]
\[
N_F = G \cdot Pe^{\frac{2(n+1)}{x}} \cdot \left\{ x^{\frac{\alpha(2\lambda-1)-n}{2n+1}} \cdot f'' \right\}^{n+1}
\]
\[
G = \frac{kT_0\mu}{q_0^2} \left( \frac{\alpha}{L^2} \right)^{n+1}
\]

\(N_C\) is the entropy generated in the axial direction; \(N_Y\) in the normal direction to the wedge surface, and \(N_F\) due to fluid friction.

The irreversibility ratio \(\phi\) is the ratio of entropy generations due to the fluid friction to the entropy generation due to heat transfer:
\[
\phi = \frac{N_F}{(N_C + N_Y)}
\]
(17)
For \(0 < \phi < 1\), the heat transfer dominates the irreversibility ratio and the fluid friction dominates when \(\phi > 1\). The case where both the heat transfer and the fluid friction have the same contribution to the entropy generation is characterized by \(\phi = 1\).

Bejan number (Be) is the ratio of heat-transfer irreversibility to the total irreversibility due to heat transfer and fluid friction:
\[
Bejan\ number = \frac{N_F}{(N_C + N_Y + N_F)} = \frac{1}{(1 + \phi)}
\]
(18)

3. RESULTS AND DISCUSSION

The spatial distribution of the entropy generation number \(N_s\) is shown in Figs. 2–5. The entropy generation decreases in the transverse direction to the wedge surface and approaches zero at the boundary-layer edge where the effects of viscosity and heat conduction effects are absent. Augmentation of energy exchange between the flow and the walls contribute to an augmentation of entropy generation via the augmentation of temperature gradient near the walls. \(N_s\) increases with streamwise distance \((x)\), Peclet number \((Pe)\), and viscous friction number \((G)\). \(N_s\) decreases with the power law index \((n)\) and the buoyancy parameter \((\lambda)\).

Both fluid friction and heat transfer contribute to the entropy generation. The irreversibility ratio \(\phi\) indicates whether the fluid friction or heat transfer dominate in entropy production. Figures 6–9 show the distribution of...
FIG. 3: Effects of $\lambda$ and $x$ on the entropy generation rate.

FIG. 4: Effects of $Pe$ and $x$ on the entropy generation rate.
FIG. 5: Effects of $G$ and $x$ on the entropy generation rate.

FIG. 6: Effects of $n$ and $x$ on the irreversibility ratio.
FIG. 7: Effects of $\lambda$ and $x$ on the irreversibility ratio.

FIG. 8: Effects of $Pe$ and $x$ on the irreversibility ratio.
the irreversibility ratio within the boundary layer. Pseudoplastic fluids \( (n < 1) \) display higher values of \( \phi \) than dilatant fluids \( (n > 1) \). This is due to the difference in the rheological character of the non-Newtonian fluids. The irreversibility ratio \( \phi \) increases with \( x, \lambda, \text{Pe}, \) and \( G \). The Bejan number \( \text{Be} \) profiles are shown in Figs. 10–13. Bejan number increases with \( n \) and \( \lambda \) and decreases with \( x, \text{Pe}, \) and \( G \).

**FIG. 9:** Effects of \( G \) and \( x \) on the irreversibility ratio.

**FIG. 10:** Effects of \( n \) and \( x \) on the Bejan number.
FIG. 11: Effects of $\lambda$ and $x$ on the Bejan number.

FIG. 12: Effects of Pe and $x$ on the Bejan number.
4. CONCLUDING REMARKS

In this paper we have applied the second law of thermodynamics for the combined convective heat transfer in a non-Newtonian boundary-layer flow over a wedge embedded in a porous medium. The criterion for the existence of similarity solutions for the velocity and temperature fields is derived. The entropy generation number $N_s$, irreversibility ratio $\phi$, and the Bejan number $Be$ are computed for a range of values of the viscosity index ($n$), streamwise distance ($x$), buoyancy parameter ($\lambda$), and viscous frictional parameter ($G$). The results show that the Bejan number increases with the viscosity index and the buoyancy parameter. Maximum entropy generation is located near the walls where the Nusselt number is maximum or thermal heat exchange between the flow and the surface is maximum. The analysis of entropy generation by examining the influence of the pertinent parameters in this study would be a useful analytical tool for engineering design and performance evaluation of such heat exchange devices based on the second law of thermodynamics, which is essential for more effective use of available energy by reducing the destruction of useful potential work.

REFERENCES


