Combined convection flow in triangular wavy chamber filled with water–CuO nanofluid: Effect of viscosity models

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ABSTRACT

This work is focused on the numerical modeling of steady laminar combined convection flow in a vertical triangular wavy enclosure filled with water–CuO nanofluid. The left and right vertical walls of the cavity take the form of a triangular wavy pattern. The bottom and top horizontal walls are mechanically driven. The lower and upper surfaces move to the right and left direction at the same constant speed respectively. They maintain constant temperature lower than both vertical walls. Two different nanofluid models namely, the Brinkman model and the Pak and Cho correlation are employed. The developed equations are given in terms of the Navier Stokes and the energy equation and are non-dimensionalized and then solved numerically subject to appropriate boundary conditions by the Galerkin’s finite-element method. Comparisons with published work are performed and found to be in good agreement. A parametric study is conducted and a selective set of graphical results is presented. The effects of the Reynolds number, Richardson number and the nanoparticles volume fraction on the flow and heat transfer characteristics in the cavity are displayed to compare the predictions obtained by the two different nanofluid models. Heat transfer enhancement can be obtained significantly due to the presence of nanoparticles. The rate of heat transfer is accentuated moderately by falling the Richardson number and rising the Reynolds number as well as the solid volume fraction.

1. Introduction

A nanofluid is a base fluid with suspended metallic nanoparticles. Because traditional fluids used for heat transfer applications such as water, mineral oils and ethylene glycol have a rather low thermal conductivity, nanofluids with relatively higher thermal conductivities have attracted enormous interest from researchers due to their potential in enhancement of heat transfer with little or no penalty in pressure drop. Kebinski et al. [1] analyzed the possible mechanisms of enhancing thermal conductivity and suggested that the size effect, the clustering of nanoparticles and the surface adsorption could be the major reason of enhancement, while the Brownian motion of nanoparticles contributes much less than other factors. This is because Brownian motion of nanoparticles is too slow to transport significant amount of heat through a nanofluid and this conclusion was also supported by their results of molecular dynamics simulation. Control volume finite element simulation of MHD forced and natural convection in a vertical channel with a heat-generating pipe was conducted by Nasrin and Alim [2]. They concluded that rate of heat transfer was obtained optimum in the absence of both MHD and Joule heating effects. Wang et al. [3] used a fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles and found that it predicted well the trend for variation of the effective thermal conductivity with dilute suspension of nanoparticles.

The convective heat transfer characteristic of nanofluids depends on the thermo-physical properties of the base fluid and the ultra fine particles, the flow pattern and flow structure, the volume fraction of the suspended particles, the dimensions and the shape of these particles. The utility of a particular nanofluid for a heat transfer application can be established by suitably modeling the convective transport in the nanofluid [4]. Several studies of convective heat transfer in nanofluids have been reported in recent years. Jou and Tzeng [5] performed a numerical study of the heat transfer performance of nanofluids inside two-dimensional rectangular enclosures. Their results indicated that increasing the volume fraction of nanoparticles produced a significant enhancement of the average rate of heat transfer. Santra et al. [6] conducted a study of heat transfer augmentation in a differentially heated square cavity filled with copper–water nanofluid using the models proposed by Maxwell-Garnett and Bruggeman. Their results showed that the Bruggemann model predicted higher heat transfer rates than the Maxwell-Garnett model. Hwang et al. [7] carried out a theoretical investigation of the thermal characteristics of natural convection of an alumina-based nanofluid in a rectangular cavity heated from below using Jang and Choi’s model for predicting the effective thermal conductivity of nanofluids and various models for predicting the effective viscosity. Free convection heat transfer in horizontal and vertical...
rectangular cavities filled with nanofluids was investigated by Wang et al. [8].

Ho et al. [9] numerically studied natural convection of nanofluid in a square enclosure considering the effects due to uncertainties of viscosity and thermal conductivity. Oztok and Abu-Nada [10] analyzed heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using nanofluids with various types of nanoparticles. They found that the use of nanofluids caused heat transfer enhancement and that this enhancement was more pronounced at a low aspect ratio than at a high one. Abu-Nada [11] carried over on the application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step and studied the effects of variable viscosity and thermal conductivity of CuO-water and Al₂O₃-water nanofluid on heat transfer enhancement in natural convection [12,13].


Mixed convection in a lid-driven cavity flow problems are encountered in a variety of thermal engineering applications including cooling of electronic devices, lubrication technologies, high-performance building insulation, multi-shield structures used for nuclear reactors, food processing, glass production, solar power collectors, drying technologies and others. Numerous studies on single or double lid-driven cavity flow and heat transfer involving different cavity configurations, various fluids and imposed temperature gradients have been continually published in the literature. Tiwari and Das [18] investigated numerically heat transfer augmentation in a lid-driven cavity filled with a nanofluid and found that the presence of nanoparticles in a base fluid is capable of increasing the heat transfer capacity of the base fluid. Mutthamilsevan et al. [19] reported on the heat transfer enhancement of copper–water nanofluids in a lid-driven enclosure with different aspect ratios. Oztop and Dagtekin [20] investigated mixed convection in two sided lid driven differentially heated square cavity. Guran [21] studied the effect of Reynolds number on streamline bifurcations in a double-lid-driven cavity with free surfaces.

Recently, Ghasemi and Aminossadati [22] performed natural convection heat transfer in an inclined enclosure filled with a water–CuO nano-fluid where the transport equations for a Newtonian fluid were solved numerically with a finite volume approach using the SIMPLE algorithm. Effect of Brownian and thermophoretic diffusions of nanoparticles on nonequilibrium heat conduction in a nanofluid layer with periodic heat flux was investigated by Zhang et al. [23]. Roslan et al. [24] conducted buoyancy-driven heat transfer in nanofluid-filled trapezoidal enclosure with variable thermal conductivity and viscosity. They found that the effect of the viscosity was more dominant than the thermal conductivity. At the same year, Raisi et al. [25] numerically studied forced convection of laminar nanofluid in a microchannel with both slip and no-slip conditions. The effects of pertinent parameters such as Reynolds number, solid volume fraction, and slip velocity coefficient on the thermal performance of the microchannel were studied in this article.

Very recently, Parvin et al. [26] analyzed thermal conductivity variation on natural convection flow of water–alumina nanofluid in an annulus. They found significant heat transfer enhancement due to the presence of nanoparticles and it was accentuated by increasing the nanoparticles volume fraction and Prandtl number as well as large Grashof number. Entropy generation due to natural convection in a partially open cavity with a thin heat source subjected to a nanofluid was conducted by Mahmoudi et al. [27]. Their results indicated that when open boundary was located up, the fluid flow augmented and hence the heat transfer and Nusselt number increased and total entropy generation decreased. Nasrin [28] investigated influences of physical parameters on mixed convection in a horizontal lid driven cavity with undulating base surface where flow and heat transfer characteristics were presented in terms of streamlines, isotherms, average Nusselt number (Nu) and maximum temperature (θm) of the fluid.

The objective of this work is to study steady combined convection in double-lid driven and triangular wavy cavity filled with a nanofluid (water with CuO nanoparticles) using two different nanofluid viscosity models and to illustrate the effects of these models on the enhancement/reduction of the heat transfer rate due to the presence of nanoparticles.

2. Problem formulation

Fig. 1 shows a schematic diagram and related boundary conditions of the vertical triangular wavy chamber. A Cartesian coordinate system is used with the origin at the lower left corner of the computational area. The height and the length of the cavity are given by \( H \) and \( L \). The vertical wavy walls are heated and maintained at a constant temperature \( T_h \) higher than the horizontal cold wall temperature \( T_i \). The top and bottom surfaces are moving along positive and negative x-axis, respectively with a uniform velocity \( u_t \). The gravitational force acts in the vertically downward direction. The fluid in the cavity is a
water-based nanofluid containing CuO nanoparticles. The nanofluid is assumed incompressible and the flow is conceived as laminar and two-dimensional. It is idealized that water and nanoparticles are in thermal equilibrium and no slip occurs between the two media. The thermo-physical properties of the nanofluid are assumed to be constant except for the density variation, which is approximated by the Boussinesq model. The left vertical triangular wavy surface has non-dimensional amplitude \((A=0.05)\), number of undulation \((\lambda=5)\) and satisfies the equation \(x = y, 0 \leq y \leq 0.1, 0.2 - y, 0.1 \leq y \leq 0.2\) with period 0.2.

The governing equations for laminar, steady-state, lid-driven convection in an enclosure filled with a nanofluid in terms of the Navier–Stokes formulation are given as:

**Equation continuity:**

\[
d\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

**x-momentum equation:**

\[
\rho_f \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_f \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

**y-momentum equation:**

\[
\rho_f \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_f \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \rho_f \phi_f (T - T_i)
\]  

**Energy equation:**

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  

where:

\[
\rho_f \phi_f = (1 - \phi) \rho_f + \phi \rho_s
\]  

\[
\rho_c \phi_f = (1 - \phi) \rho_c + \phi \rho_s
\]  

\[
\rho_c \phi = (1 - \phi) \rho_c + \phi \rho_s
\]  

\[
\alpha_f = \kappa_f / (\rho_f \lambda_f)
\]  

The effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnett model:

\[
\frac{\kappa_f}{\kappa_f} = \frac{\kappa_s + 2\kappa_f - 2\phi (\kappa_f - \kappa_s)}{\kappa_s + 2\kappa_f + \phi (\kappa_f - \kappa_s)}
\]  

This model is found to be appropriate for studying heat transfer enhancement using nanofluids (Abu-Nada [11]; Abu-Nada and Oztop [29] and Akbarinia and Behzadmehr [30]). In addition, the viscosity of the nanofluid can be approximated as viscosity of a base fluid \(\mu_f\) containing dilute suspension of fine spherical particles. In the current study, two different models are used to model the viscosity of the nanofluids, which are the Brinkman model [31] and the Pak and Cho correlation [32]. These correlations are given, respectively as

\[
\mu_{eff} = \frac{\mu_f}{(1 - \phi)^2} s
\]  

\[
\mu_{eff} = \mu_f \left( 1 + 39.11 \phi + 533.9 \phi^2 \right)
\]  

Substituting the following dimensionless variables:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\nu}, \quad V = \frac{vL}{\nu}, \quad P = \frac{pL^2}{\rho_f \nu^4}, \quad \theta = \frac{T - T_i}{T_h - T_i},
\]  

into Eqs. (1)-(4), the dimensionless governing equations become

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

\[
\frac{\partial U}{\partial T} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{-\nu_f \partial p}{\partial X} + \frac{1}{\text{Re} \text{Pr}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  

\[
\frac{\partial V}{\partial T} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{-\nu_f \partial p}{\partial Y} + \frac{1}{\text{Re} \text{Pr}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + R \left( 1 - \phi \right) \beta_f \theta + \phi \beta_s \frac{\partial T}{\partial X}
\]  

\[
\frac{\partial \theta}{\partial T} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

where \(Pr = \frac{\nu_f}{\alpha_f}, \quad Ri = \frac{\alpha_f L}{\nu_f \rho_f C_p F}, \quad Ra = \frac{\rho_s (1 - \phi) L^2}{\beta_f \nu_f} \) and \(Re = \frac{uL}{\nu_f} \) are the Prandtl number, Richardson number, Rayleigh number and Reynolds number respectively.

The appropriate dimensionless boundary conditions can be written as:

1- On the left wall:

\[
U = V = 0, \quad q = 1
\]  

2- On the right wall:

\[
U = V = 0, \quad q = 1
\]  

3- On the top wall:

\[
U = -1, \quad V = 0, \quad \frac{\partial \theta}{\partial Y} = 0
\]  

4- On the bottom wall:

\[
U = 1, \quad V = 0, \quad \frac{\partial \theta}{\partial Y} = 0
\]

The fluid motion is displayed using the stream function \(\psi\) obtained from the velocity components \(U\) and \(V\). The relationships between the
stream function and the velocity components for two dimensional flows are

\[ U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \quad (14) \]

3. Numerical implementation

After solving for \( U, V \) and \( \theta \), more useful quantities for engineering applications are obtained. For example, the Nusselt number can be expressed as:

\[ Nu = \frac{hL}{k_f} \quad (15) \]

where the heat transfer coefficient is computed from

\[ h = \frac{q_w}{T_w - T_i} \quad (16) \]

The thermal conductivity of the nanofluid is expressed as:

\[ k_{nf} = -\frac{q_w}{\partial T/\partial n} \quad (17) \]

Here \( n \) is dimensional distances either along \( x \) or \( y \) direction. Substituting Eqs. (16) and (17) into Eq. (15) and using the dimensionless quantities, the local Nusselt number along the left vertical wavy wall can be written as:

\[ Nu = \left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial N} \quad (18) \]

where \( \frac{\partial \theta}{\partial N} = \sqrt{\left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2} \) and \( (k_{nf}/k_f) \) is calculated using Eq. (5).

**Table 1**

Comparison of base fluid solutions with previous works in an enclosure for \( Re = 100 \) and \( Pr = 0.7 \).

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Tiwari and Das [18]</th>
<th>Lin and Violi [36]</th>
<th>Ha and Jung [37]</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra = ( 10^3 )</td>
<td>( u_{max} = 3.640 )</td>
<td>( 3.597 )</td>
<td>–</td>
<td>3.759</td>
</tr>
<tr>
<td>( y )</td>
<td>0.804</td>
<td>0.819</td>
<td>–</td>
<td>0.881</td>
</tr>
<tr>
<td>( v_{max} )</td>
<td>3.7026</td>
<td>3.690</td>
<td>–</td>
<td>3.7982</td>
</tr>
<tr>
<td>( x )</td>
<td>0.1780</td>
<td>0.181</td>
<td>–</td>
<td>0.2075</td>
</tr>
<tr>
<td>( Nu )</td>
<td>1.0871</td>
<td>1.118</td>
<td>1.072</td>
<td>1.125</td>
</tr>
<tr>
<td>Ra = ( 10^4 )</td>
<td>( u_{max} = 16.1439 )</td>
<td>16.158</td>
<td>–</td>
<td>16.254</td>
</tr>
<tr>
<td>( y )</td>
<td>0.822</td>
<td>0.819</td>
<td>–</td>
<td>0.894</td>
</tr>
<tr>
<td>( v_{max} )</td>
<td>19.6650</td>
<td>19.648</td>
<td>–</td>
<td>19.7962</td>
</tr>
<tr>
<td>( x )</td>
<td>0.110</td>
<td>0.112</td>
<td>–</td>
<td>0.171</td>
</tr>
<tr>
<td>( Nu )</td>
<td>2.195</td>
<td>2.243</td>
<td>2.070</td>
<td>2.313</td>
</tr>
<tr>
<td>Ra = ( 10^5 )</td>
<td>( u_{max} = 34.3009 )</td>
<td>36.732</td>
<td>–</td>
<td>35.5246</td>
</tr>
<tr>
<td>( y )</td>
<td>0.856</td>
<td>0.858</td>
<td>–</td>
<td>0.905</td>
</tr>
<tr>
<td>( v_{max} )</td>
<td>68.7646</td>
<td>68.288</td>
<td>–</td>
<td>68.8265</td>
</tr>
<tr>
<td>( x )</td>
<td>0.05935</td>
<td>0.063</td>
<td>–</td>
<td>0.07325</td>
</tr>
<tr>
<td>( Nu )</td>
<td>4.450</td>
<td>4.511</td>
<td>4.464</td>
<td>4.662</td>
</tr>
</tbody>
</table>

**Fig. 2.** Grid independency study for \( Re = 100, Ri = 0.1 \) and \( \phi = 0.04 \).

**Fig. 3.** Code validation of present work with Tiwari and Das [18] at \( Ri = 1, \phi = 0\% \), \( Pr = 6.2 \), \( Gr = 10^7 \).

**Fig. 4.** Mesh generation of double lid driven chamber.
Finally, the average Nusselt number is determined from:

\[ Nu = \frac{1}{S} \int_{S_0}^{S} Nu \, dS \quad (19) \]

where \( S, N \) are the non-dimensional length and coordinate along the heated triangular surface, respectively.

For convenience, a normalized average Nusselt number is defined as the ratio of average Nusselt number at any volume fraction of nanoparticles to that of pure water that is:

\[ Nu'(\phi) = \frac{Nu(\phi)}{Nu(\phi = 0)} \quad (20) \]

3.1. Numerical technique

In analysis such as computational fluid dynamics (CFD), nanofluids can be assumed to be single phase fluids. The classical theory of single phase fluids can be applied, where physical properties of nanofluid

Table 2

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (water)</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>997.1</td>
<td>6510</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>0.613</td>
<td>18</td>
</tr>
<tr>
<td>( \beta ) (1/K)</td>
<td>( 2.2 \times 10^{-4} )</td>
<td>( 0.085 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Fig. 5. Effect of \( Re \) on streamlines (a) Brinkman model [31] (b) Pak and Cho correlation [32] at \( Ri = 0.1 \) and \( \phi = 4\% \) (solid lines for nanofluid and dashed lines for base fluid).

Fig. 6. Effect of \( Re \) on isotherms (a) Brinkman model [31] (b) Pak and Cho correlation [32] at \( Ri = 0.1 \) and \( \phi = 4\% \) (solid lines for nanofluid and dashed lines for base fluid).
are taken as a function of properties of both constituents and their concentrations.

The momentum and energy balance equations are the combinations of mixed elliptic–parabolic system of partial differential equations that have been solved by using the Galerkin weighted residual finite element technique. The six node triangular element is used in this work for the development of the finite element equations. All six nodes are associated with velocities as well as temperature. Only three corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation. Firstly, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin’s method. The integration involved in each term of these equations is performed by using Gauss’s quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using reduced integration technique \[33,34\] and Newton–Raphson method \[35\]. Finally, these linear equations are solved by applying Triangular Factorization method.

4. Grid testing and Code validation

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution. Various mesh combinations are explored for the case of \(Ri=0.1, \ Re=100, \ \phi=4\%\). The present code is tested for grid independence by calculating the average Nusselt number on the left triangular corrugated wall. Five different non-uniform grid systems with the following number of elements within the resolution field: 1588, 2765, 3593, 4795 and 5682 are examined. The numerical scheme is carried out for highly precise key in the average Nusselt number. The Streamlines and isotherms for the Brinkman model \[31\] and Pak and Cho correlation \[32\] are given in Figs 7 and 8 respectively. (a) Brinkman model \[31\] (b) Pak and Cho correlation \[32\] at \(Ri=1\) and \(\phi=4\%\) (solid lines for nanofluid and dashed lines for base fluid).
number \( (Nu) \) for the aforementioned elements to develop an understanding of the grid fineness as shown in Fig. 2. The scale of \( Nu \) for 4795 elements shows a little difference with the results obtained for the other elements. In harmony with this, it is found that a grid system of 4795 elements ensures a grid independent solution. Another grid-independence study is performed using CuO–water nanofluid. It is confirmed that the same number of grid elements ensures a grid-independent solution.

The present numerical solution is further validated by comparing the current code results for streamlines and isotherms at \( Ri = 1, \phi = 0\% \), \( Pr = 6.2 \), \( Gr = 10^4 \) with the graphical representation of Tiwari and Das \[18\] which was reported for heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. Fig. 3 shows the above stated comparison. As shown in Fig. 3, the numerical solutions (present work and Tiwari and Das \[18\]) are in good agreement.

4.1. Mesh generation

In the finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volumes, etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Fig. 4 displays the finite element mesh of the present physical domain.

In addition, Table 1 shows a comparison with various studies ([18,36,37]) in the literature. It is seen from this table that the present results match better with the previously published results.

Fig. 9. Effect of \( Re \) on streamlines (a) Brinkman model \[31\] (b) Pak and Cho correlation \[32\] at \( Ri = 10 \) and \( \phi = 4\% \) (solid lines for nanofluid and dashed lines for base fluid).

Fig. 10. Effect of \( Re \) on isotherms (a) Brinkman model \[31\] (b) Pak and Cho correlation \[32\] at \( Ri = 10 \) and \( \phi = 4\% \) (solid lines for nanofluid and dashed lines for base fluid).
4.2. Thermo-physical properties

The thermo-physical properties of the base fluid (water) and nanoparticles (CuO) are taken from Zi-Tao Yu et al. [38] and tabulated in Table 2.

5. Results and discussion

Numerical results for the streamlines and isotherms for various values of the Reynolds number (\(Re = 10, 100, 200, 300\)), Richardson number (\(Ri = 0.1, 1, 10\)), and the solid volume fraction (\(\phi = 1\%, 4\%, 7\%, 10\%) with fixed Prandtl number (\(Pr = 7\)) are shown. In addition, the values of the average Nusselt number \(Nu\) have been calculated for different \(Re, Ri\) and \(\phi\) for both clear water and water–CuO nanofluid using two viscosity models. Therefore, the findings of this study provide more information on the heat transfer characteristics of nanofluids.

Figs. 5(a)–(b) and 6(a)–(b) present representative contour maps for the streamlines and isotherms inside a vertical triangular corrugated chamber filled with a clear water (\(\phi = 0\%) and water–CuO nanofluid having 4\% particle volume fraction for various values of the Reynolds number \(Re (= 10, 100, 200 and 300)\) at \(Ri = 0.1\) (purely forced convection regime) using the Brinkman model [31] and the Pak and Cho correlation [32], respectively. Fig. 5(a)–(b) indicate that the buoyancy effect is overwhelmed by the mechanical or shear effect due to the movement of the top lid and the flow features are similar to those of a viscous flow of a non-stratified fluid in a lid-driven cavity. The streamlines behavior in a two-dimensional double lid-driven cavity is characterized by a single anti-clockwise revolving cell occupying the cavity with the eye of the cell being in the center of cavity. Two small eddies appear inside this cell near the top and bottom surfaces of the chamber at \(Re = 10\). Due to the variation of \(Re\), these tiny vortices disappear sequentially. The streamlines of Pak and Cho correlation [32] are not similar to Brinkman model [31] as well as base fluid (water). From Fig. 6(a)–(b), it is shown that the isotherm pattern is the same for the two models along with the base fluid at the lowest value of Reynolds number. The isothermal lines cluster heavily near the vertical surfaces of the cavity which indicates temperature gradients in the normal direction in this region and that the hot fluid moves vertically towards the horizontal.
walls of the chamber. In the center of the chamber, the temperature gradients are weak and this implies that the temperature differences are very small in the interior region of the chamber due to the vigorous effects of the mechanically-driven circulations. An irregular shape is observed in the temperature field for $Re = 300$ in Fig. 6(a).

Furthermore, for moderate values of $Ri$ ($= 1$, combined convection-dominated regime), Figs. 7(a)–(b) and 8(a)–(b) indicate that the buoyancy effect is of relatively comparable magnitude of the shear effect due to the sliding top and bottom walls lid. There is no variation in the velocity and temperature fields for $Re = 10$ in comparison with Figs. 5(a)–(b) and 6(a)–(b). But at $Re = 100$, the streamlines move faster and tend to get squeezed in the center part of the cavity forming a two-eye cell with one recirculating eye moving upward towards the upper horizontal wall and another moving downward towards the lower horizontal wall. In addition, a clockwise secondary vortex develops near the horizontal wall and another moving downward towards the lower horizontal wall. The isotherms take the triangular wavy pattern close to the vertical walls. The dis-similarity becomes more visible in between the two models for higher values of $Re$. The isotherms spread upward and downward indicating moderate temperature gradients in the normal direction. The thermal current activities are almost similar for the base fluid and the Brinkman model [31].

Moreover, for large values of $Ri$ ($Ri = 10$, natural convection-dominated regime), the flow and thermal fields are depicted in Figs. 9(a)–(b) and 10(a)–(b) with $\phi = 4\%$. Both fields are exactly the same form for the lowest $Re = 10$ in comparison with the forced and mixed convection effects. The buoyancy effect is dominant and it is expected that a multi-cellular trend where the two-eye cell splits in two or more cells. Finally, there are three different vortices formed in the neighborhood of the top, bottom, and the left vertical surfaces in the chamber for the largest value of the Reynolds number ($Re = 300$). On the other hand, the isotherms become well distributed in the whole cavity and tend to become more vertically distributed as the Richardson number $Ri$ increases.

The effect of the solid volume fraction $\phi$ ($= 1\%, 4\%, 7\%$ and $10\%$) for the considered two models [31,32] along with the base fluid ($\phi = 0\%$) on the velocity and temperature profiles are displayed in Figs. 11(a)–(b) and 12(a)–(b) with $Ri = 0.1$ and $Re = 100$. As the volume fraction of CuO nanoparticles increases from $1\%$ to $10\%$, the streamline and isotherm contours tend to get affected significantly. The streamlines in this case using the Brinkman model [31] show an unnoticeable change. In general, the streamlines move at a slower rate and become more vertically stretched and they are more so for the results obtained using the Pak and Cho correlation [32] than for those obtained by employing the Brinkman model [31]. The size of the small eddies created in the streamlines becomes larger successively and they take similar pattern to other lines due to the variation of $\phi$. In addition, the isotherms corresponding to $\phi = 10\%$ move more upward towards the upper horizontal wall of the cavity causing increased thermal gradients there than for $\phi = 0\%$ (clear water). This is true for both models. However, the increase in the thermal gradients at the upper horizontal wall is much higher for the Pak and Cho correlation [32] than for the Brinkman model [31]. This means that higher heat transfer rates are predicted by the Pak and Cho correlation [32] than for the Brinkman model [31] as will be seen in the subsequent figures.

Fig. 13(i)–(iv) present typical profiles for the $x$-component of velocity $U$, $y$-component of velocity $V$, temperature profile $\theta$ at mid cavity width and the local Nusselt number (left vertical wavy surface)

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Fig. 13. Typical profiles of (i) $x$-component, (ii) $y$-component of velocity, (iii) temperature at mid cavity width and (iv) local Nusselt number at $Ri = 0.1$, $Re = 100$, $\phi = 4\%$. 

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for a triangular wavy chamber at $\text{Ri}=0.1$ and $\text{Re}=100$, $\phi=0\%$ (water) and $\phi=4\%$ [31,32]. It is seen from Fig. 13(i) that the $U$ velocity component results for the Brinkman model [31] illustrate very insignificant change from those of clear water whereas the Pak and Cho correlation [32] predicts an important effect. Fig. 13(ii) shows that the $V$ velocity component results for the Brinkman model [31] have a little difference from clear water. However, the $V$ velocity component increases appreciably using the Pak and Cho correlation [32]. The result for the temperature profile in Fig. 13(iii) expresses that $\theta-Y$ profile obtains the form of parabola. The values using Brinkman model [31] are almost the same as those for clear water. But it is affected more by using the Pak and Cho correlation [32] than by using the Brinkman model [31]. It is observed from Fig. 13(iv) that the local Nusselt number decreases in most of the $Y$-axis ($Y<0.2$). In the range $0.2<Y<0.8$, the local Nusselt number mimics the form of triangular wavy pattern. Beside this region it increases up to $Y=1$. It is predicted that the local Nusselt number rises with the presence of nanoparticles for both the models [31,32]. Moreover, it is observed that the Pak and Cho correlation [32] significantly over-predicts the local Nusselt number than that corresponding to the Brinkman model [31].

Fig. 14(i)–(iv) display the effect of $\text{Re}$, $\text{Ri}$, $\phi$ on the average Nusselt number ($\text{Nu}$) and the normalized average Nusselt number ($\text{Nu}^\ast$) for various values of the volume fraction of nanoparticles, respectively. It is clearly seen that $\text{Nu}$ grows up and down with the escalating values of $\text{Re}$ and $\text{Ri}$, respectively for both of the two models [31,32] and the base fluid. As discussed above, it is clearly seen that the values of $\text{Nu}$ and $\text{Nu}^\ast$ enhance as the value of the nanoparticles volume fraction $\phi$ rises from 1% to 10% using both models [31,32]. Furthermore, it is seen from Fig. 14(i) that the percent increases in $\text{Nu}$ for $\text{Re}$ at $\text{Ri}=0.1$ using the Pak and Cho correlation [32], Brinkman model [31] and the base fluid are 14.1%, 9.3% and 8.0%, respectively. $\text{Nu}$ for $\text{Ri}$ at $\text{Re}=100$ using the Pak and Cho correlation [32], Brinkman model [31] and the base fluid deceases in 12.3%, 5.8% and 5.1%, respectively. However, $\text{Nu}$ and $\text{Nu}^\ast$ for $\phi$ at $\text{Ri}=0.1$ and $\text{Re}=100$ using the Pak and Cho correlation [32] and the Brinkman model [31] increase in 29.4% and 24.9%, respectively.

6. Conclusion

The problem of steady laminar combined convective flow and heat transfer of a nanofluid made up of water and CuO in double-lid driven cavity was considered. Two nanofluid models namely the Brinkman model [31] and the Pak and Cho correlation [32] were employed. It was found that significant heat transfer enhancement can be obtained due to the presence of nanoparticles and that this was accentuated by increasing the nanoparticles volume fraction at moderate and large Reynolds number using both nanofluid models. However, for $\text{Ri}\leq4$, the Pak and Cho correlation [32] predicts that the presence of nanoparticles caused reductions in the heat transfer rate. The percent increase in the average Nusselt number using the Pak and Cho correlation [32] is higher than the Brinkman model [31] due to the variation of nanoparticles volume fraction.
References


