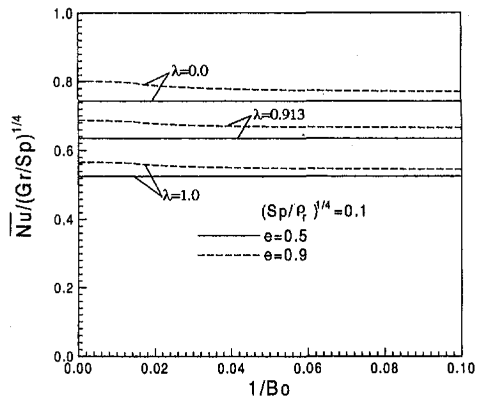

Fig. 4 Effect of surface tension on dimensionless mean heat transfer coefficient

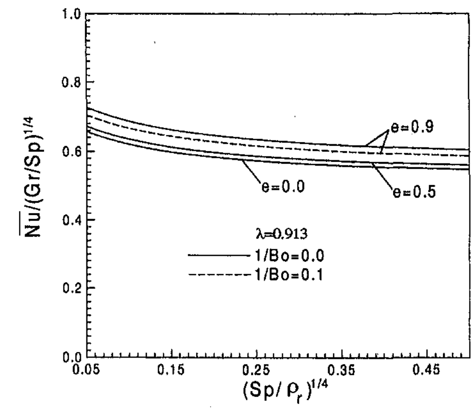

Fig. 5 Effect of superheating on dimensionless mean heat transfer coefficient

proaching unity, δ^* is becoming thinner in the lower half of the tube due to the favorable surface tension effect (pressure gradient), while δ^* is becoming thicker in the rear half of the tube due to the adverse surface tension effect. Meanwhile, owing to the surface tension effect, the vapor film will separate at smaller ϕ_s in the rear half of the elliptical tube. For example, the separation angle ϕ_S for $e = 0.9$ and $1/\text{Bo} = 0.1$ is 2.228 rad. Comparing various cases of $(Sp/\rho_r)^{1/4}$, one may see that, for a larger degree of superheating, the film of boiling vapor becomes thicker in the rear half of the tube.

3.2 Mean Nusselt Number: Effect of λ and *e*. Fig. 3 shows that for the case of $(Sp/\rho_r)^{1/4} = 0.1$ and $1/\text{Bo} = 0$, the mean Nusselt number is varying with λ and is increasing with *e*. It is found that when $\lambda = 0.913$ and $e = 0$ (circular tube), $\overline{\text{Nu}}(\text{Gr}/Sp)^{-1/4} = 0.62$, which is the same mean value of Bromley's (1950) experimental data. Compared with circular tube at the same condition, the mean heat transfer of elliptical tube is enhanced by about a 16.65 percent increase as *e* is approaching unity. It is to be noted that when $e = 1$ (vertical plate), one should use $D_e = 2l/\pi$ in Gr and obtain 0.943 for $\lambda = 0$ (zero interfacial shear); $\overline{\text{Nu}}(\text{Gr}/Sp)^{-1/4} = 0.665$ for $\lambda = 1.0$ (zero interfacial velocity) almost coincides with those obtained from Bromley (1950).

3.3 Mean Nusselt Number: Effect of Surface Tension. Fig. 4 indicates the effect of surface tension on the dimensionless mean heat transfer coefficient. For small values of *e*, it is obvious that the surface tension effect is almost negligible. For large values of *e* (about 0.9), the influence of surface tension on the mean Nusselt number is more significant but is still small. When $e = 0.9$ the mean Nusselt number is reduced by about 2.94 percent as 1/Bo increases from 0 to 0.1.

3.4 Mean Nusselt Number: Effect of Vapor Superheating. As seen in Fig. 5, the larger the degree of superheating, the more the mean heat transfer reduces. The cause for this is that the thickness of vapor film is increased with the increase of superheating degree. Such a decrease in mean heat transfer is similar to that obtained from Nishikawa and Ito (1966) by employing a two-phase boundary layer analysis.

4 Concluding Remarks

1 The present solutions are accurate for elliptical tubes with intermediate diameters. However, for very large or small tubes, the result should be modified by adopting the Breen and Westwater (1962) model.
2 The present note is quite straightforward and is easily applied to a first approximation for film boiling on an inclined tube since a vertical plane passing through a circular tube yields an ellipse.
3 In the present analysis, the surface tension effect has been given first to the film boiling heat transfer from elliptical tubes.

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A Note on Unsteady Hydromagnetic Free Convection From a Vertical Fluid Saturated Porous Medium Channel

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Nomenclature

B_0 = magnetic induction
C = inertia coefficient for porous medium
C_f = skin friction coefficient at heated wall

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Da = Darcy number, $K/(\epsilon H^2)$
F = dimensionless fluid vertical velocity
g = gravitational acceleration
Gr = Grashof number, $CK^{3/2}\rho^2 g\beta\Delta T/\mu^2$
H = channel width
K = permeability of porous medium
M = Hartmann number, $B_0 H(\epsilon\sigma/\mu)^{1/2}$
Nu = Nusselt number
Pr = Prandtl number, $\mu/(\rho\alpha_e)$
r_v = dimensionless suction or injection velocity, $\mu V_w/(\epsilon\rho g\beta H^2\Delta T)$
R_w = wall Reynold's number, $\rho V_w H/(\epsilon\mu)$
t = time
T = dimensional fluid temperature
T_0 = unheated wall temperature
u = dimensional vertical velocity
v = dimensional horizontal velocity
V_w = suction or injection velocity
x, y = Cartesian coordinates as shown on Fig. 1

Greek symbols

α_e = effective thermal diffusivity
β = coefficient of thermal expansion
ΔT = temperature increase
ϵ = porosity of porous medium
η = dimensionless horizontal distance
μ = fluid dynamic viscosity
ρ = fluid density
σ = fluid electrical conductivity
τ = dimensionless time
θ = dimensionless fluid temperature

Introduction

Increasingly in the past decade there has been considerable interest in investigating free convection flows in porous media (see, for instance, Nakayama et al., 1990; Chen and Lin, 1995). This interest stems from various industrial applications such as thermal insulation systems, enhanced oil recovery, regenerative heat exchangers, petroleum reservoirs, geothermal reservoirs, and filtration. Also, there has been a renewed interest in studying the magnetohydrodynamic effects on free convection of electrically conducting fluids from surfaces embedded in porous and nonporous media (see, for instance, Pop and Watanabe, 1994; Aldoss et al., 1995).

The transient phenomena of such flows has also been the subject of many investigators. For example, Ingham and Brown (1986) reported series expansion solutions for the problem of transient free convection on a suddenly heated vertical plate in a porous medium, respectively. Recently, Chen et al. (1989) and Nakayama et al. (1990) considered similar flat plate problems with non-Darcy porous medium inertial effects. More recently, the most fundamental problem of transient non-Darcy free convection between vertical impermeable plates in a fluid saturated porous medium with suddenly heated and suddenly cooled walls was treated by Nakayama et al. (1993). They reported closed-form approximate solutions for small and large times as well as numerical solutions for the intermediate times.

It is of interest in this paper to consider the problem discussed by Nakayama et al. (1993) (with only one of the walls subjected to sudden change in temperature) to study the effects of suction and injection at the walls as well as the influence of the presence of a magnetic field force normal to the direction of motion on the flow and heat transfer characteristics of the problem. It is assumed that the fluid is incompressible and electrically conducting. In addition, the magnetic Reynolds number is assumed small so that the induced magnetic field is neglected.

Problem Formulation

Consider transient hydromagnetic buoyancy induced flow and heat transfer through a vertical fluid saturated porous medium channel of infinite extent and having a width of H. Initially, both the channel walls as well as the porous medium are kept at the same temperature T_0. One of the channel walls is then suddenly heated by a temperature increase ΔT while the temperature of the other wall is kept unchanged. Uniform fluid suction and injection are imposed at the left side and right-side channel walls, respectively. Due to the sudden heating of the left-side channel surface, a fluid vertical motion is induced due to density gradients causing a buoyancy force. As the flow is induced, a magnetic field of uniform strength B_0 is applied normal to the flow direction. Let the x-axis be directed upwards along the channel walls and the y-axis be normal to it. The porous medium is assumed to have uniform porosity and permeability. The dimensionless governing equations and initial and boundary conditions can be written as (see, Vafai and Tien, 1981; Nakayama et al., 1993; Chen and Lin, 1995)

$$\frac{\partial F}{\partial \tau} - \Pr\frac{\partial^2 F}{\partial \eta^2} - \Pr R_w\frac{\partial F}{\partial \eta} + \Pr\left(M^2 + \frac{1}{\mathrm{Da}}\right)F + \frac{\mathrm{Gr}}{\mathrm{Da}^2}\Pr F\sqrt{F^2 + r_v^2} - \Pr\theta = 0 \quad (1)$$

$$\frac{\partial \theta}{\partial \tau} - \frac{\partial^2 \theta}{\partial \eta^2} - \Pr R_w\frac{\partial \theta}{\partial \eta} = 0 \quad (2)$$

$$F(0, \eta) = 0, \quad \theta(0, \eta) = 0, \quad F(\tau, 0) = 0,$$
$$\theta(\tau, 0) = 1, \quad F(\tau, 1) = 0, \quad \theta(\tau, 1) = 0 \quad (3a\text{–}f)$$

where

$$\tau = \alpha_e t/H^2, \quad \eta = Y/H, \quad F = \mu u/(\epsilon\rho g\beta H^2\Delta T),$$
$$\theta = (T - T_0)/\Delta T, \quad \Pr = \mu/(\rho\alpha_e),$$
$$R_w = \rho V_w H/(\epsilon\mu), \quad \mathrm{Da} = K/(\epsilon H^2),$$
$$\mathrm{Gr} = CK^{3/2}\rho^2 g\beta\Delta T/\mu^2, \quad M^2 = \epsilon\sigma B_0^2 H^2/\mu, \quad \text{and}$$
$$r_v = \mu V_w/(\epsilon\rho g\beta H^2\Delta T)$$

are the dimensionless time, vertical distance, fluid velocity and temperature, the Prandtl number, wall Reynold's number, Darcy's number, Grashof number, square of the Hartmann number, and dimensionless suction velocity, respectively.

Important physical parameters for this type of flow are the skin-friction coefficient and the wall heat transfer at the heated wall (herein called the Nusselt number). These are defined in dimensionless form as

$$C_f = \frac{\partial F}{\partial \eta}(\tau, 0), \quad \mathrm{Nu} = -\frac{\partial \theta}{\partial \eta}(\tau, 0). \quad (4a, b)$$

Steady-State Solutions. For sufficiently large time, the dependent variables of the problem will no longer change with time, and the flow is assumed to have reached steady-state conditions. In the absence of inertial porous medium effects (sometimes called Forchheimer effects), that is for slow flow, a closed form solution of the governing equations is possible. The neglect of the porous medium inertial effects ($C = 0$) causes the Grashof number to vanish. It should be noted that setting $\mathrm{Gr} = 0$ does not only mean slow flow ($C = 0$) but it can also mean that the flow is not dominated by natural convection since $\Delta T = 0$ in this case. Therefore, the steady-state Darcian flow and heat transfer problem becomes

$$F'' + R_w F' - \left(M^2 + \frac{1}{\mathrm{Da}}\right)F - \theta = 0 \quad (5)$$

$$\theta'' + \Pr R_w\theta' = 0 \quad (6)$$

$$F(0) = 0, \quad F(1) = 0, \quad \theta(0) = 1, \quad \theta(1) = 0 \quad (7a\text{–}d)$$

where a prime denotes ordinary differentiation with respect to η.

Without going into detail, it can be shown that Eqs. (5) and (6), subject to Eqs. (7), have the following solutions:

$$\theta = C_1 + C_2 \exp(-\mathrm{Pr} R_w \eta) \quad (8)$$

$$F = A + B \exp(-\mathrm{Pr} R_w \eta) + C_3 \exp(\lambda_1 \eta) + C_4 \exp(\lambda_2 \eta) \quad (9)$$

where

$$\lambda_1 = \frac{-R_w + \left(R_w^2 + 4\left(M^2 + \frac{1}{\mathrm{Da}}\right)\right)^{1/2}}{2},$$

$$\lambda_2 = \frac{-R_w - \left(R_w^2 + 4\left(M^2 + \frac{1}{\mathrm{Da}}\right)\right)^{1/2}}{2} \quad (10a, b)$$

$$C_1 = \frac{-\exp(-\mathrm{Pr} R_w)}{1 - \exp(-\mathrm{Pr} R_w)}, \quad C_2 = \frac{1}{1 - \exp(-\mathrm{Pr} R_w)},$$

$$A = \frac{C_1}{M^2 + \frac{1}{\mathrm{Da}}}$$

$$B = \frac{C_2}{\mathrm{Pr} R_w (1 - \mathrm{Pr}) + M^2 + \frac{1}{\mathrm{Da}}},$$

$$C_3 = \frac{A(\exp(\lambda_2) - 1) + B(\exp(\lambda_2) - \exp(-\mathrm{Pr} R_w))}{\exp(\lambda_1) - \exp(\lambda_2)}$$

$$C_4 = \frac{A(\exp(\lambda_1) - 1) + B(\exp(\lambda_1) - \exp(-\mathrm{Pr} R_w))}{\exp(\lambda_2) - \exp(\lambda_1)}. \quad (11a\text{–}f)$$

Some special limiting cases can now be obtained. For the case of impermeable walls ($R_w = 0$), the steady-state solutions become

$$\theta = 1 - \eta \quad (12)$$

$$F = \frac{1}{m^2}\left(1 - \eta - \frac{\sinh(m(1 - \eta))}{\sinh(m)}\right),$$

$$m = \left(M^2 + \frac{1}{\mathrm{Da}}\right)^{-1/2} \quad (13a, b)$$

$$C_f = \frac{1}{m^2}(m \coth(m) - 1), \quad \mathrm{Nu} = 1 \quad (14a, b)$$

where these solutions reduce exactly to those reported by Nakayama et al. (1993) in the absence of the magnetic field ($M = 0$). In addition, the closed-form solutions reported in this section were compared favorably with some of the analytical solutions given by Vafai and Kim (1989) and some of those reported later by Nield et al. (1996) for forced convection in porous medium channel with various thermal boundary conditions. It should be mentioned that the form of the analytical solutions reported herein does not compare directly with those of Nield et al. (1996) because the free convection effect (θ in Eq. (5)) must be dropped out.

Time-Dependent Solutions. It is possible to obtain a closed-form solution for the dimensionless energy equation (2) subject to the corresponding initial and boundary conditions ($3b$, d, f) by the separation of variables method. This can be shown to be

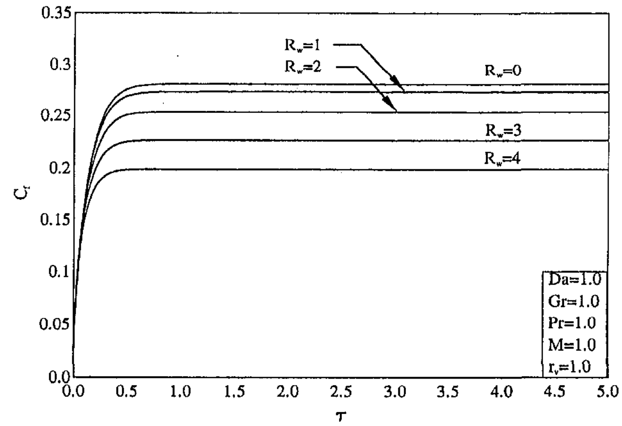

Fig. 1 Effects of R_w on temporal development of skin friction

$$\theta(\tau, \eta) = \frac{\exp(-\mathrm{Pr} R_w(\eta - 1)) - 1}{\exp(\mathrm{Pr} R_w) - 1} - 2 \sum_{n=1}^{\infty} \frac{n\pi}{\lambda_n^2} \exp\left(\frac{-\mathrm{Pr} R_w}{2}\eta - \lambda_n^2 \tau\right) \sin(n\pi\eta) \quad (15)$$

where $\lambda_n^2 = n^2\pi^2 + (\mathrm{Pr} R_w/2)^2$ are the square Eigenvalues for this problem. As the wall Reynolds number R_w approaches zero, the solution for $\theta(\tau, \eta)$ given by Nakayama et al. (1993) for impermeable walls is recovered.

Equation (15) leads to the following transient development solution of the Nusselt number:

$$\mathrm{Nu}(\tau) = \frac{\mathrm{Pr} R_w \exp(\mathrm{Pr} R_w)}{\exp(\mathrm{Pr} R_w) - 1} + 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2}{\lambda_n^2} \exp(-\lambda_n^2 \tau) \quad (16)$$

which is also consistent with that given by Nakayama et al. (1993) for $R_w = 0$.

Equation (1) governing the flow or the velocity development in the porous medium channel is nonlinear and must be solved numerically. The standard, implicit, iterative, tridiagonal finite-difference method discussed by Blottner (1970) is employed herein for the solution of the whole initial-value problem. The exact solutions reported earlier served as a vehicle for calibration of the numerical solutions. The computational domain was divided up into 201 nodes in the η direction and 301 nodes in the τ direction. Constant η step sizes ($\Delta\eta = 0.005$) and variable τ step sizes ($\Delta\tau_1 = 0.01$ with a growth factor of 1.01) were employed in the present work. These step sizes were chosen after many numerical experimentations to assess grid independence.

Figures 1 and 2 present transient skin friction and Nusselt number profiles for various values of the wall Reynold's number R_w, respectively. As the wall Reynold's number increases, that is increasing the injection at the right-side wall and the suction at the left-side wall, a distinctive peak in the velocity profile similar to that reported by Chamkha (1994), starts to form close to the left-side wall and the volumetric flow rate up the channel decreases. This is obvious since as the fluid is injected into the channel; a force normal to the flow direction is created which tends to push the fluid particles towards the left-side wall causing the fluid to slow down and the boundary friction to reduce. This is clearly shown in Fig. 1. For the case of $R_w = 0$ (impermeable walls), the temperature profile is linear, as shown by the closed-form solution reported earlier (Eq. (12)). However, as the suction (or injection) is increased, a faster decay in temperature from the left-side hot wall to the right-side wall is observed. This fast decay in temperature causes the slope of the temperature profile at the hot wall to increase, which results

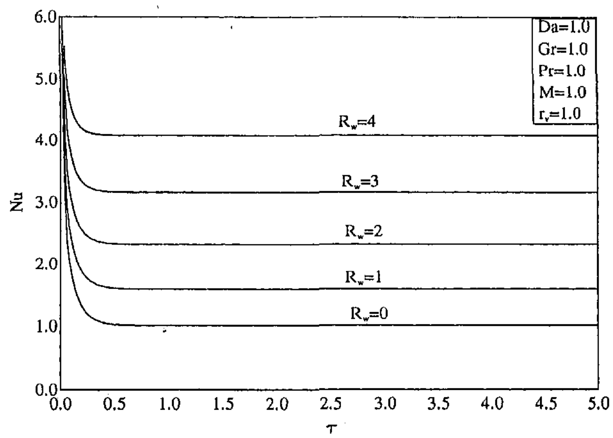

Fig. 2 Effects of R_w on temporal development of skin friction

in an increase in the Nusselt number, as clearly evident from Fig. 2. It should be mentioned that in obtaining the results associated with $R_w = 0$, r_v is set to zero since both of the parameters R_w and r_v are related to V_w. Thus, when $V_w = 0$, then both R_w and r_v must be zero.

Conclusion

The problem of unsteady hydromagnetic non-Darcy free convection from a fluid saturated porous medium supported by two infinitely long porous vertical plates is considered. One of the plates is suddenly heated while the other is maintained at the initial temperature. Fluid suction and injection are imposed at the hot and other wall, respectively. The mathematical model accounts for non-Darcy inertial effects of the porous medium and for the Hartmann effects of Magnetohydrodynamics. Several closed form and limiting solutions were obtained under steady-state conditions. In addition, the energy equation, which is independent of the flow equations, was solved analytically by the separation of variables method. The full initial-value problem was solved numerically by an implicit, tridiagonal, finite-difference methodology. Few of the obtained results were illustrated graphically to show special features of the solutions.

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Interaction of Surface Radiation and Free Convection in an Enclosure With a Vertical Partition

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Nomenclature

A = aspect ratio, H/d
d = total width of the enclosure, m
F_{ij} = diffuse view factor between area elements i and j
g = acceleration due to gravity, m/s^2
H = height of the enclosure, m
J = radiosity of wall element, W/m^2
k = thermal conductivity of the fluid, W/m·K
N_{RC} = radiation conduction interaction parameter, $\sigma T_H^4 d/[k(T_H - T_C)]$
Nu = local Nusselt number based on d (bar indicates the mean value, appropriate subscript indicates convective or radiative as the case may be), $qd/[k(T_H - T_C)]$
Pr = Prandtl number of the fluid, ν/α
q = heat flux, W/m^2
Ra = Rayleigh number based on d, $g\beta(T_H - T_C)d^3/(\nu\alpha)$
T = temperature, K
T_R = temperature ratio, T_C/T_H
x = vertical coordinate, m
X = dimensionless vertical coordinate, x/d
y = horizontal coordinate, m
Y = dimensionless horizontal coordinate, y/d
W = vorticity, s

Greek Symbols

α = thermal diffusivity of the fluid, m^2/s
β = coefficient of expansion, 1/K
ϵ = total hemispherical emissivity
ν = kinematic viscosity of the fluid, m$^2/s$
ϕ = dimensionless temperature, $(T - T_C)/(T_H - T_C)$
ψ' = stream function, m^2/s
ψ = nondimensional stream function, ψ'/α

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