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HYDROMAGNETIC MIXED CONVECTION STAGNATION FLOW WITH SUCTION AND BLOWING

Ali J. Chamkha
Department of Mechanical and Industrial Engineering,
Kuwait University
P. O. Box 5969
Safat, 13060 KUWAIT

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ABSTRACT

This paper deals with steady, two-dimensional, mixed convection flow of an electrically-conducting and heat-absorbing fluid near a stagnation point on a semi-infinite vertical permeable surface at arbitrary surface heat flux variations in the presence of a magnetic field. Similarity equations are derived and solved numerically by an implicit and accurate finite-difference method. Graphical solutions for the local skin-friction coefficient and the local Nusselt number are presented and discussed for various parametric conditions. These results are presented to illustrate the influence of the Hartmann number, wall mass transfer coefficient, heat absorption coefficient, Prandtl number and the mixed convection or buoyancy parameter. © 1998 Elsevier Science Ltd

Introduction

There have been considerable interest in investigating plane and axisymmetric flow near a stagnation point on a surface. This interest stems from various possible applications of this type of flow in many processes such as cooling of electronic devices by fans, cooling of nuclear reactors, and many aerodynamic and hydrodynamic processes. Early work on plane stagnation-point flow was considered by Hiemenz [1] who analyzed the Navier Stokes equations exactly. Goldstein [2] produced the thermal solution for the Hiemenz [1] problem. Later, the axisymmetric stagnation-point flow and heat transfer was considered by Homann [3] and Sibulkin [4], respectively. After this initial work, various aspects of stagnation-point flows have been investigated (see, for instance, Wang [5], Ramachandran et al. [6], Takhar et al. [7], Kumari et al. [8], Merkin and Mahmood [9] and Chamkha [10]). Although the importance of electrically-conducting and heat-generating or absorbing fluids have been known in many applications such as in nuclear reactors and geothermal systems, no work have been reported on the influence of magnetic fields on stagnation-point flows and heat transfer of electrically-conducting and heat-generating or absorbing fluids. Representative work on steady hydromagnetic flow over a flat plate can be found in the early papers by Tan and Wang [11] and Na [12]. Vajravelu and Nayfeh [13] and Chamkha [14] have considered heat generation or absorption effects in their work on natural convection from a cone and a wedge. In addition, inspite of the significance of stagnation-point flow with surface suction or blowing from both theoretical and technological points of view, little work have been reported on this subject (see, for instance Burde [15]).

Beside the forced flow, the possible presence of buoyancy effects due to density changes in stagnation-point flow situations have lead to some work on mixed convection at a stagnation point. Recent works on this topic are given by Gorla and Hassanien [16] and Amin and Riley [17]. Gorla and Hassanien [16] have reported numerical solutions for mixed convection in stagnation flows of micropolar fluids over vertical surfaces with non-uniform surface heat flux. Amin and Riley [17] have considered mixed convective flow of a Newtonian fluid at a two-dimensional stagnation point on a heated horizontal non-isothermal boundary.

Motivated by the above referenced work and the possible industrial applications, it is of interest in this paper to consider steady, two-dimensional, mixed-convection flow of a Newtonian and electrically-conducting fluid at the stagnation point of a vertical semi-infinite permeable surface at arbitrary heat flux wall conditions and in the presence of a magnetic field and heat absorption effects. A similarity transformation is used to simplify the numerical effort and a numerical solution for the problem is obtained by the finite-difference methodology.

Problem Formulation

Consider steady, laminar, two-dimensional, hydromagnetic, mixed convection stagnation-point flow impinging on a heated vertical semi-infinite permeable surface. Far from the surface, the free stream is moving with a uniform velocity U_∞ and is at a constant temperature T_∞. Let x denote the distance along the plate and y be the distance normal to it. A uniform magnetic field is applied in the y direction causing a flow resistive force in the x direction. The permeable plate or surface is subjected to an arbitrary heat flux $q_w(x)$ and uniform fluid suction or blowing. Figure 1 shows the flow model and the coordinate system. It is assumed that the fluid is Newtonian, viscous, electrically conducting, and generates or absorbs heat at a uniform rate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field will be neglected. Under these circumstances and taking into account the Boussinesq and the boundary-layer approximations, the governing equations can be written as

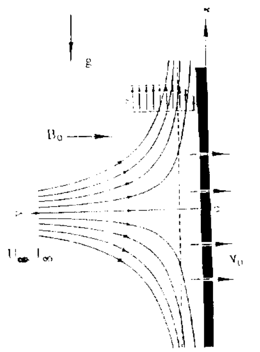

FIG. 1

Flow Model and Coordinate System

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+v\frac{\partial^2 u}{\partial y^2}+Zg\beta(T-T_\infty)-\frac{\sigma B_o^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q_o}{\rho c_p}(T-T_\infty) \tag{3}$$

where u, v, p, and T are the fluid x-component of velocity, y-component of velocity, pressure, and temperature, respectively. ρ, v, c_p, and α are the fluid density, kinematic viscosity, specific heat at constant pressure, and thermal diffusivity, respectively. σ, B_o and Q_o are the fluid electrical conductivity, magnetic induction, and dimensional heat generation or absorption coefficient, respectively. g and β are the acceleration due to gravity and coefficient of thermal expansion, and Z is a parameter such that $Z=1$ denotes buoyancy-assisting or aiding flow for which $T_w > T_\infty$ and $Z=-1$ corresponds to buoyancy-opposing flow for which $T_w < T_\infty$.

The boundary and limiting conditions for this problem are:

$$u(x,0)=0,\ \ v(x,0)=-v_o,\ \ \frac{\partial T}{\partial y}(x,0)=-\frac{q_w(x)}{k}=-\frac{Ax^n}{k} \tag{4}$$

$$u(x,\infty)=U_\infty,\ \ T(x,\infty)=T_\infty$$

where v_o is the suction or injection velocity, k is the fluid thermal conductivity, and A and n are constants.

It is convenient to make Equations (1) through (4) dimensionless by using

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \psi = \sqrt{\nu a}\; x\, f(\xi, \eta), \quad \eta = \sqrt{\frac{a}{\nu}}\, y$$
$$\xi = \frac{g\beta q_w}{a^2 k}\, Re_x^{-1/2} = \frac{Gr_x}{Re_x^{5/2}}, \quad T = (\frac{q_w x}{k})\, Re_x^{-1/2}\, \theta + T_\infty \tag{5}$$

where a is a proportionality constant, $Re_x = U_\infty\, x / \nu$ is the local Reynolds number, and $Gr_x = g\beta q_w\, x^4 / (\nu^2 k)$ is the local Grashof number.

Using Equations (5) and performing the necessary operations yield

$$f''' + ff'' - (f')^2 - M^2 f' + Z\xi\theta + 1 = 0 \tag{6}$$

$$\frac{\theta''}{Pr} + f\theta' + \phi\theta - n f'\theta = 0 \tag{7}$$

$$f(\xi,0) = f_w, \quad f'(\xi,0) = 0, \quad \theta'(\xi,0) = -1$$
$$f'(\xi,\infty) = 1, \quad \theta(\xi,\infty) = 0 \tag{8}$$

where a prime denotes ordinary differentiation with respect to η and $M^2 = \sigma B_o^2 / (\rho a)$, $Pr = \nu/\alpha$, $\phi = Q_o / (\rho c_p a)$ and $f_w = v_o / \sqrt{\nu a}$ are the square of the Hartmann number, Prandtl number, dimensionless heat generation or absorption coefficient and the dimensionless wall mass transfer coefficient, respectively. It should be noted here that positive values of f_w indicate fluid suction at the surface while negative values of f_w correspond to fluid blowing or injection at the wall. In addition, positive values of ϕ mean heat generation (source) and negative values of ϕ mean heat absorption (sink).

In Equation (7), n is a constant indicating the thermal surface boundary condition such that $n = 0$ corresponds to constant heat flux and $n = 1$ corresponds to linear distribution of wall heat flux with the axial or tangential distance x.

The skin-friction coefficient C and the local Nusselt number Nu_x are important physical properties for this problem. They can be defined in dimensionless form as

$$C = 2\, Re_x^{-1/2}\, f''(\xi,0) \tag{9}$$

$$Nu_x = \frac{Re_x^{1/2}}{\theta(\xi,0)} \tag{10}$$

Results and Discussion

Equations (6) and (7) are coupled non-linear ordinary differential equations which possess no closed-form solution. Therefore, a numerical solution of these equations is sought. The implicit, iterative, finite-difference method discussed by Blottner [18] has shown to be successful and accurate for solving this type of equations. For this reason, it is employed in the present work.

The computational domain consisted of 195 nodal points of non-uniform distribution employed to accommodate steep changes in velocity and temperature in the immediate vicinity of the wall. Numerical experimentations performed to assess grid independence and accuracy of the results led to the choice of an initial step size $\Delta\eta_1$ of 0.001 and a growth factor K of 1.03 such that $\Delta\eta_{i+1} = K\,\Delta\eta_i$. The convergence criterion used for this problem was based on the relative difference between the current and the previous iterations. When this difference reached 10^{-5}, the solution was assumed converged and the iteration process was terminated. More details of the numerical method are given by Blottner [18].

To check on the accuracy of the numerical results, a comparison of $f''(0)$ and $1/\theta(0)$ with those reported by Gorla and Hassanien [16] for a Newtonian fluid and linear non-uniform wall heat flux $(n = 1)$ is performed. The result of this comparison is given in Table 1. It is clearly seen that favorable agreement between the results is obtained. This lends confidence to the accuracy of the numerical results to be reported subsequently. A set of representative numerical results is illustrated graphically in Figs. 2 through 9 to show the influence of the strength of magnetic field, wall suction or blowing, and heat absorption characteristics.

TABLE 1

Comparison of $f''(0)$ and $1/\theta(0)$ with those reported by Gorla and Hassanien [16]
$f_w = 0$, $M = 0$, $n = 1$, $\phi = 0$, and $\xi = 1$

	Z = 1				Z = −1			
Pr	f''(0) present	f''(0) [16]	1/θ(0) present	1/θ(0) [16]	f''(0) present	f''(0) [16]	1/θ(0) present	1/θ(0) [16]
0.7	1.83289	1.83388	0.77896	0.77761	--	--	--	--
10	1.37107	1.37115	1.90785	1.90666	1.08094	1.08077	1.81124	1.80961
50	1.28414	1.29247	3.27866	3.28553	1.17936	1.18850	3.21847	3.22589
100	1.26577	1.28475	4.14028	4.16429	1.19874	1.21971	4.09142	4.11785

Figures 2 and 3 depict the influence of the buoyancy parameter ξ and the Hartmann number M on the local skin-friction coefficient C and the local Nusselt number Nu_x for constant $(n = 0)$ and linear $(n = 1)$ wall heat flux conditions and buoyancy-assisted flow, respectively. Increases in the buoyancy parameter ξ have the tendency to increase the slope of the x-component of velocity and to decrease the fluid temperature at the wall. This has the direct effect of increasing both the local skin-friction coefficient and Nusselt number as is evident from Equations (9) and (10) and Figs. 2 and 3. Application of a magnetic field $(M \neq 0)$ causes heating and retardation in the fluid velocity. This yields simultaneous reductions in C and Nu_x. This is depicted in the decreases in both C and Nu_x as M increases presented in Figs. 2 and 3.

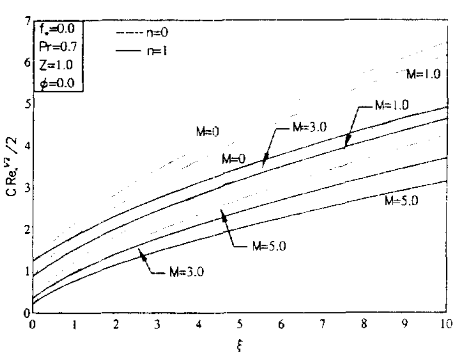

FIG. 2
Effects of ξ and M on Local Skin-Friction Coefficient

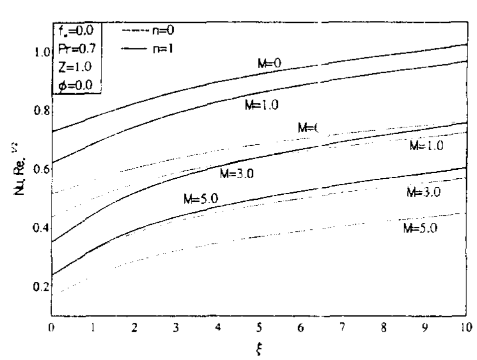

FIG. 3
Effects of ξ and M on Local Nusselt Number

Figures 4 and 5 present the same parameters as Figs. 2 and 3 except that the wall mass transfer coefficient f_w is varied instead of M. Imposition of fluid suction $(f_w > 0)$ at the plate surface causes decreases in the wall shear stress and wall temperature. This results in decreasing the local skin-friction coefficient and increasing the local Nusselt number as shown in Figs. 4 and 5, respectively. On the other hand, the opposite result is obtained for fluid blowing or injection ($f_w < 0$) at the wall, namely, an increase in C and a decrease in Nu_x.

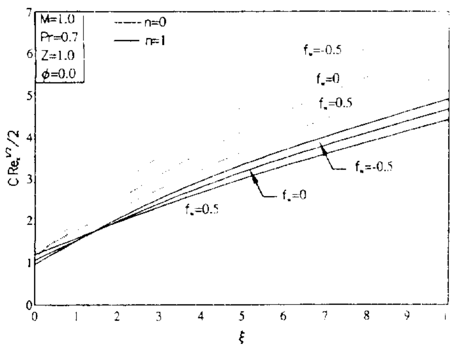

FIG. 4

Effects of ξ and f_w on Local Skin-Friction Coefficient

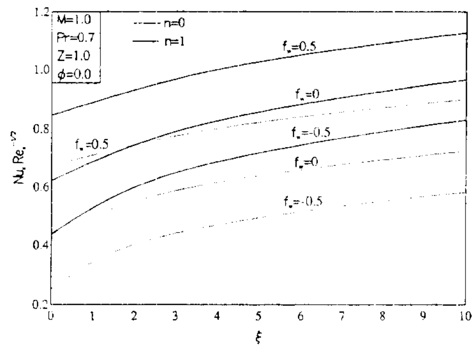

FIG. 5

Effects of ξ and f_w on Local Nusselt Number

The effects of the fluid Prandtl number Pr on C and Nu_x for both buoyancy-assisted ($Z = 1$) and buoyancy-opposing ($Z = -1$) flows are displayed in Figs. 6 and 7, respectively. As expected, increases in the values of Pr result in reducing C and increasing Nu_x for any finite value of the buoyancy parameter ξ. For large values of Pr ($Pr = 50$), almost linear relations between C and Nu_x with ξ are predicted. Also, while increases in ξ cause increases in C and Nu_x for buoyancy-assisting flows ($Z = 1$) as discussed before, an opposite behavior is predicted for buoyancy-opposing flows ($Z = -1$). For the buoyancy-opposing case shown on Figs. 6 and 7, no solution was possible beyond approximately $\xi = 2.3$ as the value of C was approaching zero suggesting the approach to a singularity where flow separation may exist as discussed by Merkin [19].

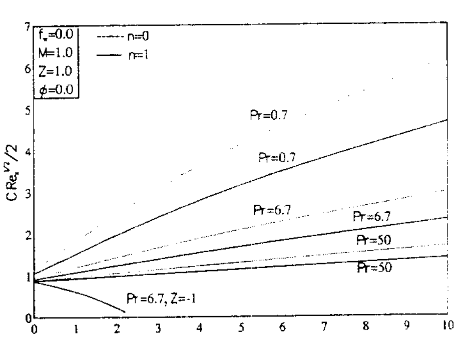

FIG. 6

Effects of ξ, Pr and Z on Local Skin-Friction Coefficient

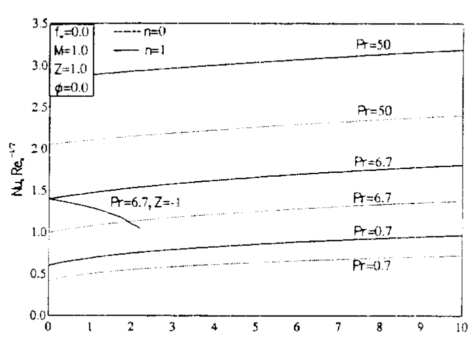

FIG. 7

Effects of ξ, Pr and Z on the Local Nusselt Number

Finally, the influence of heat absorption ($\phi < 0$) on both C and Nu_x is illustrated in Figs. 8 and 9, respectively. It is seen from these figures that as the heat-generation or absorption coefficient ϕ is decreased, lower values of C and higher values of Nu_x are predicted. For heat generation conditions ($\phi > 0$) limited solutions were possible for small values of ξ ($\xi < 1$). For this reason, no figures for $\phi > 0$ are presented. It should be mentioned that, in all of the results reported above for buoyancy-assisting flow situations ($Z = 1$), higher values of C and lower values of Nu_x for constant wall heat flux ($n = 0$) than those associated with linear non-uniform wall heat flux under the same parametric conditions are predicted.

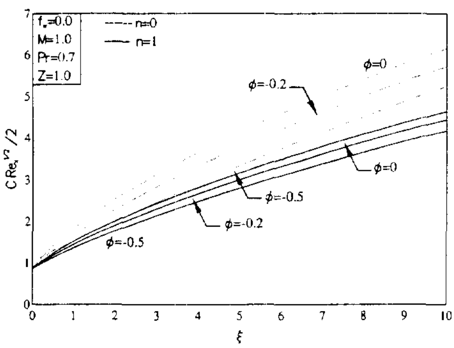

FIG. 8
Effects of ξ and ϕ on Local Skin-Friction Coefficient

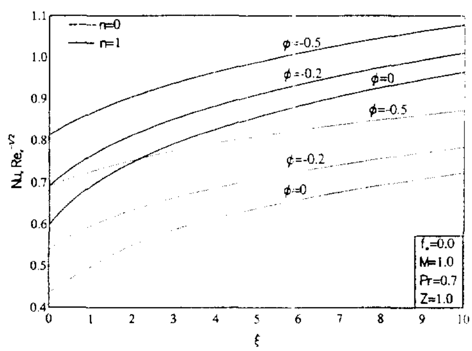

FIG. 9
Effects of ξ and ϕ on Local Nusselt Number

Conclusion

The problem of steady, laminar, two-dimensional mixed-convection flow of a Newtonian, electrically-conducting and heat-absorbing fluid at a stagnation point in the presence of a magnetic field and wall mass transfer is considered. A similarity transformation is used to convert the governing partial differential equations into ordinary ones. A numerical solution of the problem based on the finite-difference methodology is obtained for buoyancy-aiding and buoyancy-opposing flow situations and uniform and linear non-uniform wall heat flux thermal boundary conditions. It was found that both the local skin-friction coefficient and the local Nusselt number increased with increasing buoyancy effects for buoyancy-assisting flows while they decreased for buoyancy-opposing flows. Also, both of these two physical parameters decreased as the strength of the applied magnetic field was increased. In addition, it was concluded that both constant wall suction and fluid heat absorption produced lower skin-friction values and higher Nusselt number values. Furthermore, reduced wall friction effects and enhanced wall heat transfer rates were obtained for a linear non-uniform wall heat flux condition than for a uniform wall heat flux condition. The same was predicted as the Prandtl number was increased.

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