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UNSTEADY HYDROMAGNETIC FLOW AND HEAT TRANSFER FROM A NON-ISOTHERMAL STRETCHING SHEET IMMERSED IN A POROUS MEDIUM

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ABSTRACT

This paper considers unsteady, laminar, boundary-layer flow and heat transfer of an electrically-conducting and heat-generating (or absorbing) fluid along a horizontal stretching plate at variable temperature embedded in a porous medium in the presence of a magnetic field. The governing equations are transformed and solved numerically. The obtained results for a special case of the problem are compared with previously published work and found to be in excellent agreement. A parametric study illustrating the influence of various physical parameters is performed. Numerical results for the steady-state velocity and temperature profiles as well as the time histories for the skin-friction and wall heat transfer coefficients are presented graphically and discussed. © 1998 Elsevier Science Ltd

Introduction

The problem discussed in this paper is that of unsteady laminar flow and heat transfer of an electrically-conducting and heat-generating (or absorbing) fluid from a non-isothermal stretching semi-infinite plate adjacent to a porous medium in the presence of a uniform magnetic field. This type of flow and heat transfer situation finds application in a number of manufacturing and technological processes. Examples of these processes include glass blowing, hot rolling, extrusion of films and plates, continuous casting, cooling of metallic sheets, cooling of electronic chips, crystal growing, melt spinning and many others.

The classical problem of steady flow on a stretched surface extruded from a slit was first considered by Sakiadis [1,2] who developed a numerical solution using a similarity transformation. Many other authors have investigated the steady problem with the plate moving with a linear velocity and various wall thermal conditions (see, for instance, Crane [3], Grubka and Bobba [4]). Banks [5] and Ali [6,7] have considered power-law velocity variation of the stretched surface. Chiam [8] has reported solutions for steady hydromagnetic flow over a surface stretching with a power-law velocity.

There has been some work done on unsteady flow due to a stretching flat surface (see, for example, Surma Devi et al. [9] and Lakshmish et al. [10], Takhar and Nath [11], and Pop and Na [12]). Heat generation effects become important in applications dealing with chemical reactions, heat rejection processes, and cooling of electronic equipment (see, Vajravelu and Nayfeh [13]). The possible presence of a porous medium adjacent to surfaces lead to many studies in that area (see, Vafai and Tien [14] and Chamkha [15]).

Motivated by all the works mentioned above, it is of interest to extend the work of Pop and Na [12] by including such effects as magnetic field effects, heat generation or absorption, and Darcy effects of porous media. In addition, the thermal problem is also considered in the present work for the case where the plate surface temperature varies linearly with the distance along the plate. The flow is assumed laminar and the magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected.

Governing Equations

Consider unsteady, incompressible, hydromagnetic, forced convection, boundary-layer flow of an electrically-conducting and heat-generating fluid over a non-isothermal flat plate immersed in a uniform porous medium stretched with a linear velocity with the tangential coordinate x. A uniform magnetic field is applied in the transverse direction y normal to the plate. The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy modified to account for the presence of the porous medium, the magnetic field and the heat generation effects. These can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}B_0^2\,u-\frac{\nu}{K}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_e\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c}(T-T_\infty) \tag{3}$$

where t, x and y represent time, tangential distance, and transverse or normal distance, respectively. u, v, and T are the fluid tangential velocity, normal velocity, and temperature, respectively. ν, ρ, σ, and c are the fluid kinematic viscosity, density, electrical conductivity, and specific heat, respectively. B_0, T_∞, K, α_e, and Q_0 are the magnetic induction, ambient temperature, porous medium permeability, effective thermal diffusivity of the porous medium, and the heat generation/absorption coefficient, respectively. It should be mentioned here that the plate and the porous medium are both in local equilibrium. In addition, positive values of Q_0 indicate heat generation (source) and negative values of Q_0 correspond to heat absorption (sink).

The corresponding initial and boundary conditions for this problem can be written as

$$\begin{aligned}&u(0,x,y)=ax,\quad v(0,x,y)=0,\quad T(0,x,y)=T_\infty+Ax\\&u(t,x,0)=ax,\quad v(t,x,0)=0,\quad T(t,x,0)=T_w=T_\infty+Ax\\&u(t,x,\infty)=0,\quad T(t,x,\infty)=T_\infty\end{aligned} \tag{4}$$

where a and A are positive constants.

It is convenient to nondimensionalize and transform Equations (1) through (4) by using

$$\begin{aligned}&t=\frac{\tau}{a},\quad y=2\sqrt{\nu t}\;\eta,\quad u=ax\frac{\partial f}{\partial \eta}(\tau,\eta)\\&v=-2a\sqrt{\nu t}\;f(\tau,\eta),\quad T=T_\infty+(T_w-T_\infty)\,\theta(\tau,\eta)\end{aligned} \tag{5}$$

Substituting Equations (5) into Equations (1) through (3) results

$$f'''+2\eta f''-4\tau\frac{\partial f'}{\partial \tau}+4\tau(ff''-(f')^2-(M^2+Da^{-1})f')=0 \tag{6}$$

$$\theta''+Pr(2\eta\theta'-4\tau\frac{\partial \theta}{\partial \tau}+4\tau(f\theta'-f'\theta-\phi\theta))=0 \tag{7}$$

where Equation (1) is identically satisfied. In Equations (6) and (7), a prime indicates differentiation with respect to η and $M=B_0\sqrt{\sigma/(\rho a)}$, $Da^{-1}=\nu/(Ka)$, and $\phi=Q_0/(\rho c a)$ are the Hartmann number, the inverse Darcy number, and the dimensionless heat generation/absorption coefficient, respectively. The transformed initial and boundary conditions become

$$
\begin{aligned}
&f'(0,\eta)=1,\ \ f(0,\eta)=0,\ \ \theta(0,\eta)=1\\
&f'(\tau,0)=1,\ \ f(\tau,0)=0,\ \ \theta(\tau,0)=1\\
&f'(\tau,\infty)=0,\ \ \theta(\tau,\infty)=0
\end{aligned}
\tag{8}
$$

Of special significance for this type of flow are the skin-friction coefficient and the wall heat transfer coefficient. These physical parameters can be defined in dimensionless form as

$$
C_f = \frac{-\mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}}{\dfrac{\mu a x}{2\sqrt{\nu t}}} = -f''(\tau,0) \tag{9}
$$

$$
q_w = \frac{-k \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{\dfrac{k(T_w - T_\infty)}{2\sqrt{\nu t}}} = -\theta'(\tau,0) \tag{10}
$$

where k is the thermal conductivity.

Results and Discussion

The initial-value problem represented by Equations (6) through (8) is nonlinear and possesses no analytical solution. Therefore, a numerical solution is sought for this problem. The standard implicit, iterative, finite-difference method discussed by Blottner [16] is chosen for the solution of Equations (6) and (7) subject to Equations (8).

The computational domain is divided into 196 by 196 nodes in the τ and η directions, respectively. Since the changes in the dependent variables are large in the immediate vicinity of the plate while these changes decrease greatly as the distance above the plate increases, variable step sizes in the η direction are used. For the same reason, variable step sizes in the τ direction are also employed. The initial step sizes employed were $\Delta\eta_1 = 0.001$ and $\Delta\tau_1 = 0.001$ and the growth factors were $K_\eta = 1.03$ and $K_\tau = 1.03$ such that $\Delta\eta_n = K_\eta\, \Delta\eta_{n-1}$ and $\Delta\tau_m = K_\tau\, \Delta\tau_{m-1}$. The convergence criterion used was based on the relative difference between the current and the previous iterations which was set to 10^{-5} in the present work. For more details on the numerical procedure, the reader is advised to read the paper by Blottner [16].

Figures 1 through 10 represent typical flow and heat transfer numerical results. These results are obtained to illustrate the influence of the magnetic field, porous medium, Prandtl number, and the heat generation or absorption effect.

Figures 1 and 2 show typical steady-state fluid tangential velocity V and temperature θ profiles for various values of the magnetic Hartmann number M, respectively. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of flow. This has a tendency to reduce the fluid tangential velocity and increase its temperature. This is indicative from the decreases and increases in V and θ as M increases shown in figures 1 and 2, respectively.

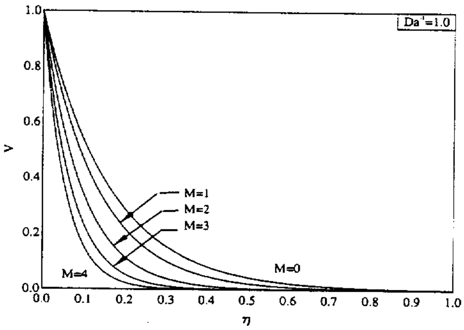

FIG. 1
Effects of M on Steady-State Tangential Velocity Profiles

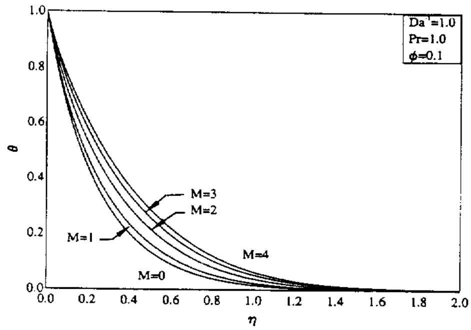

FIG. 2
Effects of M on Steady-State Temperature Profiles

Figures 3 and 4 illustrate the transient development of the skin-friction coefficient C_f and the wall heat transfer coefficient q_w for different values of M, respectively. For convenience, all the results for C_f and q_w in these and all subsequent figures are divided by the factor $2\sqrt{\tau}$.

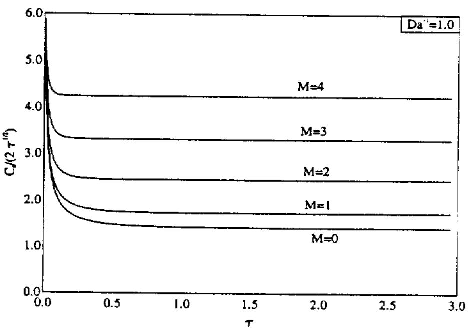

FIG. 3
Effects of M on Skin-Friction Coefficient

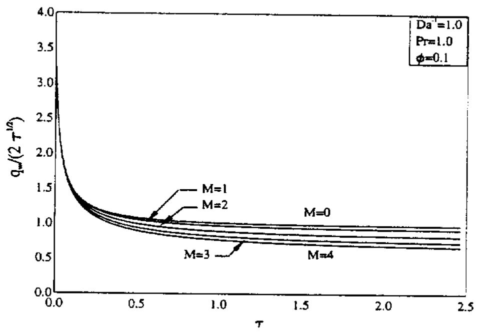

FIG. 4
Effects of M on Wall Heat Transfer Coefficient

This is done in a way similar to Pop and Na [12] since the steady-state results are proportional to this factor. As mentioned before, increases in M cause respective decreases in V and increases in . This results in increasing and decreasing the slopes of the tangential velocity and temperature, respectively. This has the direct effect of increasing C_f and decreasing q_w due to increases in M as depicted in figures 3 and 4, respectively. It is noticed from these figures that the steady-state conditions at large values of are reached rather quickly. This is related to the parametric values employed to obtain the numerical results.

Figures 5 and 6 present the influence of the inverse Darcy number Da^{-1} on the steady-state tangential velocity profiles and the time history of the skin-friction coefficient, respectively. It is obvious that the presence of a porous medium ($Da^{-1} \neq 0$) causes higher restriction to the fluid flow which, in turn, slows its motion. As a result of this, the shear stress at the plate surface increases for the same reasons discussed earlier for figure 3. The decreases in V and increases in C_f as Da^{-1} increases are illustrated clearly in figures 5 and 6, respectively. It should be mentioned that the changes in θ and q_w as a result of increasing Da^{-1} are small and, therefore, not shown herein for brevity.

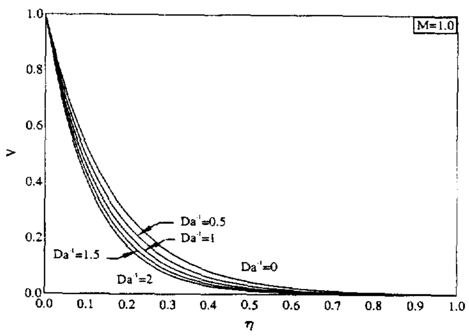

FIG. 5

Effects of Da^{-1} on Steady-State Tangential Velocity

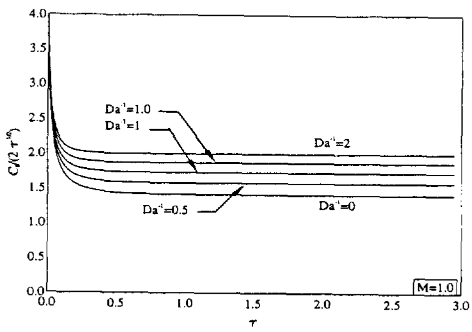

FIG. 6

Effects of Da^{-1} on Skin-Friction Coefficient

Figures 7 and 8 depict the influence of the fluid Prandtl number Pr on the behavior of the steady profiles of θ and the transient profiles of q_w, respectively. Increasing the value of Pr tends to reduce the thermal boundary layer along the plate. This reduces the fluid temperature at every point above the plate surface and increases the wall heat transfer. These facts are clearly shown in figures 7 and 8, respectively. Since the flow problem is independent of the thermal problem, changes in Pr do not affect the flow characteristics.

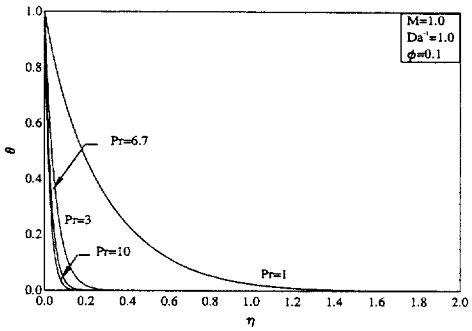

FIG. 7
Effects of Pr on Steady-State Temperature Profiles

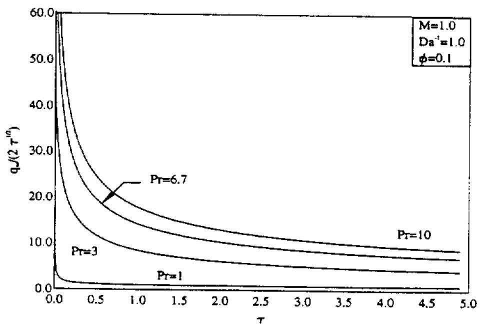

FIG. 8
Effects of Pr on Wall Heat Transfer Coefficient

The effects of heat generation or absorption ($\phi \neq 0$) on θ and q_w are illustrated in figures 9 and 10, respectively. It is clear from these figures that θ decreases and q_w increases as the heat generation or absorption coefficient increases.

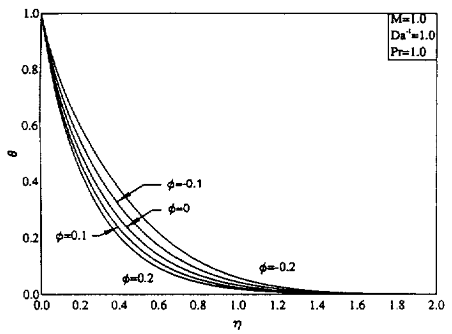

FIG. 9
Effects of ϕ on Steady-State Temperature Profiles

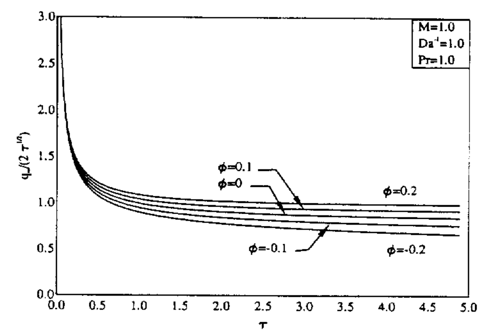

FIG. 10
Effects of ϕ on Wall Heat Transfer Coefficient

It should be mentioned that in the absence of the magnetic field ($M = 0$) and the porous medium ($Da^{-1} = 0$) the solution of the flow problem is in excellent agreement with that of Pop and Na [12].

Conclusion

The problem of unsteady, laminar, hydromagnetic boundary-layer flow and heat transfer of an electrically-conducting and heat-generating (or absorbing) fluid over a non-isothermal horizontal stretching plate embedded in a porous medium was formulated, transformed and solved numerically by the finite-difference method. Comparisons with previously published work were performed and found to be in excellent agreement. It was found that the shear stress at the plate surface increased as either of the Hartmann number or the inverse Darcy number increased. In addition, the wall heat transfer was increased as either of the Prandtl number or the heat generation coefficient was increased. However, the heat transfer at the wall was decreased as the strength of the magnetic field was increased.

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