International Journal of Fluid Mechanics Research, Vol. 27, Nos. 2–4, 2000

Analytical Solutions for Hydromagnetic Free Convection of a Participate Suspension from an Inclined Plate with Heat Absorption

H. M. Ramadan[1] and **A. J. Chamkha**[2]
[1]Kuwait Airways Corporation, Operations Department
Salmeya, Kuwait
[2]Department of Mechanical and Industrial Engineering, Kuwait University
Safat, Kuwait

Continuum equations governing steady, laminar, free convection flow of a particulate suspension over an infinite, permeable, inclined, and isothermal flat plate with magnetic field and fluid heat absorbing effects are developed. The equations account for particulate viscous effects which are absent from most two-phase fluid-particle models. Analytical solutions are developed for inviscid and viscous particle phase situations. Graphical results for the velocity and temperature profiles for both phases as well as their skin-friction coefficients and the Nusselt number are presented and discussed.

* * *

Introduction

Large quantities of studies are available in the published literature for the steady free convection of an electrically-conducting fluid in the presence of a magnetic field (Moreau, 1990, Sacheti et al., 1994, Garandet et al., 1992, Gebhartet et al., 1988, Sparrow and Cess, 1961, Vajravelu and Nayfeh, 1992, Riley, 1964, Raptis and Singh, 1983, and Hossain, 1992). The interest in such flows stems from the wide industrial applications that these flows are involved in (see for example Moreau, 1990). Gebhart et al. (1988) indicated that early interest in such flows arose in astrophysics, geophysics, and controlled nuclear fusion. The study of heat absorption effects in moving fluids is also gaining wide attention in view of several physical problems faced in industry such as those dealing with chemical reactions and those concerned dissociating fluids. Example of this type of problems can be found in the paper by Chamkha (1996a).

The common ground for most of these studies is that they are solved and analyzed by assuming a pure fluid with no contaminants that will effect the heat transfer or the resultant type of fluid flow. While this assumption approximates the reality in many cases quite well especially for low contamination levels, it is not, however, valid in a lot of other cases in which the contaminants in the fluid play a major role in altering the resultant flow and heat transfer characteristics. In many world environments, like the authors own environment (Kuwait), dust storms or fine dust suspension in the air

ISSN 1064-2277

are encountered for many months during the year. In fact, while this introduction was written, the outside horizontal visibility reported to be 2000 meters in dust. These fine particles of dust penetrate the enclosures and the various devices, and have a serious impact on the performance of many equipments. This example is only one possible case of fluid-particle suspension situation where a pure fluid assumption does not accurately represent the reality. Another example is the damping of the undesirable convection movements by magnetic fields during manufacturing crystals. Once the crystals start to form, the fluid-particle interaction has a given value dependent on the size of the crystals, and in turn the crystal can be approximated as a particulate suspension with a given influence on the type and the characteristics of flow and heat transfer.

A survey of the technical literature concluded that no work has been done on the problem of hydromagnetic free or natural convection heat transfer from surfaces for a particulate (fluid-particle) suspension. Recently, the present authors (1998) have reported analytical solutions for free convection flow of a particulate suspension past an infinite permeable vertical plate. In those solutions the particle-phase density distribution was assumed to be uniform across the domain of interest. This assumption greatly simplified the governing equations which in turn facilitated the development of the analytical solutions. Among other things, a detailed study of the influence of plate tilting of the problem considered in (1998) has been reported by Ramadan (1997). The above referenced work has been extended further (Ramadan and Chamkha, 1999) by assuming the particle-phase density distribution to be a variable and considering the effect of plate's inclination. These assumptions resulted in a set of nonlinear ordinary differential equations from which only the particle-phase density distribution possessed an analytical solution. The remaining equations had to be solved numerically. The present work is a continuation to the above effort. In the present work the effects of a transverse magnetic field and heat absorption are considered and the objective is to develop analytical solutions to describe the required variables. Two sets of analytical solutions will be presented depending on the assumption of inviscid or viscous particle-phase flow. The particle-phase viscosity can be thought of as a natural consequence of the averaging processes involved in representing a discrete system of particles as a continuum (see, for example. Drew, 1983 and Drew and Segal, 1971). Also, the particle-phase viscous effects can be used to model particle-particle interaction and particle-wall interaction in relatively dense suspensions. These effects have been investigated previously by many authors such as Tsuo and Gidaspow (1990) and Gadiraju et al. (1992). In all the solutions that will be presented next, the particle-phase density is assumed to be constant throughout the domain of interest.

Problem Description and Governing Equations

An infinite permeable flat plate maintained at a constant wall temperature T_w, and subjected to constant wall suction, is immersed in a stagnant isothermal fluid at temperature T_α. The fluid contains a uniform distribution of solid spherical particle suspension in thermal equilibrium with the fluid. All particles are assumed to have uniform shape and size and are endowed by a viscosity and a diffusivity. Apart from the fluid influence, the gravity is the only force that can affect the particle phase motion. The steady state, free convection, laminar flow for this situation will be studied in which the particle-phase density is assumed to be uniform across the whole domain.

The plate is free to tilt around its lower base in the range of ± 60° from the vertical position. A constant fluid suction is imposed across the plate surface. An energy source, within the plate itself, will maintain the plate wall temperature at a constant value T_w, at all times. T_w, will be greater than T_∞ at all times. Fig. 1 shows the general configuration of the problem under consideration.

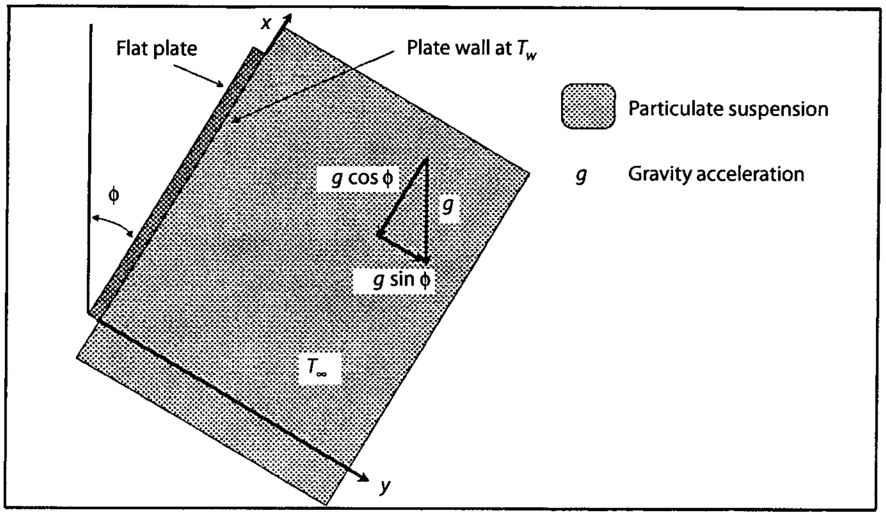

Fig. 1. The general configuration of the problem.

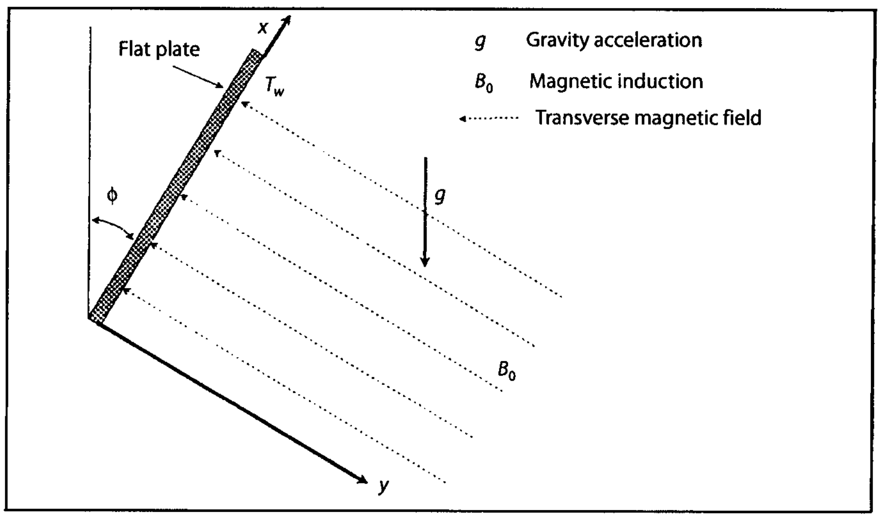

Fig. 2. The transverse magnetic field orientation.

The fluid is assumed to be Newtonian and has constant properties in the range of the study conditions except for the density in the buoyancy term. Both the fluid and the particle phases are modeled as interacting continua as fully discussed by Marble (1970). The thermal diffusivity of the particulate phase is assumed constant throughout. The volume fraction of the suspended particles relative to the fluid will be considered small as done by Marble (1970).

In addition, the fluid is assumed to be affected by a magnetic field and has heat absorption effects. The magnetic field is assumed to be normal to the plate surface regardless of the plate orientation as shown in Fig. 2. The induced magnetic and electrical fields are assumed to be negligible and the electrical displacement and convection are assumed to be small. In addition, the Hall effect and the magnetic Reynolds number are assumed to be small. Only the fluid phase will be affected by the presence of the magnetic field since it is assumed to be electrically conducting. The particle phase however, will not be affected by that field directly since it is assumed to be electrically non-conducting (insulators). Nevertheless, it will be influenced indirectly by the magnetic field due to the interphase drag mechanism between the phases.

The governing equations for the problem under consideration are based on the balance laws of mass, linear momentum, and energy for both the fluid and the particle phases. Since the plate is assumed to be infinite, all dependent variables will be functions of the independent variable y. Taking all the previous assumptions into consideration, the governing equations can be written as

$$\frac{dv}{dy} = 0 \tag{1}$$

$$\mu\frac{d^2u}{dy^2} - \rho v\frac{du}{dy} - \rho_p N(u-u_p) - \rho g\cos\phi - \frac{\sigma_0 B_0^2}{\rho}u = 0 \tag{2}$$

$$k\frac{d^2T}{dy^2} - \rho c v\frac{dT}{dy} + \rho_p c_p N_T(T_p - T) + q_0(T - T_\infty) = 0 \tag{3}$$

$$D_p\frac{d^2\rho_p}{dy^2} - \frac{d(\rho_p v_p)}{dy} = 0 \tag{4}$$

$$\nu_p\frac{d}{dy}\left(\rho_p\frac{du_p}{dy}\right) - \rho_p v_p\frac{du_p}{dy} + \rho_p N(u-u_p) - \rho_p g\cos\phi = 0 \tag{5}$$

$$2\nu_p\frac{d}{dy}\left(\rho_p\frac{dv_p}{dy}\right) - \rho_p v_p\frac{dv_p}{dy} + \rho_p N(v-v_p) - \rho_p g\sin\phi = 0 \tag{6}$$

$$\frac{k_p}{\rho_p}\frac{d}{dy}\left(\rho_p\frac{dT_p}{dy}\right) - \rho_p c_p v_p\frac{dT_p}{dy} - \rho_p c_p N_T(T_p - T) = 0 \tag{7}$$

The boundary conditions are

$$v(0) = -v_w$$

$$u(0) = 0 \qquad u(\infty) = 0$$

$$T(0) = T_w \qquad T(\infty) = T_\infty$$

$$T_p(0) = T_w \qquad T_p(\infty) = T_\infty \tag{8}$$

Two boundary conditions for $u_p(0)$ will be investigated:

$$u_p(0) = \omega_s \frac{du_p}{dy}\bigg|_{y=0} \quad \text{or} \quad u_p(0) = u_{p0} = \text{const} \tag{9}$$

and

$$u_p(\infty) = -\frac{g}{N}\cos\phi \tag{10}$$

where ω_s is a dimensional particle-phase wall slip coefficient. To date, the exact form of boundary conditions to be satisfied by a particle phase at a given surface is unknown. Since the particle phase may resemble a rarefied gas and undergoes slip at a boundary, then a boundary condition borrowed from rarefied gas dynamics as done by previous authors (Soo, 1989 and Chamkha, 1996) will be employed in this study. However, because of this uncertainty another boundary condition is considered in which the particle-phase x-component of velocity has a constant value over the plate wall.

The hydrostatic gradient pressure in Eq. (2) is approximated as

$$\frac{dP}{dx} = \rho_{p_\infty} N u_{p_\infty} - \rho_\infty g \cos\phi \tag{11}$$

Where

$$u_{p_\infty} = -\frac{g\cos\phi}{N} \tag{12}$$

This approximation can be easily obtained by evaluating Eq. (1) through (7) at $y = \alpha$. Using Boussinesq approximation [21] to couple the fluid momentum equation to the temperature field and substituting Eqs. (11) and (12), Eq. (2) can be written as

$$\mu\frac{d^2u}{dy^2} - \rho_\infty v\frac{du}{dy} - \rho_p N(u - u_p) + \rho_{p_\infty} g\cos\phi + \rho_\infty g\bar{\beta}(T - T_\infty)\cos\phi - \frac{\sigma_0 B_0^2}{\rho_\infty}u = 0 \tag{13}$$

where $\bar{\beta}$ is the volume expansion coefficient.

Equations (1), (3) through (7), and (13) constitute the governing equations for the present problem. These equations represent a generalization of the dusty-gas equations discussed by Marble (1970) to include particle-phase viscous, diffusive and buoyancy effects.

To nondimensionalize the above governing equations, the following are used

$$Y = \frac{y\text{Gr}^{1/4}}{L} \qquad U = \frac{uL}{\nu\text{Gr}^{1/2}} \qquad V = \frac{vL}{\nu\text{Gr}^{1/4}}$$

$$U_p = \frac{u_pL}{\nu\text{Gr}^{1/2}} \qquad V_p = \frac{v_pL}{\nu\text{Gr}^{1/4}} \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\theta_p = \frac{T_p - T_\infty}{T_w - T_\infty} \qquad Q_p = \frac{\rho_p}{\rho_{p_\infty}} \qquad S = \frac{q_0L^2}{\mu c\text{Gr}^{1/2}}$$

$$\kappa = \frac{\rho_{p_\infty}}{\rho_\infty} \qquad \alpha = \frac{NL^2}{\nu\text{Gr}^{1/2}} \qquad H = \frac{gL^3}{\nu^2\text{Gr}}$$

$$\gamma = \frac{c_p}{c} \qquad \varepsilon = \frac{N_T L^2}{\nu \mathrm{Gr}^{1/2}} \qquad \beta = \frac{\nu_p}{\nu}$$

$$\mathrm{M}^2 = \frac{\sigma_0 B_0^2 L^2}{\mu \mathrm{Gr}^{1/2}} \qquad \mathrm{Pr} = \frac{\mu c}{k} \qquad \mathrm{Pr}_p = \frac{\mu_p c_p}{k_p}$$

$$\mathrm{Sc} = \frac{D_p}{\nu} \qquad \mathrm{Gr} = \frac{g\bar{\beta}(T_w - T_\infty)L^3}{\nu^2} \tag{14}$$

When the above defined quantities are substituted in Eqs. (1), (3) through (7), and (13) and after simplifying, the following dimensionless equations will result

$$\frac{dV}{dY} = 0 \tag{15}$$

$$\frac{d^2U}{dY^2} - V\frac{dU}{dY} - \alpha\kappa Q_p(U - U_p) - \mathrm{M}^2 U + (\kappa H + \theta)\cos\phi = 0 \tag{16}$$

$$\mathrm{Pr}^{-1}\frac{d^2\theta}{dY^2} - V\frac{d\theta}{dY} + \gamma\varepsilon\kappa Q_p(\theta_p - \theta) + S\theta = 0 \tag{17}$$

$$\mathrm{Sc}\frac{d^2Q_p}{d^2Y} - V_p\frac{dQ_p}{dY} = 0 \tag{18}$$

$$\beta Q_p\frac{d^2U_p}{dY^2} + \beta\frac{dQ_p}{dY}\frac{dU_p}{dY} - V_pQ_p\frac{dU_p}{dY} + \alpha Q_p(U - U_p) - HQ_p\cos\phi = 0 \tag{19}$$

$$2\beta\frac{d}{dY}\left(Q_p\frac{dV_P}{dY}\right) - Q_pV_p\frac{dV_p}{dY} + \alpha Q_p(V - V_P) + H\mathrm{Gr}^{1/4}Q_p\sin\phi = 0 \tag{20}$$

$$\beta\mathrm{Pr}_p^{-1}Q_p\frac{d^2\theta_p}{dY^2} + \beta\mathrm{Pr}_p^{-1}\frac{dQ_p}{dY}\frac{d\theta_p}{dY} - V_pQ_p\frac{d\theta_p}{dY} + \varepsilon Q_p(\theta - \theta_p) = 0 \tag{21}$$

The boundary conditions are modified as follows

$$U(0) = U(\infty) = 0 \qquad \theta(0) = 1.0 \qquad \theta(\infty) = 0$$

$$U_p(\infty) = -\frac{H}{\alpha}\cos\phi \qquad \theta_p(0) = 1.0 \qquad \theta_p(\infty) = 0 \qquad V(0) = -\mathrm{Rv} \tag{22}$$

The two boundary conditions for $U_p(0)$:

$$U_p(0) = U_{p0} \quad \text{or} \quad U_p(0) = \omega\left.\frac{dU_p}{dY}\right|_{Y=0} \tag{23}$$

where $\mathrm{Rv} = v_w L/(\nu \mathrm{Gr}^{1/4})$ and $\omega = \omega_s \mathrm{Gr}^{1/4}/L$ are the dimensionless fluid-phase suction velocity and the particle-phase slip coefficient. It should be noted that Rv and ω are assumed to be constants.

For the sake of obtaining analytical results for the present problem, the particle-phase density is assumed to be constant throughout the domain of interest and its value equals to that of the ambient condition, i.e. $Q_p = 1$. The consequence of this assumption on the fluid-phase equations is evident. The particle-phase governing equations are modified as follows

$$\frac{dV_p}{dY} = 0 \tag{24}$$

$$\beta\frac{d^2U_p}{dY^2} - V_p\frac{dU_p}{dY} + \alpha(U - U_p) - H\cos\phi = 0 \tag{25}$$

$$\alpha(V - V_p) + \mathrm{Gr}^{1/4}H\sin\phi = 0 \tag{26}$$

$$\beta \mathrm{Pr}_p^{-1}\frac{d^2\theta_p}{dY^2} - V_p\frac{d\theta_p}{dY} + \varepsilon(\theta - \theta_p) = 0 \tag{27}$$

The fluid-phase skin-friction coefficient C_f, the particle-phase skin-friction coefficient C_{fp} and the Nusselt number are defined, respectively, as

$$C_f = \frac{\mu\left.\frac{du}{dy}\right|_{y=0}}{\frac{1}{2}\rho\left(\frac{\nu}{L}\right)^2\mathrm{Gr}^{3/4}} = 2\left.\frac{dU}{dY}\right|_{Y=0} \tag{28}$$

$$C_{fp} = \frac{\mu_p\left.\frac{du_p}{dy}\right|_{y=0}}{\frac{1}{2}\rho\left(\frac{\nu}{L}\right)^2\mathrm{Gr}^{3/4}} = 2\kappa\beta\left.\frac{dU_p}{dY}\right|_{Y=0} \tag{29}$$

$$\mathrm{Nu} = \frac{hL}{k\mathrm{Gr}^{1/4}} = -\left.\frac{d\theta}{dY}\right|_{Y=0} \tag{30}$$

Analytical Solutions

In this section, analytical solutions of the problem will be reported for the cases of both inviscid and viscous particle phases. Since the governing equations are linear and the methods for solving such equations are known, and for brevity, no details on how the analytical solutions to be reported below are obtained will be given.

Integrating Eq. (15) and applying the boundary condition for V yields

$$V = -\mathrm{Rv} \tag{31}$$

Similarly, by integrating Eq. (24) and using Eq. (26) to determine the integration constant

$$V_p = \frac{H}{\alpha}\mathrm{Gr}^{1/4}\sin\phi - \mathrm{Rv} = \Gamma \tag{32}$$

Inviscid Particle Phase Solutions ($\beta = 0$). The solutions of the governing equations under the inviscid particle phase ($\beta = 0$) assumption can be shown to be:

$$\theta = \exp(\pi_1 Y) \tag{33}$$

$$\theta_p = \left(1 + \frac{\Pi}{\varepsilon}\pi_1\right)^{-1}\exp(\pi_1 Y) \tag{34}$$

$$U = W_1\left(1 + \frac{\Pi}{\alpha}\Omega_1\right)\exp(\Omega_1 Y) + W_4\left(1 + \frac{\Pi}{\alpha}\pi_1\right)\exp(\pi_1 Y) \tag{35}$$

$$U_p = W_1\exp(\Omega_1 Y) + W_4\exp(\pi_1 Y) - \frac{H}{\alpha}\cos\phi \tag{36}$$

$$N_u = -\left.\frac{d\theta}{dY}\right|_{Y=0} = -\pi_1 \tag{37}$$

$$C_f = 2\left.\frac{dU}{dY}\right|_{Y=0} = 2W_1\left(1 + \frac{\Pi}{\alpha}\Omega_1\right)\Omega_1 + 2W_4\left(1 + \frac{\Pi}{\alpha}\pi_1\right)\pi_1 \tag{38}$$

where π_1 is a solution of the following cubic equation such that π_1 is the only negative root (all roots must be real).

$$\pi^3 + \left[\frac{\varepsilon + \mathrm{PrRv}\Pi}{\Pi}\right]\pi^2 + \mathrm{Pr}\left[S - \gamma\kappa\varepsilon + \frac{\varepsilon\mathrm{Rv}}{\Pi}\right]\pi + \frac{\varepsilon\mathrm{Pr}S}{\Pi} = 0 \tag{39}$$

Ω_1 is a solution of the following cubic equation such that Ω_1 is the only negative root (all roots must be real).

$$\Omega^3 + \left[\frac{\alpha + \Pi\mathrm{Rv}}{\Pi}\right]\Omega^2 + \left[\frac{\alpha\mathrm{Rv}}{\Pi} - \alpha\kappa - \mathrm{M}^2\right]\Omega - \frac{\alpha\mathrm{M}^2}{\Pi} = 0 \tag{40}$$

$$W_4 = \frac{-\alpha\cos\phi}{\Pi\left\{\pi_1^3 + \left(\frac{\alpha}{\Pi} + \mathrm{Rv}\right)\pi_1^2 + \left(\frac{\mathrm{Rv}\alpha}{\Pi} - \alpha\kappa - \mathrm{M}^2\right)\pi - \frac{\alpha}{\Pi}\mathrm{M}^2\right\}} \tag{41}$$

$$W_1 = -W_4\left[\frac{1 + \frac{\Pi}{\alpha}\pi_1}{1 + \frac{\Pi}{\alpha}\Omega_1}\right] \tag{42}$$

Viscous Particle Phase Solutions ($\beta \neq 0$). The solutions of the governing equations under the viscous particle phase assumption can be shown to be:

$$\theta = D_1\Gamma_1\exp(I_1Y) + D_2\Gamma_2\exp(I_2Y) \tag{43}$$

$$\theta_p = D_1\exp(I_1Y) + D_2\exp(I_2Y) \tag{44}$$

$$U = L_1P_1\exp(\Lambda_1Y) + L_2P_2\exp(\Lambda_2Y) + L_3P_5\exp(I_1Y) + L_4P_6\exp(I_2Y) \tag{45}$$

$$U_p = P_1\exp(\Lambda_1Y) + P_1\exp(\Lambda_2Y) + P_5\exp(I_1Y) + P_6\exp(I_2Y) - \frac{H}{\alpha}\cos\phi \tag{46}$$

$$\mathrm{Nu} = -(D_1\Gamma_1I_1 + D_2\Gamma_2I_2) \tag{47}$$

$$C_f = 2(P_1L_1\Lambda_1 + P_2L_2\Lambda_2 + P_5L_3I_1 + P_6L_4I_2) \tag{48}$$

$$C_{fp} = 2\kappa\beta(P_1\Lambda_1 + P_2\Lambda_2 + P_5I_1 + P_6I_2) \tag{49}$$

where I_1 and I_2 are the solution of the following quartic equation such that I_1 and I_2 are the only two negative roots (all roots must be real).

$$I^4 - \left[\frac{\mathrm{Pr}_p\Pi - \beta\mathrm{RvPr}}{\beta}\right]I^3 - \left[\frac{\varepsilon\mathrm{Pr}_p + \mathrm{PvPrPr}_p\Pi + \gamma\kappa\beta\varepsilon\mathrm{Pr} - \beta\mathrm{Pr}S}{\beta}\right]I^2$$

$$-\frac{\varepsilon\mathrm{Pr}_p}{\beta}\left[\mathrm{PrRv} - \gamma\kappa\mathrm{Pr}\Pi + \frac{\Pi}{\varepsilon}\mathrm{Pr}S\right]I - \frac{\varepsilon}{\beta}\mathrm{PrPr}_pS = 0 \tag{50}$$

Λ_1 and Λ_2 are the solution of the following quartic equation such that Λ_1 and Λ_2 are the only two negative roots (all roots must be real).

$$\Lambda^4 + \left[\mathrm{Rv} - \frac{\Pi}{\beta}\right]\Lambda^3 - \frac{\alpha}{\beta}\left[1 + \frac{\mathrm{Rv}\Pi}{\alpha} + \kappa\beta + \frac{\beta}{\alpha}\mathrm{M}^2\right]\Lambda^2$$

$$-\frac{\alpha}{\beta}\left[\mathrm{Rv} - \kappa\Pi - \frac{\Pi}{\alpha}\mathrm{M}^2\right]\Lambda + \frac{\alpha}{\beta}\mathrm{M}^2 = 0 \tag{51}$$

$$\Gamma_1 = 1 + \frac{\Pi}{\varepsilon}I_1 - \frac{\beta}{\varepsilon\mathrm{Pr}_p}I_1^2 \qquad \Gamma_2 = 1 + \frac{\Pi}{\varepsilon}I_2 - \frac{\beta}{\varepsilon\mathrm{Pr}_p}I_2^2 \tag{52}$$

$$D_1 = \frac{\frac{\beta}{\mathrm{Pr}_p}I_2^2 - \Pi I_2}{\Pi(I_1 - I_2) + \frac{\beta}{\mathrm{Pr}_p}(I_2^2 - I_1^2)} \tag{53}$$

$$D_2 = \frac{\Pi I_1 - \frac{\beta}{\mathrm{Pr}_p} I_1^2}{\Pi(I_1 - I_2) + \frac{\beta}{\mathrm{Pr}_p}(I_2^2 - I_1^2)} \tag{54}$$

$$P_5 = \frac{\alpha D_1 \Gamma_1 \cos\phi}{\beta\left\{I_1^4 + \left(\mathrm{Rv} - \frac{\Pi}{\beta}\right)I_1^3 - \frac{\alpha}{\beta}\left(1 + \frac{\mathrm{Rv}\Pi}{\alpha} + \kappa\beta + \frac{\beta}{\alpha}\mathrm{M}^2\right)I_1^2 - \frac{\alpha}{\beta}\left(\mathrm{Rv} - \kappa\Pi - \frac{\Pi}{\alpha}\mathrm{M}^2\right)I_1 + \frac{\alpha}{\beta}\mathrm{M}^2\right\}} \tag{55}$$

$$P_6 = \frac{\alpha D_2 \Gamma_2 \cos\phi}{\beta\left\{I_2^4 + \left(\mathrm{Rv} - \frac{\Pi}{\beta}\right)I_2^3 - \frac{\alpha}{\beta}\left(1 + \frac{\mathrm{Rv}\Pi}{\alpha} + \kappa\beta + \frac{\beta}{\alpha}\mathrm{M}^2\right)I_2^2 - \frac{\alpha}{\beta}\left(\mathrm{Rv} - \kappa\Pi - \frac{\Pi}{\alpha}\mathrm{M}^2\right)I_2 + \frac{\alpha}{\beta}\mathrm{M}^2\right\}} \tag{56}$$

$$L_1 = 1 - \frac{\beta}{\alpha}\Lambda_1^2 + \frac{\Pi}{\alpha}\Lambda_1 \qquad L_2 = 1 - \frac{\beta}{\alpha}\Lambda_2^2 + \frac{\Pi}{\alpha}\Lambda_2 \tag{57}$$

$$L_3 = 1 - \frac{\beta}{\alpha}I_1^2 + \frac{\Pi}{\alpha}I_1 \qquad L_4 = 1 - \frac{\beta}{\alpha}I_2^2 + \frac{\Pi}{\alpha}I_2 \tag{58}$$

If $U_p(0) = U_{p0}$

$$P_1 = \frac{L_2\left(U_{p0} - P_5 - P_6 + \frac{H}{\alpha}\cos\phi\right) + P_5 L_3 + P_6 L_4}{L_2 - L_1} \tag{59}$$

$$P_2 = \frac{L_1\left(U_{p0} - P_5 - P_6 + \frac{H}{\alpha}\cos\phi\right) + P_5 L_3 + P_6 L_4}{L_1 - L_2} \tag{60}$$

If $U_p(0) = \omega \left.\frac{dU_p}{dY}\right|_{Y=0}$

$$P_1 = \frac{Z_2(P_5 L_3 + P_6 L_4) + L_2\left(\frac{H}{\alpha}\cos\phi - P_5 Z_3 - P_6 Z_4\right)}{L_2 Z_1 - L_1 Z_2} \tag{61}$$

$$P_2 = \frac{Z_1(P_5 L_3 + P_6 L_4) + L_1\left(\frac{H}{\alpha}\cos\phi - P_5 Z_3 - P_6 Z_4\right)}{L_1 Z_2 - L_2 Z_1} \tag{62}$$

where

$$Z_1 = 1 - \omega\Lambda_1 \qquad Z_2 = 1 - \omega\Lambda_2 \tag{63}$$

$$Z_3 = 1 - \omega I_1 \qquad Z_4 = 1 - \omega I_2 \tag{64}$$

It should be mentioned that if M, S and ϕ are all formally equated to zero, all of the analytical results reported herein reduce to the results reported earlier by Chamkha and Ramadan (1998). In addition, as $\beta \rightarrow 0$, the analytical solutions for the viscous particle-phase case given by Eq. (43) through (64) will approach the corresponding solutions for the inviscid case given by Eqs. (33) through (42).

Results and Discussion

The analytical solutions listed above show one very important limitation. It manifests itself clearly in Eq. (34). To analyze Eq. (34), assume that the plate is tilted forward in an angle ϕ. Consider first the term between brackets. It is required for the solution formulas above that the resultant root, π_1, to be negative. If the value of V_p (namely Γ) is allowed to be positive, the quantity between brackets in Eq. (34) will be less than one (since ε is always positive). The required coefficient is the inverse of this quantity, which will be greater than one. If θ_p is now evaluated at the wall (Y = 0), the result will be that θ_p is greater than one, an impossible physical situation.

To state the limitation explained above, the value of Γ must always be negative or at maximum equal to zero. Stated differently, the value of the fluid suction at the wall, Rv, must be equal to or greater than the gravity resultant velocity of V_p. This limitation applies only if the plate is tilted forward, and seizes to exist if the plate is vertical or tilted backward.

To understand why this limitation exists, it is beneficial to visualize the physical orientation of the plate. If the plate is tilted forward, it will serve as a natural obstacle for the particles coming directly from above. Furthermore, the particles are also continuously sucked along with the fluid through the plate's permeable wall. If, in addition, the value of Γ (the velocity component of the

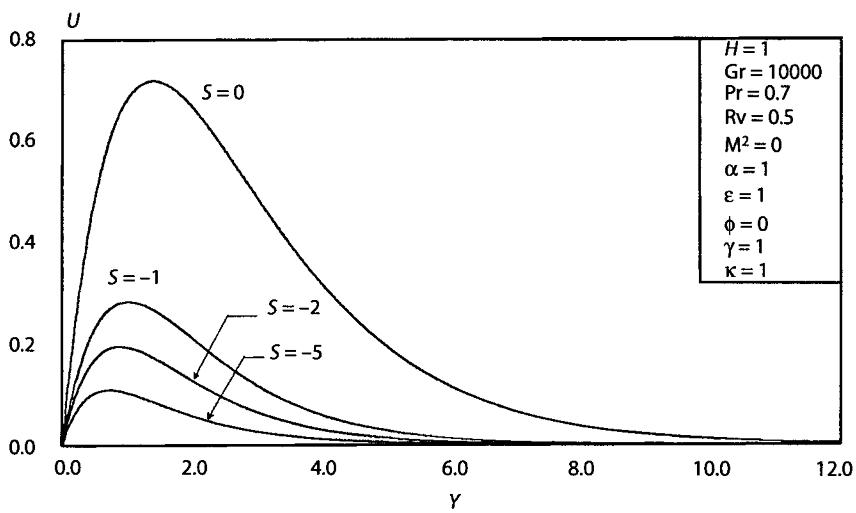

Fig. 3. Effect of heat absorption on the fluid velocity profiles.

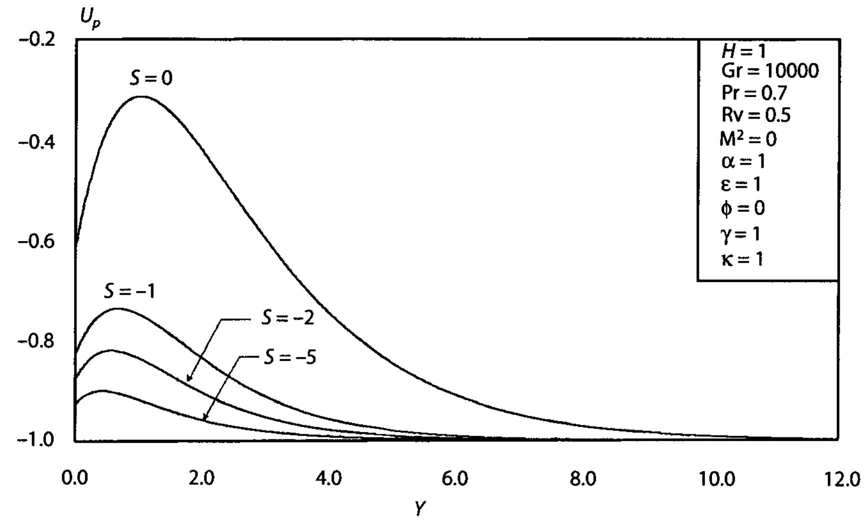

Fig. 4. Effects of heat absorption on the particle-phase velocity profiles.

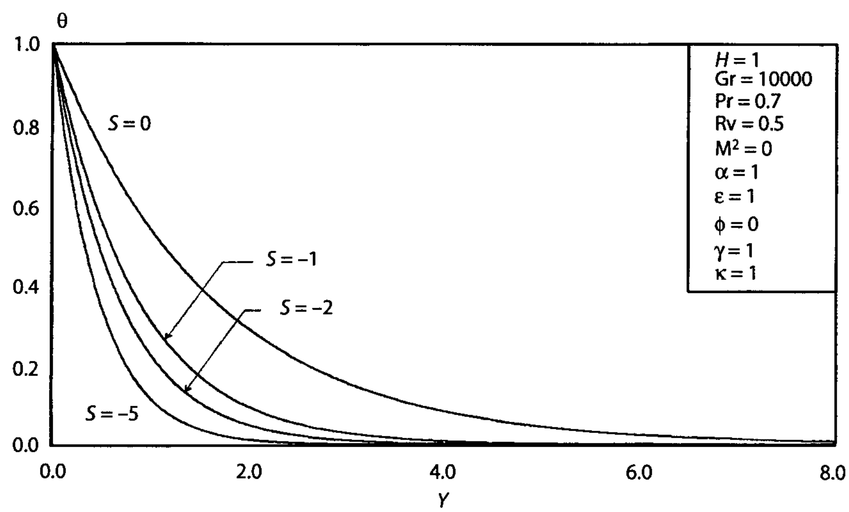

Fig. 5. Effects of heat absorption on the fluid temperature profiles.

particle-phase normal to the plate) is positive, i.e. away from the plate, this means that the area next to the plate will eventually be cleaned of all particles. And most importantly the assumption of constant particle suspension density, which was made in deriving the solutions, can not be sustained anymore. The conclusion is that the negative value of Γ will ensure the flow of particles towards the plate to compensate for the particles that are shaded away by the forward plate tilting, or being sucked through the wall vent's. Thus, this limitation is a direct result of the assumption that was made earlier in deriving the equations and the solutions, namely Q_p = constant = 1.0, and by observing this limitation, the validity of the results is assured.

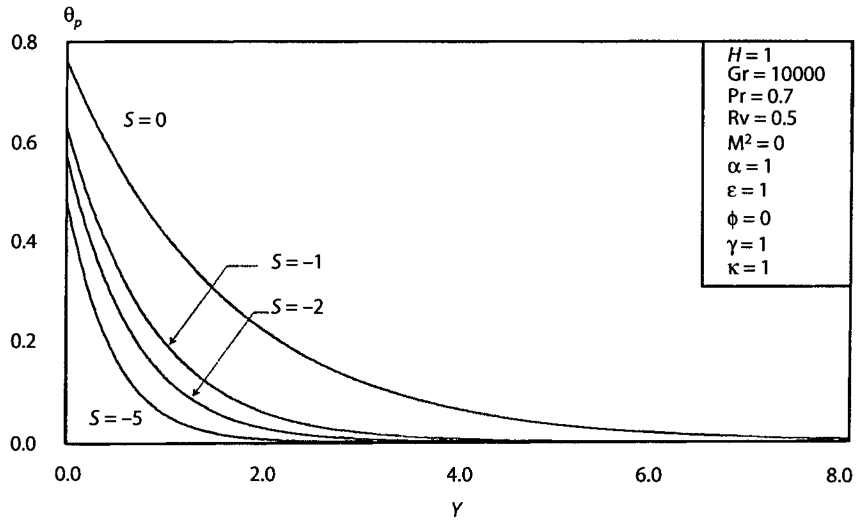

Fig. 6. Effects of heat absorption on the particle-phase temperature profile.

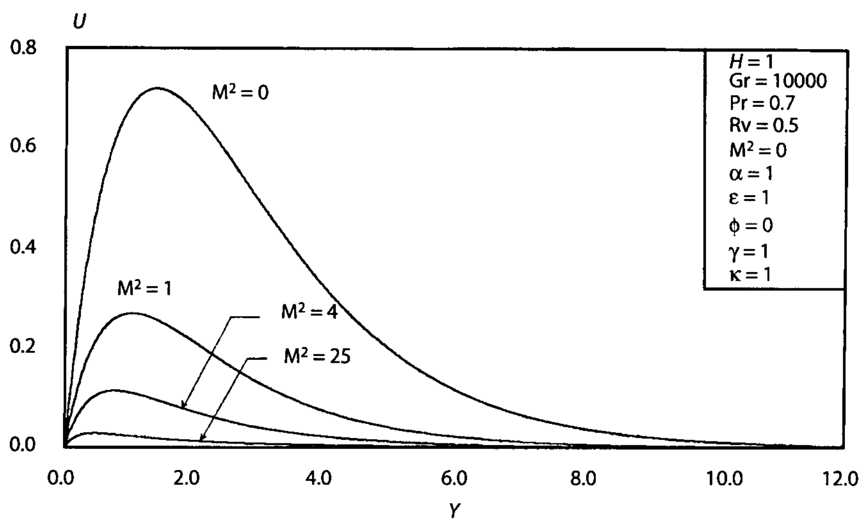

Fig. 7. Effects of Hartmann number on the fluid velocity profiles.

Numerical evaluations of the analytical solutions reported earlier for both the inviscid and viscous particle-phase cases are performed and reported in graphical form in Fig. 3 through Fig. 17. Fig. 3 through Fig. 9 correspond to the case of non-viscous ($\beta = 0$) particle phase while Fig. 10 through Fig. 17 report on the viscous ($\beta \neq 0$) particle-phase case in addition to the inviscid case.

The effects of heat absorption, S, are shown in Fig. 3 through Fig. 6. In general the effect of heat absorption is to damp the flow and the temperature profiles of both phases and to reduce the thicknesses of both the velocity and the thermal layers. This result is expected since, with heat absorbing liquid, less energy is available to enhance the buoyancy effects in the fluid.

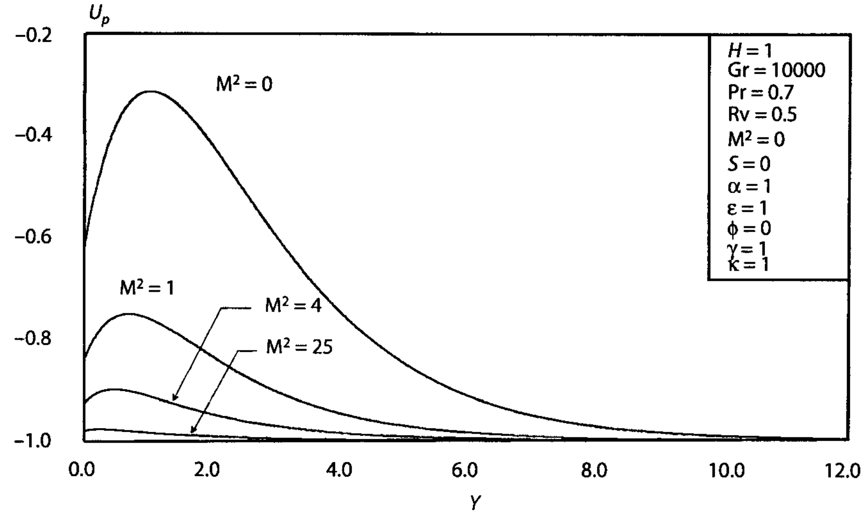

Fig. 8. Effects of Hartmann number on the particle-phase velocity profiles.

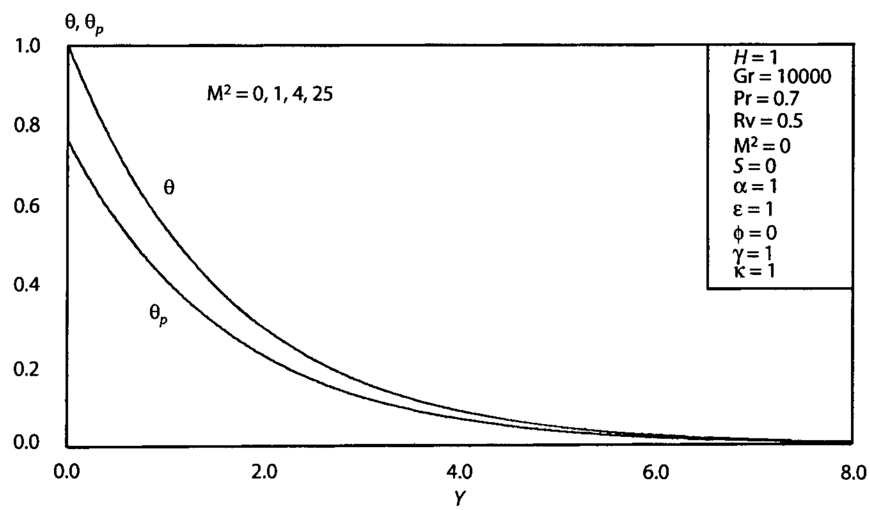

Fig. 9. Effects of Hartmann number on the temperature profiles.

The effects of the square of the Hartmann number, M^2, are shown in Fig. 7 through Fig. 9. Increasing the Hartmann number has also the effect of damping the velocity profiles of both the fluid and particle phases. This is because the application of a transverse magnetic field normal to the flow direction will result in a resistive force similar to the drag force, Lorentz force, which tends to resist the fluid flow and thus reducing its velocity. In the absence of viscous and magnetic dissipation, the Hartmann number has no effect on the thermal layers and the temperature profiles as shown in Fig. 9. Thus, the Hartmann number has no effect on the Nusselt number as it can be concluded from Fig. 9.

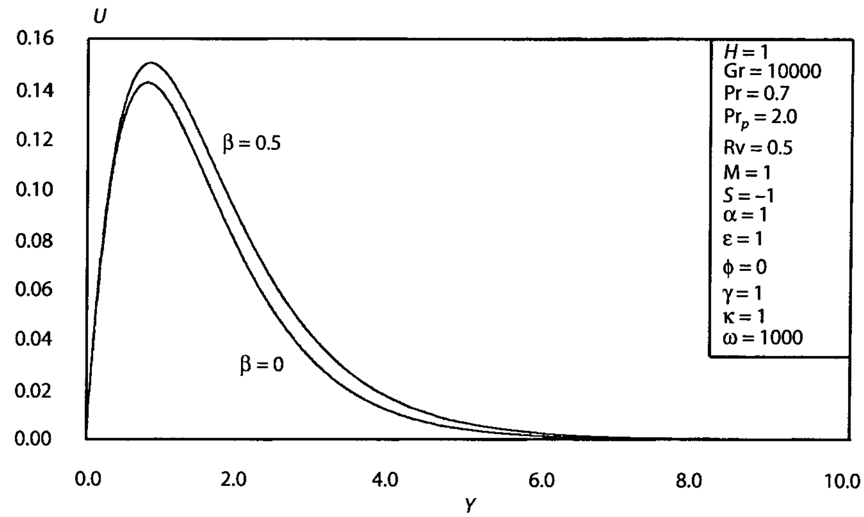

Fig. 10. Effects of β on the fluid velocity profiles.

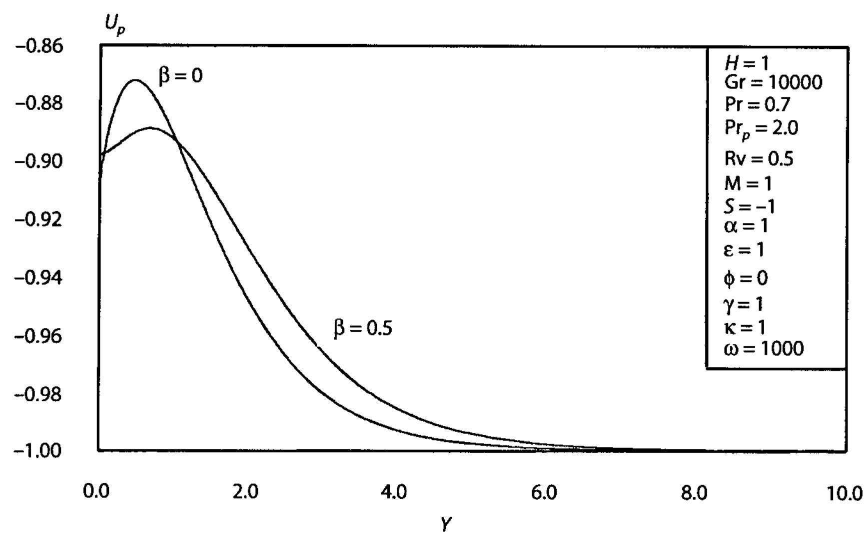

Fig. 11. Effects of β on the particle-phase velocity profiles.

A comparison between the results obtained for inviscid particle phase and those obtained for viscous particle phase is made in Fig. 10 through Fig. 17. Although the intuition suggests that because the particle phase becomes more viscous with the introduction of β, the fluid will face more resistance, i.e. drag, and hence become more damped, however. Figure 10 suggests otherwise. The answer to this lies in the energy Eq. (27). With the introduction of β, the particle-phase heat conduction term appears in the energy Eq. (27). In this case the particles, which are in continuous

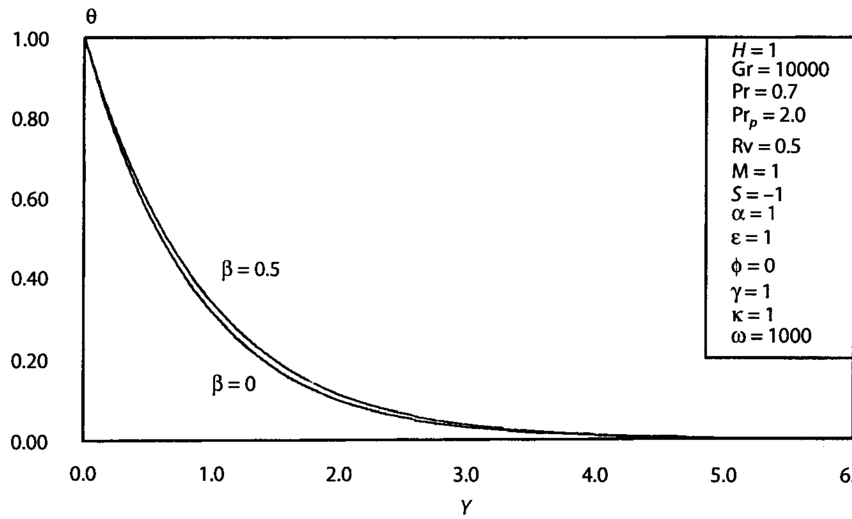

Fig. 12. Effects of β on the fluid temperature profiles.

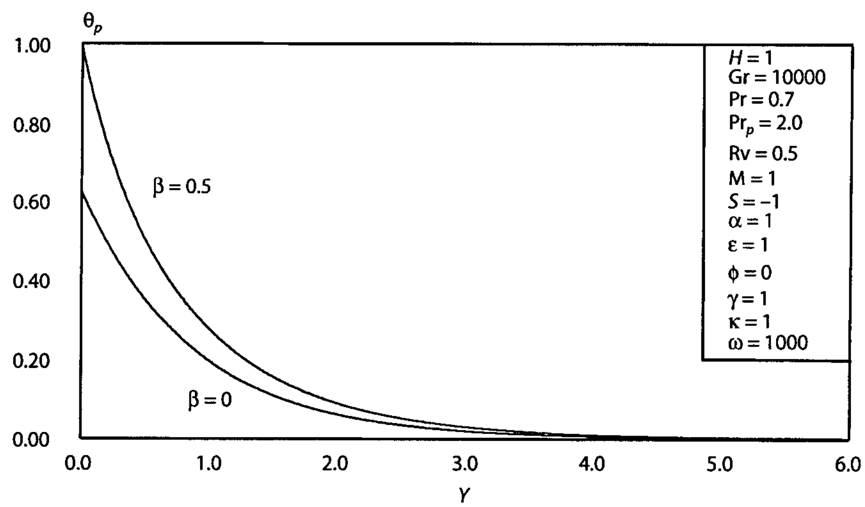

Fig. 13. Effects of β on the particle-phase temperature profiles.

interaction with each other and with the fluid, will transfer heat more efficiently with other particles and also with the fluid due to the heat conduction influence between the particles themselves. This helps to enhance the buoyancy effect in the fluid which overcomes the extra drag which is introduced with the finite value of β. This conclusion is supported by Fig. 12 and Fig. 13, which show clearly the enhancement of temperature profiles with the presence of β. The particle-phase velocity profiles in Fig. 11 show the damping effect which β has on the particle phase of the flow.

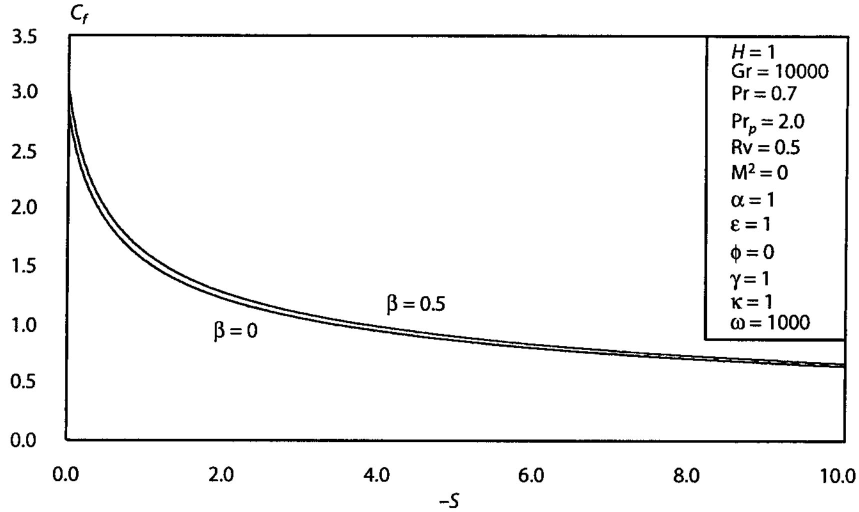

Fig. 14. Effects of heat absorption and β on the fluid-phase coefficient of friction.

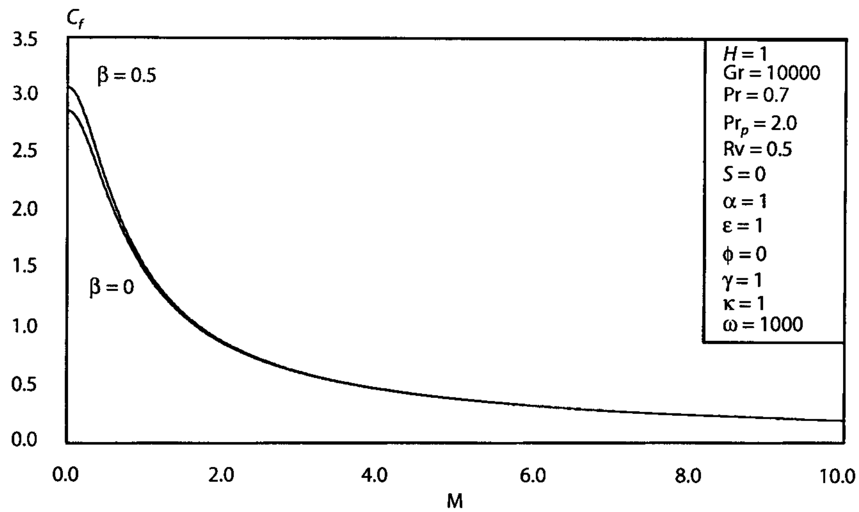

Fig. 15. Effects of Hartmann number and β on the fluid-phase coefficient of friction.

The effects of the particle-phase viscosity, heat absorption and the magnetic field on the coefficients of friction of both the fluid and the particle phases are shown in Fig. 14 through Fig. 16, respectively. The reduction of C_f with increases in both the heat generation and the magnetic field effects is due to the damping effect that they have on the flow. In addition, the increases in the values of C_f as β increases is due to the increase in the slope of the fluid-phase velocity at the wall as is clear from Fig. 10. Figure 16 shows that C_{fp} increases as either M or the absolute value of S increases.

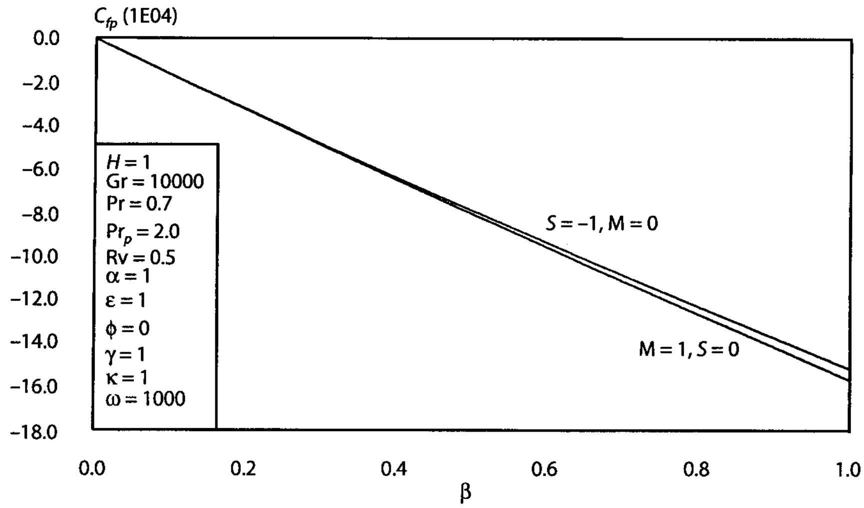

Fig. 16. Effects of heat absorption, Hartmann number and β on the particle-phase coefficient of friction.

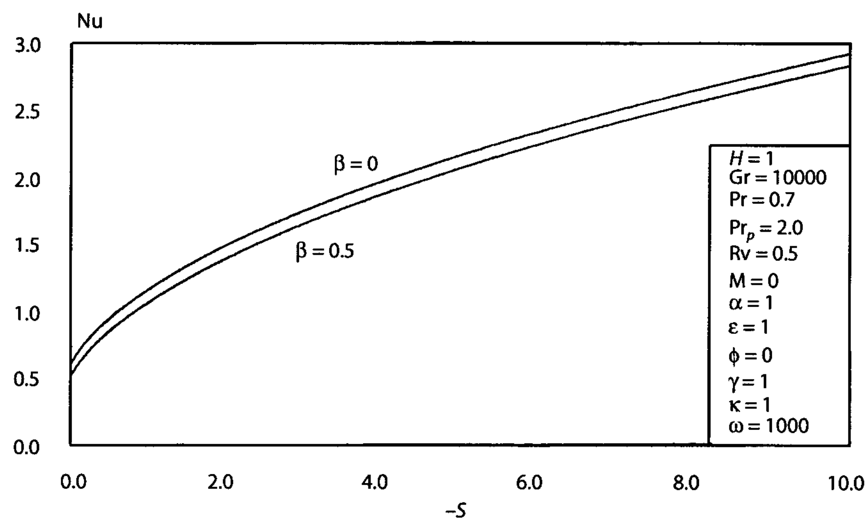

Fig. 17. Effects of heat absorption and β on Nusselt number.

Finally, the effects of heat absorption and particle-phase viscosity on Nusselt number is shown in Fig. 17. The Nusselt number increases with increases in the heat absorption coefficient. This is due to the fact that as the heat absorption increases, the plate tend to supply more energy through its wall to maintain the constant temperature of the wall, and hence increases the Nusselt number. It is also seen from this figure that Nu decreases as β increases. This is due to the particle heat conduction effect which is introduced with the introduction of β as explained earlier. This tends to reduce the slope of the fluid temperature profile at the plate surface predicted as β increases.

Conclusion

The mathematical modelling for free convection of a particulate suspension over an infinite, inclined, permeable and isothermal plate with magnetic field and fluid heat absorption effects was studied. The model accounts for particle-phase viscous effects and fluid buoyancy effects. The particle-phase density distribution was assumed to be uniform across the domain of interest. Analytical solutions were developed for the variables of interest for both inviscid and viscous particle-phase situations. It was found that the introduction of the particle-phase viscosity enhanced the buoyancy effects due to the particle-particle heat conduction across the domain of interest. The general effects of the magnetic field and the heat absorbing fluid was found to damp the flow and the thermal profiles. The Nusselt number however, was found to be enhanced with the fluid heat absorption effects. The coefficients of friction for both phases were found to decrease with increases in the magnetic field and/or the fluid heat absorption effects.

NOMENCLATURE

B_0	magnetic induction
c	fluid-phase specific heat
C_f	fluid-phase skin coefficient of friction
D	diffusion coefficient
g	gravitational acceleration
Gr	Grashof number
H	dimensionless gravitational acceleration
k	fluid-phase thermal conductivity
L	characteristic length
M	Hartmann number
N	interphase momentum transfer coefficient
N_T	interphase heat transfer coefficient
Nu	Nusselt number
P	fluid-phase pressure
Pr	fluid-phase Prandtl number
q_0	heat absorption coefficient
Q_p	dimensionless particle-phase density
Rv	fluid-phase wall suction velocity
S	dimensionless heat absorption coefficient
Sc	inverse Schmidt number
T	fluid-phase temperature
u	fluid-phase x-component of velocity
U	fluid-phase dimensionless tangential velocity
v	fluid-phase y-component of velocity
V	fluid-phase dimensionless normal velocity
x, y	Cartesian coordinates
Y	dimensionless normal distance

Symbols

α	velocity inverse Stokes number
β	particle-phase to fluid viscosity ratio

$\bar{\beta}$ volume expansion coefficient
γ specific heats ratio
ε thermal inverse Stokes number
θ fluid-phase dimensionless temperature
κ particle loading
μ fluid-phase dynamic viscosity
ν fluid-phase kinematic viscosity
ρ fluid-phase density
σ_0 fluid-phase electrical conductivity
ϕ tilt angle
ω particle-phase dimensionless wall slip coefficient
ω_s particle-phase dimensional wall slip coefficient

Subscripts

p particle phase
w plate wall
∞ very large distance away from the plate surface (ambient condition)

REFERENCES

Bejan, A., 1995, *Convection Heat Transfer*, 2nd edition, John Wiley and Sons, Inc.

Chamkha, A. J. and Ramadan, H. M., 1997, Analytical Solutions for Free Convection of a Particulate Suspension Past an Infinite Vertical Surface, *Int. J. Engng. Science*, **36**, pp. 49–60.

Chamkha, A. J., 1996a, Non-Darcy Hydromagnetic Free Convection From a Cone and a Wedge in Porous Media, *Int. Commun. Heat Mass Transfer*, **23**, pp. 875–887.

Chamkha, A. J., 1996b, Compressible Dusty-Gas Boundary-Layer Flow over a Flat Surface, *ASME Journal of Fluids Engineering*, **118**, pp. 179–185.

Drew, D. A. and Segal, L. A., 1971, Analysis of Fluidized Beds and Foams Using Averaged Equations, *Studies in Applied Mathematics*, **50**, pp. 233–252.

Drew, D. A., 1983, Mathematical Modeling of Two-Phase Flow, *Annual Review of Fluid Mechanics*, **15**, pp. 261–291.

Gadiraju, M., Peddieson, J., and Munukutia, S., 1992, Exact Solutions for Two-Phase Vertical Pipe Flow, *Mechanics Research Communications*, **19**, pp. 7–13.

Garandet, J. P., Alboussiere, T., and Moreau, R., 1992, Buoyancy Driven Convection in a Rectangular Enclosure with a Transverse Magnetic Field, *Int. J. Heat Mass Transfer*, **35**, pp. 741–749.

Gebhart, B., Jaluria, Y., Mahajan, R. L., and Sammakia, B., 1988, Buoyancy – Induced Flows and Transport, *Hemisphere Publishing Corporation*, New York.

Hossain, M. A., 1992, Viscous and Joule Heating Effect on MHD-Free Convection with Variable Plate Temperature, *Int. J. Heat Mass Transfer*, **35**, pp. 3485–3487.

Marble, F. E., 1970, Dynamics of Dusty Gases, *Annual Review of Fluid Mechanics*, **2**, pp. 297–446.

Moreau, R., 1990, Magnetohydrodynamics, *Kluwer Academic Publishers.*

Ramadan, H. M. and Chamkha, A. J., 1999, Two-Phase Free Convection Flow over an Infinite Permeable Inclined Plate with Non-Uniform Particle-Phase Density, *Int. J. Engng. Science*, **37**, pp. 1351–1367.

Ramadan, H. M., 1997, *Free Convection Flow of a Particulate Suspension over an Inclined Isothermal Surface*, *M. S. Thesis*, Kuwait University.

Raptis, A. and Singh A. K., 1983, MHD Free Convection Flow Past an Accelerated Vertical Plate, *Int. Commun. Heat Mass Transfer*, **10**, pp. 313–321.

Riley, N., 1964, Magnetohydrodynamics Free Convection, *Fluid Mech.*, **18**, pp. 577–586.

Sacheti, N. C., Chandran, P., and Singh, A. K., 1994, An Exact Solution for Unsteady Magnetohydrodynamics Free Convection Flow with Constant Heat Flux, *Int. Commun. Heat Mass Transfer*, **21**, pp. 131–142.

Soo, S. L., 1989, *Particulates and Continuum Multiphase Fluid Dynamics*, Hemisphere Publishing Corporation.

Sparrow, E. M. and Cess, R. D., 1961, Effect of Magnetic Field on Free Convection Heat Transfer, *Int. J. Heat Mass Transfer*, **3**, pp. 267–274.

Tsuo, Y. P. and Gidaspow, D., 1990, Computation of Flow Patterns in Circulating Fluidized Beds, *AIChE Journal*, **36**, pp. 88–896.

Vajravelu, K. and Nayfeh, J., 1992, Hydromagnetic Convection on a Cone and a Wedge, *Int. Commun. Heat Mass Transfer*, **19**, pp. 701–710.

□◆◆◆□