



THERMAL CONVECTION IN A PARTICLE-LADEN BOUNDARY LAYER FLOW PAST A FLAT PLATE

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Introduction

It has been reported by many investigators (see, for instance, Prahba and Jain [1], and Osipov [2]) that boundary-layer flow of a particle-fluid suspension over a semi-infinite impermeable flat plate exhibits a singular behavior. This is because the particle-phase wall density becomes infinite at some distance downstream of the plate's leading edge. One possible interpretation of this fact would be that the particle phase is becoming so dense there that it can no longer be treated as a fluid. In a previous contribution, Chamkha [3] reported a more refined model and showed that thermophoresis, or correspondingly Brownian diffusion is capable of removing this discontinuous behavior. Also, Chamkha and Peddieson [4] showed that this singularity is a feature of the original dusty-gas model (a model for the continuum description of suspensions having small volume fraction, see Marble [5]) by the imposition of fluid-phase wall suction. It should be pointed out that when the dusty-gas theory discussed by Marble [5] is extended to suspensions having finite volume fraction and applied to the present problem, no discontinuities exist (see for instance Chamkha and Peddieson [6], and Chamkha [7]). It should be mentioned that wall suction is a very practical way of delaying boundary-layer transition to turbulence and has been applied in the aerospace industry. It is of interest in this paper to study the thermal boundary-layer response of a two-phase particulate suspension flow past a semi-infinite permeable plate using the hydrodynamic flow solutions reported by Chamkha and Peddieson [4]. The fluid phase is assumed incompressible and has constant properties. The particle phase is assumed to consist of small solid spherical particles of uniform size with no mutual collision and the particle-phase volume fraction is assumed to be small.

Governing Equations

Consider the two-dimensional, steady, laminar, boundary-layer flow of a two-phase particulate suspension past a semi-infinite permeable flat plate. The plate is coincident with the

half plane $y=0$, $x \geq 0$ and the flow far from the plate is a uniform stream in the x direction parallel to the plate with both phases in equilibrium.

The governing equations for this investigation are based on the description of both phases as two interacting continua (through drag and heat transfer) obeying separate balance laws of mass, linear momentum, and energy (see, for instance Marble [5], and Soo [8]). Taking into account the assumptions made earlier, the boundary-layer form of the equations of motion are given by

$$\begin{aligned} \partial_x u + \partial_y v &= 0, \quad u \partial_x u + v \partial_y u - \nu \partial_{yy} u + (\rho_p / \rho)(u - u_p) / \tau_v = 0 \\ u \partial_x T + v \partial_y T - (k / (\rho c)) \partial_{yy} T + (\rho_p c_p / (\rho c))(T - T_p) / \tau_T \\ &- (\nu / c)(\partial_y u)^2 - (\rho_p / (\rho c))(u_p - u)^2 / \tau_v = 0 \quad (1) \\ \partial_x (\rho_p u_p) + \partial_y (\rho_p v_p) &= 0, \quad u_p \partial_x u_p + v_p \partial_y u_p + (u_p - u) / \tau_v = 0 \\ u_p \partial_x v_p + v_p \partial_y v_p + (v_p - v) / \tau_v &= 0, \quad u_p \partial_x T_p + v_p \partial_y T_p + (T_p - T) / \tau_T = 0 \end{aligned}$$

In Equations (1) x is the tangential coordinate and y is the normal coordinate. ρ , ν , k , c , u , v , and T are the fluid-phase density, kinematic viscosity, thermal conductivity, specific heat at constant pressure, tangential velocity, normal velocity, and temperature, respectively. ρ_p , c_p , u_p , v_p , and T_p are the particulate-phase in-suspension density, specific heat, tangential velocity, normal velocity, and temperature, respectively. $\tau_v (= (\rho_s d^2) / (18 \rho \nu))$ (where ρ_s is the density of particulate material and d is the particle diameter), and $\tau_T (= (\rho_s d^2 c_p) / (12 k))$ are the respective velocity and temperature relaxation times. The reader is reminded that Equations (1) are only applicable for dilute particle concentrations (small particulate volume fraction in the suspension) in the sense that no inter-particle interactions exist (see, for instance Apazidis [9]). That is, the partial pressure of particles is assumed negligible and the particle phase is assumed stress free. The particles are simply dragged along by the fluid motion through momentum exchange mechanism.

It is convenient to nondimensionalize the governing equations and transform the tangential coordinate from semi-infinite ($0 \leq x < \infty$) to finite ($0 \leq \xi \leq 1$). This is accomplished by introducing

$$x = V_\infty \tau_v \xi / (1 - \xi), \quad y = (2\nu \tau_v \xi / (1 - \xi))^{1/2} \eta$$

$$u = V_\infty F(\xi, \eta), \quad v = (\nu(1 - \xi) / (2\tau_v \xi))^{1/2} (G(\xi, \eta) + \eta F(\xi, \eta)) \tag{2}$$

$$u_p = V_\infty F_p(\xi, \eta), \quad v_p = (\nu(1 - \xi) / (2\tau_v \xi))^{1/2} (G_p(\xi, \eta) + \eta F_p(\xi, \eta))$$

$$T = T_\infty H(\xi, \eta), \quad T_p = T_\infty H_p(\xi, \eta), \quad \rho_p = \rho_{p\infty} Q_p(\xi, \eta)$$

where V_∞ , $\rho_{p\infty}$, and T_∞ are the free-stream velocity, particle-phase density, and temperature, respectively. It should be pointed out that an advantage for using the above modified Blasius transformations is that they eliminate singularities associated with the plate's leading edge.

Substitution of Equations (2) into Equations (1) and rearranging yield

$$\begin{aligned} \partial_\eta G + F + 2\xi(1 - \xi)\partial_\xi F &= 0, \quad \partial_\eta(Q_p G_p) + Q_p F_p + 2\xi(1 - \xi)\partial_\xi(Q_p F_p) = 0 \\ G_p \partial_\eta G_p - \eta F_p^2 + 2\xi(1 - \xi)F_p \partial_\xi G_p + 2\xi(G_p - G)/(1 - \xi) &= 0 \\ \partial_{\eta\eta} F - G \partial_\eta F - 2\xi(1 - \xi)F \partial_\xi F + 2\kappa \xi Q_p (F_p - F)/(1 - \xi) &= 0 \\ G_p \partial_\eta F_p + 2\xi(1 - \xi)F_p \partial_\xi F_p + 2\xi(F_p - F)/(1 - \xi) &= 0 \tag{3} \\ \partial_{\eta\eta} H - P_r G \partial_\eta H + P_r E_c (\partial_\eta F)^2 - 2\xi(1 - \xi)P_r F \partial_\xi H \\ + 2\xi \kappa P_r Q_p (\gamma \epsilon (H_p - H) + E_c (F_p - F)^2)/(1 - \xi) &= 0 \\ G_p \partial_\eta H_p + 2\xi(1 - \xi)F_p \partial_\xi H_p + 2\xi \epsilon (H_p - H)/(1 - \xi) &= 0 \end{aligned}$$

where

$$\kappa = \rho_{p\infty} / \rho, \quad P_r = \rho \nu c / k, \quad E_c = V_\infty^2 / (c T_\infty), \quad \gamma = c_p / c, \quad \epsilon = \tau_v / \tau_T. \tag{4}$$

are the particle loading, fluid-phase Prandtl number, Eckert number, specific heat ratio, and the ratio of the momentum relaxation time to the temperature relaxation time, respectively. It should be recalled that τ_v is the time necessary for the relative velocity between the two phases to reduce e^{-1} of its original value, and τ_T is the time that it takes for the temperature difference between the two phases to decrease e^{-1} of its initial value. Therefore, the ratio of these parameters is of a significant physical importance. It is a measure of the rate at which the velocity and thermal effects transit from frozen conditions at $\xi = 0$ to equilibrium conditions at $\xi = 1$.

The corresponding boundary and matching conditions for this problem are

$$\begin{aligned} F(\xi, 0) = 0, \quad G(\xi, 0) = -G_w, \quad H(\xi, 0) = H_w, \quad G_p(\xi, \infty) \rightarrow G(\xi, \infty) \quad Q_p(\xi, \infty) \rightarrow 1 \\ F(\xi, \infty) \rightarrow 1, \quad F_p(\xi, \infty) \rightarrow 1, \quad H(\xi, \infty) \rightarrow 1, \quad H_p(\xi, \infty) \rightarrow 1 \end{aligned} \tag{5}$$

where G_w and H_w are constants. In Equations (5), the second and the third equations allow, respectively, for fluid-phase suction and constant surface temperature. Obviously, the suction distribution (which is assumed constant) is unrealistic, being proportional to $x^{-1/2}$. It is, nevertheless, useful for the purpose of eliminating the singular behavior in the particle-phase density mentioned earlier. In addition, it should be noted that solutions obtained in this way represent generalizations to two-phase flow of single-phase Blasius solutions with suction.

The dimensionless wall heat flux for flat-plate boundary layer with constant wall temperature is an important thermal parameter. It is defined as follows

$$\dot{q}_w(\xi) = -\partial_\eta H(\xi, 0)/(P_r E_c). \quad (6)$$

Results

Due to the highly nonlinear nature of Equations (3), no similarity solutions (like the case of single-phase flow) appear to be possible, and therefore, a numerical solution is required. An important feature of the governing equations is that they exhibit a hyperbolic-parabolic behavior. This allows the problem to be solved as an initial-value problem with ξ playing the role of time.

Numerical solutions of Equations (3) subject to Equations (5) were obtained using an implicit finite-difference method similar to the one described by Blottner [9]. A 1000 X 196 grid (in ξ and η , respectively) was utilized. Constant step sizes in ξ ($\Delta\xi=0.001$) and variable step sizes in η (with the smallest step size ($\Delta\eta_1=0.001$) adjacent to the plate surface where significant variations from uniformity are expected) were employed. A growth factor of 1.03 was used in the η direction. It should be mentioned that the step sizes used in the present work were chosen as a result of many numerical tests (by reducing the step sizes in both directions) which were performed to insure grid independence. All derivatives with respect to ξ were represented by two-point backward difference quotients. Derivatives with respect to η in the second-order equations in η were represented by three-point central difference quotients while η differencing in the first-order equations in η was accomplished by the trapezoidal rule. In Equations (3), the third and fifth equations evaluated at $\eta=0$ were used in place of boundary conditions at the wall. The solution was obtained line by line starting at $\xi=0$ and marching downstream toward $\xi=1$. It should be pointed out that as a result of using Equations (2), exact equations were solved at

$\xi=0$ instead of assuming initial profiles for the flow variables as usually done when working with the original untransformed variables. Because the governing equations are nonlinear, iteration was used at each line of constant ξ . As a result, a tri-diagonal matrix of linear algebraic equations was created and solved numerically by the Thomas' algorithm as discussed by Blottner [10]. The iteration process was continued until convergence of the desired solution occurred within certain acceptable limits (when the difference between the current and the previous iteration is 10^{-5} in this case). Since Chamkha and Peddieson [4] reported solutions for the flow fields, attention will be focused herein on the temperature distributions and wall heat transfer.

Figures 1 and 2 show typical temperature profiles for the fluid and particulate phases at various locations along the plate, respectively. These figures illustrate the complete transition from a thermally frozen state (no interphase energy transfer) at $\xi=0$ to a thermal equilibrium state (complete interphase energy transfer) at $\xi=1$.

As mentioned before, in the case of an impermeable plate, there is a critical point ($\xi=0.5$) along the plate where the particles tend to accumulate at the surface and the density of the particles becomes infinite. This occurs as the particle-phase tangential velocity at the wall $F_p(\xi,0)$ vanishes and reaches equilibrium with the fluid phase at the wall. It is believed that a connection between the vanishing of $F_p(\xi,0)$ and the singularity in the wall particle-phase density $Q_p(\xi,0)$. The imposition of suction caused $F_p(\xi,0)$ to be finite in the vicinity of $\xi=0.5$ which prevented $Q_p(\xi,0)$ from becoming very large. This explains the peaks in figure 3 which depicts the influence of the particle loading κ on $Q_p(\xi,0)$ along the axial position ξ . Increasing κ has the tendency to increase the uniformity in the fluid-phase tangential velocity profiles and thus, through the drag mechanism, the uniformity in the particle-phase tangential velocity profiles. This causes smoothness in the profiles of $F_p(\xi,0)$ (not shown herein for brevity). It is observed that the highest peak value of $Q_p(\xi,0)$ corresponds to the least smooth profile of $F_p(\xi,0)$ (smallest value of κ). This tends to support the conjecture made earlier that the singularity in $Q_p(\xi,0)$ predicted for the impermeable plate was associated with the vanishing of $F_p(\xi,0)$.

Figure 4 shows the variations of the wall heat transfer rate \dot{q}_w with the particle loading κ and the suction parameter G_w . As κ increases, the thermal interaction between the two phases (in which the fluid gains some kinetic and thermal energy from the particles) increases and the thermal boundary layer decreases causing the wall heat transfer to increase. It can also be

observed that \hat{q}_w increases to a peak for finite values of κ and then decreases far downstream where complete thermal equilibrium occurs at $\xi=1$. The reason for this behavior is that in the initial stages of the flow, the relative velocity between the phases is high. This causes high temperature differences which result in high temperature gradients in the fluid phase at the plate's surface. However, as the flow continues further downstream, the relative velocity and the temperature difference between the phases decrease which results in lower fluid-phase wall temperature gradients. Figure 4 also indicates that \hat{q}_w increases as G_w increases as expected. In viewing the results shown in figure 4, the reader should be aware of the fact that while the heat flux remains bounded when the transformed axial distance ξ approaches zero, it is actually unbounded in terms of the original untransformed variables. This is a consequence of employing the modified Blasius transformations which eliminate the plate's leading edge related singularities.

Conclusion

The problem of thermal convection in a particle-laden laminar boundary-layer flow past a semi-infinite permeable flat plate is solved numerically using an implicit tri-diagonal finite-difference technique. Graphical results for the temperature profiles, particle-phase wall density distribution, and the wall heat flux are presented to illustrate the influence of the particle loading and the wall suction on the solutions. A continuous non-singular behavior is predicted when fluid-phase wall suction is imposed. This is true no matter how small a value of G_w is employed (see Chamkha and Peddieson [4]). It is found that increasing the amount of suction has the tendency to augment the wall heat flux. A similar effect is predicted when the particle loading is increased. It should be pointed out that no comparisons with experimental data is possible at present due to the unavailability of such data. An experimental investigation of this problem will be most valuable for evaluating the predictions reported herein and will help verify that the singular behavior in the particle-phase density for the case of no suction is a property of the dusty-gas model and not an artifact of the numerical method. It should be noticed that the small volume fraction of particles assumption inherent in the dusty-gas model utilized herein is at odds with the catastrophic growth in the particle-phase wall density predicted which may suggest that the original dusty-gas model discussed by Marble [5] is inadequate for the solution of the present

problem. Refinements to the model to include Brownian diffusion and particle-phase pressure directly proportional to the particle-phase density were pursued and in both cases continuous singularity-free solutions were predicted (see, for instance Chamkha and Peddieson [11,12] and Chamkha [3]). Evaluation of the accuracy of the different models can only be carried by experimental investigations. It is hoped that the present work stimulates interest in such activities.

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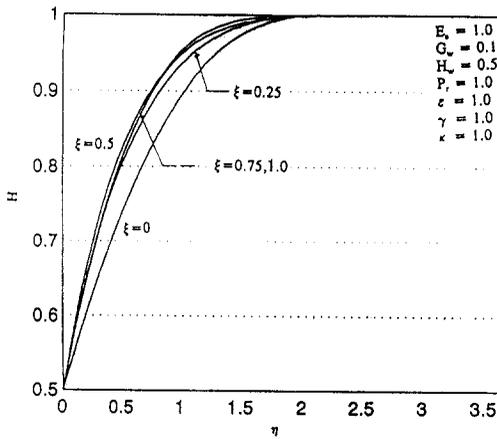


Figure 1. Fluid-Phase Temperature Profiles

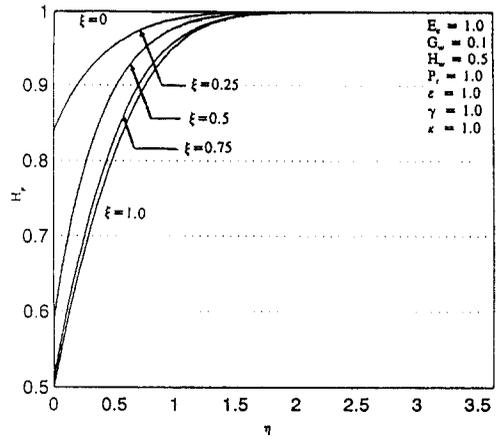


Figure 2. Particle-Phase Temperature Profiles

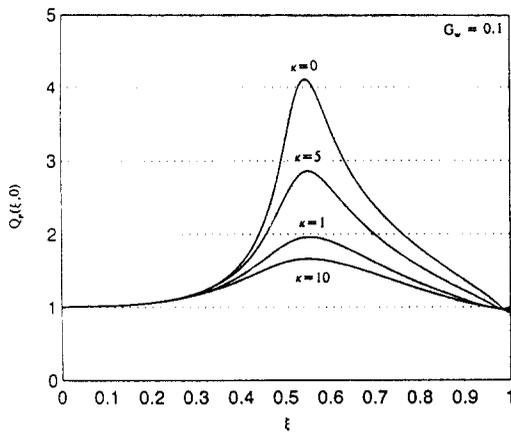


Figure 3. Wall Particle-Phase Density vs. Position

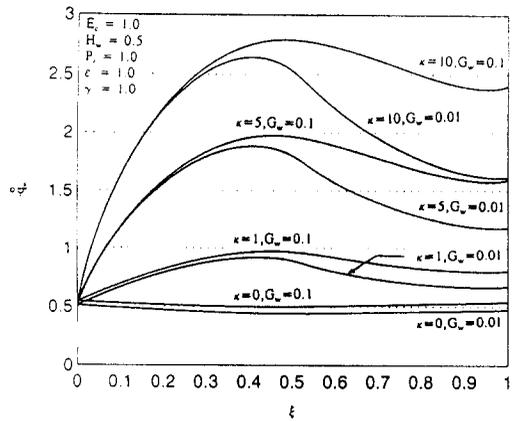


Figure 4. Wall Heat Transfer Coefficient vs. Position