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TRANSIENT POWER–LAW FLUID FLOW IN A POROUS MEDIUM CHANNEL

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Numerical solutions for transient power-law fluid flow in a porous medium with uniform porosity and permeability bounded between two infinitely long parallel permeable plates are obtained using an implicit finite-difference algorithm. The flow is induced through the application of a constant pressure gradient in the flow direction parallel to the channel walls. Uniform injection and extraction conditions are applied at the channel surfaces. Some representative results are presented graphically to illustrate the history and developments of the velocity profiles, volume flow rate, and boundary friction coefficient and to study the influence of the fluid behavior coefficient on these parameters for this type of flow.


INTRODUCTION

The problem in this paper is that of unsteady flow of a non-Newtonian power-law fluid (where the instantaneous shear stress and strain rate exhibit a nonlinear power-law function) in a porous medium channel. The channel consists of two parallel porous flat plates within a porous medium placed. Uniform fluid suction and injection are imposed at the lower and upper walls of the channel, respectively. The flow in the channel is driven by a constant pressure gradient in the X direction (see Fig. 1). Porous media bounded by fixed boundaries occur in many industrial applications ranging from filtration, rheology, fixed-bed catalytic reactors, packed-bed heat exchangers, metal processing, and pharmaceutical industry. Therefore, a better understanding of the flow behavior in such porous geometries is very beneficial in designing systems related to these industries. This is the purpose of the present paper.

Figure 1. Porous Medium Channel Schematic

An examination of the open literature reveals that there have been some work related to the present problem mostly for Newtonian fluids. Khodadadi (1991) reported analytical solutions for unsteady Newtonian fluid flow through a porous medium channel with impermeable walls. Kaviany (1985) analyzed flow and heat transfer of steady flow in a porous medium channel. Khodadadi and Kroll (1992) considered the steady-state version of the present problem for a Newtonian fluid. Chamkha (1992) extended the work of Khodadadi and Kroll (1992) to non-Newtonian fluids. Kapur (1963) considered power-law fluid flow between two parallel plates with uniform suction and injection. White (1991) reported exact solutions for the Newtonian version of Kapur’s problem with different boundary conditions. Wang (1971) analyzed pulsatile flow of a Newtonian fluid in a channel with permeable walls. Rushton (1986) and Bragg (1986) discussed flow and filtration of non-Newtonian fluids. The physical properties of the most of the non-Newtonian fluids depend greatly on temperature. However, for low temperatures these can be treated as constants as done herein. The porous medium is assumed isotropic and incompressible and the flow is assumed incompressible. The channel is assumed to be long so that all physical flow variables (except pressure) will depend on the normal coordinate and time only.

GOVERNING EQUATIONS

The governing equations for this investigation are based on the local volume averaged Navier-Stokes equations modified to incorporate the effects of the presence of the porous matrix (Khodadadi, 1991; Vafai and Tien, 1981) and non-Newtonian effects (Kapur, 1963; Rushton, 1986). For this problem the continuity equation is identically satisfied when both the suction and injection velocities are equal. The y-momentum equation indicates that there is no changes in pressure in the y direction while the x-momentum equation reduce to (Khodadadi and Kroll, 1992)

\[
\frac{\partial u}{\partial t} + \frac{m}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) - V_w \frac{\partial u}{\partial y} + \frac{mE}{\rho K} u^n + \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\left( u^2 \right)}{4K} u \left( u^2 + V_0^2 \right) \sqrt{2} = 0
\]
where $t$ is time, $y$ is the normal distance, $u$ is the velocity in the $x$ direction, $V_m$ is the velocity of the fluid being extracted or injected at the walls, $\beta P/R_s$ is the fluid pressure gradient, $\rho$, $m$ and $n$ are the fluid density, dynamic viscosity-like phenomenological coefficient, and behavior coefficient (or power-law index), respectively; $e$, $K$, and $f$ are the porous medium porosity, permeability, and inertia coefficient, respectively. The behavior coefficient $n$ determines whether the fluid is Newtonian ($n=1$) or non-Newtonian ($n \neq 1$).

The last term in Eq. 1 accounts for the inertial effects which would occur when the Reynolds number based on the pore diameter exceeds a value in the range 1-10. In this case inertial forces become comparable to the viscous forces. The was formulated some time ago by making the pressure drop across the porous medium as a quadratic function of filter face velocity. The initial and boundary conditions for this problem are

$$u(y,0) = V_m, \quad u(0,t) = 0, \quad u(W,t) = 0$$  \hspace{1cm} (2)

where $W$ is the channel width. It should be mentioned that the unsteadiness in this problem is caused by the difference in boundary conditions between the actual no slip conditions at the walls and the slip conditions assumed at $t=0$.

Eqs. 1 and 2 can be made dimensionless by using

$$t = \frac{W}{V_m}, \quad y = \frac{y}{W}, \quad u = V_m \frac{\partial P}{P_x}, \quad \frac{\partial P}{\partial x} = -\frac{\rho V_m^2}{W} G$$  \hspace{1cm} (3)

Substituting Eq. 3 into Eqs. 1 and 2 gives

$$\frac{\partial F}{\partial x} - \frac{R}{\gamma} \left( \frac{\partial F}{\partial y} \right) \frac{\partial F}{\partial \eta} + \frac{\partial F}{\partial \eta} \frac{\partial F}{\partial \eta} = -G$$  \hspace{1cm} (4)

$$F(0,0) = 1, F(0,\xi) = 0, F(1,\xi) = 0$$  \hspace{1cm} (5)

where

$$R = \frac{m V_m^n (\rho W)}{\rho}, \quad \gamma = \left( \frac{m V_m W_{m}^{n-1}}{\rho K} \right)^{1/2}$$

are the blowing inverse Reynolds number, the porous medium shape coefficient, and non-Darcian inertial factor respectively.

The volumetric flow rate in the porous medium channel and boundary friction coefficient at the lower surface are defined in dimensionless form as

$$Q(\xi) = \int_{0}^{\eta} \frac{\partial F}{\partial \eta} d\eta, \quad C(\xi) = \left( \frac{\partial F}{\partial \eta} (0,\xi) \right)^{n}$$  \hspace{1cm} (6)

**RESULTS AND DISCUSSION**

Eqs. 4 and 5 represent an initial value problem which can be solved analytically and, therefore, must be solved numerically. A numerical scheme based on the finite difference methodology (Blottnner, 1970) is chosen for that purpose. The dimensionless time derivative appearing in Eq. 4 is approximated by a two-point backward difference quotient while the space derivatives are discretized using three-point difference quotients. At each line of constant $\xi$, iteration is used to deal with the nonlinear nature of Eq. 4. This procedure corrects the nonlinear equations encountered at different lines of constant $\xi$ to linear tri-diagonal algebraic equations which can be solved by the Thomas’ algorithm (Blottnner, 1970). 200 nodes with a constant step size of 0.005 were used in the $\eta$ direction and 400 nodes with variable step size are used in the $\xi$ direction. An initial time step size $\Delta t_1$ of 0.001 and a growth factor of 1.015 were utilized. These values were chosen after many different numerical tests performed to assess grid independence. The convergence criterion was such that when the difference between the current and the previous iteration reached $10^{-3}$, convergence was achieved and the iteration process was terminated. It should be mentioned that a large number of computations were performed and many results were obtained with no numerical difficulties. For brevity, only a representative set of results will be presented graphically in Figs. 2 through 6 to show the effect of the fluid behavior coefficient $n$ and the porous medium shape coefficient $\gamma$ on the transient solutions.

Fig. 2 presents typical velocity profiles in the porous medium channel at different dimensionless time $\xi$. In this and all subsequent figures, the set parametric values given may or may not be representative of any intended application. They were only chosen to carry out the numerical computations. Initially at $\xi=0$, the fluid velocity exhibits an assumed uniform profile. As time progresses, velocity profiles with distinctive peaks close to the lower wall of the channel are observed. This phenomenon is called channeling and has been observed experimentally by Schertz and Bischoff (1969) (through velocity measurements in packed beds) and discussed extensively by Vafai (1984). This channeling effect is believed to be due to the inertial forces. This was shown by Vafai (1984) by the method of matched asymptotic expansions. Fig. 2 also shows that the maximum velocity in the porous medium channel is reduced with increasing values of $\xi$ until steady-state conditions are eventually attained.

Figs. 3 and 4 present the time histories of the volumetric flow rate in the porous medium channel $Q$ and the boundary friction coefficient at the lower wall $C$ for various values of the fluid behavior coefficient $n$, respectively. These figures indicate that both $Q$ and $C$ decrease with increasing values of $\xi$. This is expected since decreases in the fluid velocity as $\xi$ increases shown in Fig. 2 cause $Q$ and $C$ to decrease. It can be easily seen from Figs. 3 and 4 that the steady-state conditions are attained relatively quickly. Due to the uniform velocity distribution used as an initial condition, the values of $C$ at $\xi=0$ is zero followed by very large values due to the no slip condition at the wall for $\xi>0$. These large values of $C$ are not shown in Fig. 4. As $n$ decreases, the average velocity at any time in the channel (not shown herein for brevity) increases. This produces an increase in $Q$. 

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Figs. 5 and 6 depict the transient behaviors of $Q$ and $C$ for various values of $\gamma$. As the porous medium shape factor $\gamma$ increases, for a fixed values of the fluid behavior coefficient $n$, the average velocity in the channel decreases at any time. This causes the fluid flow in the channel to occur at a slower rate with less friction at the boundary. This behavior is clearly depicted in Figs. 5 and 6 by the decreases observed in $Q$ and $C$ as $\gamma$ increases.

Some comparative numerical tests were performed to provide a check on the numerical procedure. This required some modifications in the dimensionless parameters and boundary conditions. For instance, the steady-state results associated with $\delta=0$ (no porous medium inertial effects) were found consistent with those reported by Chamkha (1993). Also, the Newtonian results given by Khosravi (1991) were reproduced. Because of the various favorable comparisons performed during the course of the present study, it is concluded that the numerical algorithm used herein is adequate.
CONCLUSION

The problem of transient power-law fluid flow in a porous medium with uniform porosity and permeability bounded between two infinitely long parallel permeable plates is solved numerically. A constant pressure gradient is applied in the flow direction parallel to the channel surfaces and uniform injection and extraction conditions are imposed there. The time histories of the velocity profiles, volume flow rate, and boundary friction coefficient for this type of flow are presented graphically for pseudo-plastic, Newtonian and dilatant fluids. It was found that increases in the fluid behavior coefficient caused the fluid velocity at any time to decrease everywhere within the channel boundaries. This produced a decrease in the volume flow rate in the channel. Also, both the boundary friction coefficient at the lower surface of the channel and the flow rate were decreased when the porous medium shape factor was increased. Furthermore, the channeling phenomenon (where velocity profiles with peaks occurring close to the walls) reported by previous published results and, as a result, the numerical method was found to be adequate for the solution of the present problem.

REFERENCES