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POWER-LAW DUSTY-FLUID FLOW BETWEEN
TWO PARALLEL POROUS PLATES

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Finite-difference solutions are obtained for the problem of fully developed laminar flow of a power-law dusty fluid in a porous channel due to the application of a constant pressure gradient in the flow direction. Uniform fluid-phase injection and extraction are imposed at the channel walls. Some graphical results for the velocity profiles and volume flow rates for both the fluid and particle phases and the fluid skin-friction coefficient are presented to show interesting features of the solutions.


INTRODUCTION

Extensive mathematical modeling of fluid/solid systems is based on the continuum approach where both the fluid and the solid particulates are treated as two interacting continua (Marbrie, 1970; Ishii, 1975; and Soo, 1967). Most of this work has been focused on the treatment of the fluid as Newtonian in nature where the fluid viscous stress is linearly related to its strain rate. However, many fluid-particle suspensions of practical interest exhibit non-Newtonian behavior. It should be recalled that fluids with no initial yield stress are called pseudoplastic if the apparent viscosity decreases with an increase in shear rate. They are called dilatant if the apparent viscosity increases with increasing shear rate. These behaviors are also known as shear thinning and shear thickening, respectively. Examples of shear thinning fluids are dilute solutions of polymers, most printing ink, and paper pulp. Examples of materials that have been found to show dilatancy are starch, potassium silicate, and some cornstarch/sugar solutions (Gebhart, et al., 1988 and Skelland, 1967).

Particulate suspensions in these types of fluids is increasingly encountered in a wide variety of industries such as food processing, filtration, chemical, and pharmaceutical industries. A literature search reveals that little work has been done on modeling suspension of particles in power-law fluids. The present paper considers the flow of such a suspension between two parallel porous plates. Although the flow in this geometry may or may not have a direct practical application, it is chosen here for simplicity and to illustrate some features of this type of two-phase flow.

The flow of a Newtonian fluid in a porous channel with injection and extraction of fluid from the walls was considered by White (1991). Closed-form solutions for the non-Newtonian power-law version of White's problem with different boundary conditions were reported by Kapur (1963). Chamkha (1992; in press) solved the problem of Newtonian two-phase suspension between two parallel porous flat plates with suction and injection analytically for a constant pressure gradient. The objective of this paper is to extend the solutions reported by Chamkha (1992) to power-law fluids. The fluid phase is assumed incompressible and the particle phase is assumed to consist of small non-deformable solids of spherical shape and uniform size that are being dragged along with the fluid. In general, the parameters that describe the fluid rheology such as its consistency index may be temperature dependent. However, for low temperature differences across the transport region, the fluid's physical properties can be assumed constants. This assumption is adopted herein.

GOVERNING EQUATIONS

Consider a steady, fully developed, laminar flow of a power-law dusty fluid in a porous channel due to the action of a constant pressure gradient applied in the x-direction. A schematic of this problem is shown in Fig. 1. This type of flow is governed by the balance laws of mass and linear momentum for the fluid and particle phases (Marble, 1970; Kapur, 1963). Let equal uniform fluid extraction and injection be applied at the lower and upper walls of the channel, respectively. This is because the fluid-phase continuity equation dictates that the fluid-phase normal velocity is equal to the injection velocity. The particle-phase continuity equation requires that the particle-phase normal velocity be a constant. The y-momentum equations for both phases are identically satisfied when the particle-phase normal velocity is identical to the normal velocity for the fluid phase. The small volume fraction assumption inherent in the dusty-gas model will be retained herein and the suspension is assumed dilute in the sense that no particle-particle interaction exists. For the assumptions made earlier, the balance laws reduce to

\[
\frac{K}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) + V_x \frac{\partial u}{\partial y} + \frac{P_x}{\rho} N (u - u_p) \cdot \frac{\partial}{\partial x} = 0
\]

(1)

\[
V_w \frac{\partial u_p}{\partial y} + N (u - u_p) = 0
\]

(2)

where y is the distance normal to the flow direction, \( \rho \), \( u \), and \( u_p \) are the fluid-phase density, velocity in the x-direction, and pressure gradient, respectively. \( K \) and \( n \) are fluid phenomenological constants. \( P_x \) and \( u_p \) are the particle-phase in-suspension density and velocity in the x-direction. \( V_w \) and \( N \) are the fluid (or particle) injection (or extraction) velocity and the interphase momentum transfer coefficient, respectively.
Three boundary conditions (two for the fluid phase and one for the particle phase) are needed to solve Eqs. 1 and 2. While it is obvious that the fluid phase will not slip at the boundaries of the channel, it is not clear what constitutes a correct boundary condition for the particle phase. In general, the particle-phase momentum equation evaluated at the wall is usually used in place of a boundary condition. However, when this is done for the present problem, a physically unacceptable behavior results where particle-phase backflow at the upper wall of the channel occurs. This behavior was prevented by making the slope of the particle-phase velocity profile vanish at the upper wall (which implies that the particle phase will not slip like the fluid phase). This can be seen from Eq. 2. (Chamkha, in review). Thus, the boundary conditions employed herein are

\[ u(0) = 0, \quad u(2h) = 0, \quad \frac{du}{dy}(2h) = 0 \]  \hspace{1cm} (3)

where \( h \) is the channel half width.

To eliminate the dependence of the solutions on the dimensions of the various parameters, the following dimensionless parameters are used

\[ y = h \eta, \quad u = V_c \eta, \quad u_p = V_c \eta P(x), \quad \frac{dP}{dx} = \frac{KVP}{h \kappa}, \]  \hspace{1cm} (4)

where \( V_c \) is a characteristic velocity. The resulting dimensionless equations are

\[ \frac{d}{d\eta} \left( \frac{dE_p}{d\eta} \right) + R_e \frac{dE_p}{d\eta} + k \eta (F_p - F) + G = 0 \]  \hspace{1cm} (5)

\[ R_w \frac{dF_p}{d\eta} + \alpha (F - F_p) = 0 \]  \hspace{1cm} (6)

where

\[ R_w = \left( \frac{\rho V_c h \eta}{KV_p} \right), \quad \kappa = \left( \frac{\rho \eta}{KV_c} \right), \]

\[ \alpha = \left( \frac{\rho \eta}{KV_c} \right) \]

are the wall Reynolds number, the particle loading, and the inverse Stokes number, respectively. If \( n \) is equated to 1 in Eq. 5, the dusty-gas model discussed by Marble (1970) will be recovered.

The dimensionless boundary conditions become

\[ F(0) = 0, \quad F(2) = 0, \quad \frac{dF(2)}{d\eta} = 0 \]  \hspace{1cm} (7)

An objective of this work is to observe the variations of the flow rates of both the fluid and the particle phases as well as the skin-friction coefficient for the fluid phase as a result of changing various parameters. These can be defined in dimensionless form as

\[ Q = \int_0^2 F d\eta, \quad Q_p = \int_0^2 F_p d\eta, \quad C = \left( \frac{dF}{d\eta} \right) \]  \hspace{1cm} (8)

In the next section, the method of solution and some representative results will be presented and discussed.

RESULTS AND DISCUSSION

The mathematical model developed in the previous section cannot be solved analytically due to the nonlinearity of Eq. 5. Therefore, the governing equations were solved numerically subject to the boundary conditions in Eq. 7. A finite-difference method was devised for this purpose. The method is implicit and employs iteration to deal with the nonlinearity of Eq. 5. This method represents an extension of the method discussed by Blottner (1970) and Patankar (1980) to two-phase flows. The computational domain was divided into 200 meshes with a constant step size of 0.01. Discretization of Eq. 5 was accomplished using three point difference quotients while Eq. 6 was discretized by using the trapezoidal rule. At each stage of the iteration process linear equations were formed and solved using the Thomas algorithm (or the Potter's method; Davis, 1968). To start the solution process, a distribution for \( F \) was assumed. For this assumed distribution Eq. 6 was solved for \( F_p \). The solution for \( F_p \) was then substituted into Eq. 5 which could then be solved for \( F \). The calculated distribution of \( F \) was compared with the assumed distribution. When the difference between the calculated and the assumed distributions reached 10^{-4}, convergence was achieved and the iteration process was terminated. It should be mentioned that no numerical difficulties were encountered in producing the results. A representative set of graphical results will be presented in Figs. 1 through 5 to show the effect of the fluid behavior coefficient, \( n \), and the particle loading on the solutions.

Figs. 1 and 2 present typical velocity profiles in the channel for both the fluid and particle phases for different values of the fluid behavior coefficient, \( n \), respectively. It is apparent from Fig. 1 that the maximum velocity for a dilatant fluid (\( n > 1 \)) is higher than those of Newtonian (\( n = 1 \)) and pseudo-plastic (\( n < 1 \)) fluids. Also, the velocity profiles flatten in the enclosed region away from the walls and plug-type profiles result as \( n \) is decreased. In Fig. 2, it is observed that the peak in the particle velocity profiles occur close to the lower wall of the channel and the particle slip there is decreased with decreasing values of \( n \).

Figs. 3 through 5 depict the influence of the fluid behavior coefficient, \( n \), and the particle loading \( K \) on the fluid-phase volume flow rate \( Q \), the particle-phase volume flow rate \( Q_p \), and the fluid-phase skin-friction coefficient \( C \), respectively. In these figures each square represents a separate calculation. As mentioned earlier, as \( n \) is decreased the velocity profiles of both phases flatten and the distinctive peaks disappear. This causes both the fluid and particle volume flow rates to decrease. This is reflected in the decreases observed in both \( Q \) and \( Q_p \) as \( n \) decreases. Increases in the particle loading \( K \) have the tendency to slow the fluid motion and thus, causing \( Q \), \( Q_p \) and \( C \) to decrease. This behavior is clearly shown in Figs. 3 through 5. Fig. 5 shows that for relatively small values of \( K \) increases in the values of \( n \) cause increases in the values of \( C \).
However, it seems that there is a specific value of the particle loading (K=S) for which this behavior is reversed and C decreases as it increases. It should be mentioned that all the results associated with the Newtonian conditions (n=1) are in excellent agreement with the closed-form results reported by Chankha (1992). Also, the results associated with K=0 (single-phase flow) are comparable with those reported by Kapur (1963). This confirms that the numerical method chosen to solve this problem is adequate. Also, it should be added that, as far as the author is aware, no experimental data for this problem exist in the open literature.

CONCLUSIONS

The problem of laminar fully developed flow of non-Newtonian power-law dusty fluid exhibiting small volume fraction in a porous channel was solved numerically using an implicit finite-difference based technique. The obtained results were compared with previously published analytical solutions for the Newtonian case and were found to be in excellent agreement. Through the analysis of the numerical results, it was found that, in general, pseudo-plastic suspensions have plug-type velocity profiles and lower volume flow rates than those of dilatant suspensions under the same conditions. Also, the presence of particles cause the fluid and the particle volume flow rates in the channel and the wall friction to decrease for all types of suspensions. It is hoped that these observations be confirmed experimentally and will be helpful in modeling more complex geometries involving non-Newtonian power-law suspensions.

LITERATURE