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TRANSIENT MHD FREE CONVECTION FROM A POROUS MEDIUM SUPPORTED BY A SURFACE

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Equations governing transient magnetohydrodynamic (MHD) free convection flow of an electrically-conducting fluid in a porous medium supported by an infinite porous vertical plate are developed. The flow situation is generated by an impulsive movement of the vertical plate which is maintained at a constant heat flux. The coupled partial differential equations are solved numerically using an implicit finite-difference scheme. Graphical results for the velocity and temperature profiles and the skin-friction coefficient at different values of the Hartmann number, the Grashof number, the Prandtl number, the suction parameter, and the porous medium non-Darcian inertial parameter are presented and discussed.

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INTRODUCTION

Magnetohydrodynamic (MHD) flow and heat transfer of electrically-conducting fluids in a porous medium and over surfaces is a subject that has special importance in engineering because it has many applications in heat exchanger design, MHD accelerators, nuclear reactors, solar energy collectors, and filtration. This has led to extensive research in this area. The problem considered in this paper is that of transient free convection flow of an electrically-conducting fluid up an infinite porous vertical plate at constant heat flux embedded in a porous medium in the presence of a transverse magnetic field. The flow is generated by impulsively moving the plate in its own plane. Some related work on this subject can be found in the papers by Sacheti, et al. (1994), Singh and Sacheti (1988), Georgantopoulos, et al. (1979), Soundalgerkar and Patil (1980), Chen and Lin (1995), Gupta (1963), Wilks (1976), Watanabe and Pop (1993), Takhar and Pop (1987), Takhar and Ram (1994), and Tien and Vafai (1990). The governing equations incorporate combined effects of porous medium inertia, magnetic field, and imposed fluid wall suction. The flow is assumed incompressible and laminar. The magnetic Reynolds number is assumed small so that the induced magnetic field can be neglected. Also, the Hall affect of magnetohydrodynamics is neglected.

GOVERNING EQUATIONS

Consider transient two-dimensional flow of an electrically-conducting fluid along a vertical infinite porous plate embedded in a porous medium in the presence of a transverse magnetic field. To model this flow, a cartesian coordinate is chosen with the plate being coincident with the x-axis which is positive in the upward direction. The y- axis is taken normal to the x-axis. The magnetic field is applied in the y-direction normal to the flow direction (see Figure 1). Uniform suction is imposed at the plate surface which is electrically non-conducting and is maintained at a constant heat flux. The fluid is initially at rest and is suddenly set to motion by impulsive motion of the plate in its own plane with a uniform velocity U. The flow is assumed laminar and incompressible and the pressure gradient is neglected. Since the plate is infinite, all the physical variables will be functions of the y coordinate and time. The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy modified to include the porous medium effects, the Boussinesq effects, and the magnetic effects. Taking into account the assumptions made earlier, these can be expressed as

\[ \frac{\partial \mathbf{v}}{\partial y} = \mathbf{0} \]  (1)

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\[ \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = ev \frac{\partial^2 u}{\partial y^2} + e^2 \alpha B_2^2 \frac{\partial^2 u}{\partial x^2} - e^2 \frac{\partial^2 T}{\partial y^2} - e^2 \frac{\partial T}{\partial y} \frac{\partial u}{\partial y} \rho c_p \]

where \( t \) and \( y \) stand for time and distance away from the plate, respectively. \( u, v \), and \( T \) are the x-component of velocity, y-component of velocity, and temperature, respectively. \( \rho, \nu, \) and \( c_p \) are the fluid density, kinematic viscosity, and the specific heat at constant pressure, respectively. \( g, \beta, \) and \( T_e \) are the gravitational acceleration, volumetric coefficient of thermal expansion, and the fluid temperature at \( t = 0 \), respectively. \( \varepsilon, K, \) and \( \alpha_e \) are the porosity, permeability, medium inertia coefficient, and the effective thermal diffusivity of the medium, respectively. \( \alpha \) and \( B_2 \) are the electrical conductivity and the magnetic induction, respectively.

It should be noted that the last term of Equation (2) accounts for inertial effects of the porous medium which would occur when the medium pressure gradient is a quadratic function of velocity (see, Vafai and Tien, 1980). Also, the last two terms of Equation (3) represent the viscous and the magnetic dissipations, respectively.

The appropriate initial and boundary conditions for this problem can be written as

\[ u(0,y) = 0, v(0,y) = 0, T(0,y) = T_e \]

\[ u(t,0) = U, v(t,0) = -V \frac{\partial T}{\partial y} (t,0) = -\frac{q}{k} \]

\[ u(t,\infty) = 0, T(t,\infty) = T_e \]

where \( q \) and \( k \) are the plate constant heat flux per unit area and the effective thermal conductivity, respectively, and \( V_e \) \( (\eta > 0) \) is the suction velocity.

It is convenient to make Equations (1) through (4) dimensionless. This is accomplished by using

\[ \tau = \frac{\nu \tau}{U^2}, \quad \eta = \frac{\nu \eta}{U}, \quad u = UF \]

The dimensionless suction velocity, the inverse Darcy number, the non-Darcian inertial parameter, square of the Hartmann number, the Graashof number, the Froudi number, and the Eckert number, respectively. It should be noted that Equation (6) is obtained by integration and nondimensionlization of Equation (1).

The dimensionless initial and boundary conditions become

\[ F(0,\eta) = 0, \quad \tilde{\theta}(0,\eta) = 0 \]

\[ F(\tau,0) = 1, \quad \frac{\partial \tilde{\theta}}{\partial \eta} (\tau,0) = -1 \]

\[ F(\tau,\infty) = 0, \quad \tilde{\theta}(\tau,\infty) = 0 \]
The skin-friction coefficient is an important physical parameter for this flow situation. It can be defined as

\[ C_f = \frac{\partial F}{\partial n}(\tau, \Theta) \]  

(11)

RESULTS AND DISCUSSION

Equations (6) through (10) are non-linear and do not possess a closed-form solution. However, the steady-state problem with neglected porous medium inertial effects and viscous and magnetic dissipations can be solved analytically. This solution can be shown to be

\[ \frac{1}{r_s Pr} \exp(-r_s Pr \eta) \]

\[ F = (1 - A) \exp(-\lambda \eta) + A \exp(-r_s Pr \eta) \]

(12)

\[ \lambda = \frac{r_s}{2e} \left( \frac{r_s^2 + 4e^2(\psi + M^2)}{2e} \right)^{1/2} \]

(13)

\[ A = \frac{e^2 Gr}{r_s Pr\left(r_s^2 Pr^2 - r_s^2 Pr - e^2(\psi + M^2)\right)} \]

\[ C_f = (1 - A) \lambda + r_s Pr A \]

(14)

The complete transient equations are solved numerically via an implicit, iterative, tri-diagonal finite-difference method (see, Blottner, 1970 and Patankar, 1980). The computational domain was divided up into 201 nodes in the \( \eta \) direction and 301 nodes in the \( \tau \) direction. Variable \( \eta \) step sizes (\( \Delta \eta = 0.001 \)) with a growth factor of 1.03 and variable \( \tau \) step sizes (\( \Delta \tau = 0.01 \)) with a growth factor of 1.01 were employed in the present work. These step sizes were chosen after many numerical experiments to assess grid independence. The analytical results produced before helped in this task. Three-point backward-difference formulas were used to replace all first derivatives with respect to \( \tau \) while three-point central difference quotients were employed to discretize the governing equations. As a result, linear tri-diagonal algebraic equations were solved by the Thomas' algorithm (see, Blottner, 1970) at each line of constant \( \tau \). A convergence criterion based on the difference between the current and the previous iterations (set to \( 10^{-6} \)) was employed in the present work. The steady-state solutions given in Equations (12) through (14) are used to check on the correctness of the numerical results.

Figures 2 through 16 are representative numerical results which are obtained to show the relative effect of the different physical parameters of the porous problem on the solution.

Figures 2 through 4 present steady-state results for the velocity \( F \), temperature \( \Theta \), and transient solutions for the skin-friction coefficient \( C_f \) for various Hartmann number \( M \), respectively. Applying a transverse magnetic field normal to the flow direction creates a force in the direction opposite to flow which tends to drag the flow. This results in decreasing the fluid velocity and increasing its temperature. As the velocity decreases, the slope of its profile at the wall increases. This yields an increase in the values of \( C_f \). The decreases of \( F \) and increases of \( \Theta \) and \( C_f \) as \( M \) increases are clearly evident from figures 2 through 4.

Figures 5 through 7 illustrate the influence of the Grashof number \( G_r \) on \( F \), \( \Theta \), and \( C_f \), respectively. As the Grashof number increases, the coupling between the velocity and temperature increases. This tends to increase the velocity across the plate at every point except at the boundaries \( (\tau = 0, \eta = \infty) \), and decreases the slope of the velocity profile at \( \eta = 0 \). This produces a decrease in the values of \( C_f \). Since the viscous and magnetic dissipations are small compared to the convective terms in Equation (8), the Grashof number has a minimum effect on \( \Theta \). These facts are clearly depicted in figures 5 through 7.

Figures 8 through 10 depict the effect of the Prandtl number \( Pr \) on \( F \), \( \Theta \), and \( C_f \), respectively. Increases in the values of \( Pr \) have a tendency to reduce the hydrodynamic and thermal regions close to the plate. This causes decreases in both \( F \) and \( \Theta \) and increases in the values of \( C_f \) (as the slope of \( F \) at \( \eta = 0 \) increases) as clearly observed in figures 8 through 10.

Increasing the suction parameter \( r_s \) produces the same effect as that obtained by increasing the values of \( Pr \). Namely, decreases in \( F \) and \( \Theta \) and increases in \( C_f \) as shown in figures 11 through 13, respectively.

The effects of the non-Darcian porous medium inertial parameter \( \Gamma \) on the velocity, temperature, and the skin-friction coefficient are presented in figures 14 through 16, respectively. Increasing in the medium reactivity tends to slow the flow along the plate with increased friction at the plate surface and causes the fluid to warm up. This produces decreases in \( F \) and increases in \( \Theta \) and \( C_f \) as \( \Gamma \) increases as illustrated in figures 14 through 16.

It should be mentioned that the numerical results associated with \( r_s = 0, Ec = 0, \psi = 0, \) and \( \Gamma = 0 \) are in excellent agreement with the results reported by Sachetti, et al. (1994) and Singh and Sachetti (1998). The numerical results were also favorably checked against the analytical solutions reported earlier for steady-state conditions. These comparisons provided a confirmation on the correctness of the numerical results.
CONCLUSION

The problem of transient free convection magnetohydrodynamic (MHD) flow of an electrically-conducting fluid past an infinite porous vertical plate embedded in a uniform porous medium subject to impulsive motion and constant heat flux was considered. Closed-form solution of the steady-state version of the problem with some neglected effects was derived. The complete unsteady problem was solved numerically by an implicit finite-difference scheme. It was found that increases in either of the Hartmann number or the non-Darcian inertial parameter produced decreases in fluid velocity and increases in the fluid temperature and the skin-friction coefficient. Increases in the Graetz number resulted in increases in the fluid velocity and decreases in the skin-friction coefficient. Furthermore, increases in either of the Prandtl number or the suction parameter yielded reductions in the velocity and the temperature of the fluid and increases in the skin-friction coefficient. Comparisons with previously published results were made and found to be in excellent agreement. It should be mentioned that no comparisons with experimental data on this problem were made due to the absence of such data at present. It is hoped that the present results will serve as a stimulus for experimental work on this problem.

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Figure 1. Schematic of the problem.

Figure 2. Steady tangential velocity profiles for various Hartmann numbers.

Figure 3. Steady temperature profiles for various Hartmann numbers.
Figure 4. Transient skin friction coefficients for various Hartmann numbers.

Figure 5. Steady tangential velocity profiles for various Grashof numbers.

Figure 6. Steady temperature profiles for various Grashof numbers.

Figure 7. Transient skin friction coefficients for various Grashof numbers.

Figure 8. Steady tangential velocity profiles for various Prandtl numbers.

Figure 9. Steady temperature profiles for various Prandtl numbers.
Figure 10. Transient skin friction coefficients for various Prandtl numbers.

Figure 11. Steady tangential velocity profiles for various suction numbers.

Figure 12. Steady temperature profiles for various suction numbers.

Figure 13. Transient skin friction coefficients for various suction numbers.

Figure 14. Steady tangential velocity profiles for various inertial coefficients.

Figure 15. Steady temperature profiles for various inertial coefficients.
Figure 16. Transient skin friction coefficients for various inertial coefficients.