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SIMULATION OF COLLECTION EFFICIENCIES FOR SHALLOW FILTERS

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INTRODUCTION
The usual mathematical models of filter cloths and sheets [see, for instance, Cheremisinoff and Azbel (1983)] are based on the assumption of uniform conditions within the filter. An advantage of employing this assumption is that it leads to a description of the filtration process involving ordinary differential equations. A disadvantage is that it eliminates the distinction between inlet and outlet conditions and, thus, does not allow a meaningful definition of collection efficiency. In the present paper some ideas are borrowed from deep filtration modeling (see, for instance, Tien (1989) or Adin and Rajagopal (1989)) to produce a theory of thin filters which allows for both the calculation of collection efficiencies and a wide choice of mass transfer and momentum transfer functions while retaining most of the simplicity of the usual models. Closed form solutions are given for some idealized problems. Both constant flow rate and constant pressure drop situations are discussed.

GOVERNING EQUATIONS
The governing equations for one dimensional deep filtration are discussed by Tien (1989) and can be written in the forms

\[ \partial_t (e e) + q \partial_z e = -M (c, v, e), \quad \partial_t e = -M (c, v, e) \quad \text{(1a,b)} \]

\[ \partial_z p = -F (c, v, e), \quad v = q (t) \quad \text{(1c,d)} \]

In Eqs. 1a,b,c,d, \( e \) is the porosity accounting for collected particles, \( c \) is the volume fraction of particles in the filtrate (fluid plus uncollected particles), \( z \) denotes position, \( t \) denotes time, \( M \) characterizes the mass transfer between the uncollected particle cloud and the filter, \( F \) characterizes the linear momentum transfer between the liquid and the filter, \( v \) is the velocity of the filtrate, and \( q \) is the superficial velocity of the filtrate.

In the usual theories of filter cloths and sheets it is assumed that all dependent variables appearing in Eqs. 1a,b,c,d are independent of position. Then Eqs. 1a,b can be added and integrated to yield

\[ e (1-c) = \xi_1 (1-\xi) \quad \text{(2)} \]

where the subscript \( i \) denotes an initial value. Eq. 2 shows that \( c \) and \( e \) are not independent under these conditions. One of them (usually \( c \)) can, therefore, be eliminated from the arguments of the functions \( M \) and \( F \). It is proposed herein to postulate

\[ M = M (q, e), \quad F = F (q, e) \quad \text{(3a,b)} \]

(where Eq. 1d has been taken into account) as constitutive assumptions but to assume nothing about the dependence of the dependent variables on \( z \) and \( t \). It is further proposed to take a phenomenological approach to the specification of \( M \) and \( F \) (as is normal in deep filtration modeling) rather than restricting attention to the full, partial, and intermediate blocking models. This produces a generalization of the normal filter sheet/cloth theory as discussed below.

Consider a filter cloth or sheet of thickness \( L \). Use of Eq. 3a allows Eq. 1b to be satisfied by \( e = \xi(t) \) provided that the initial porosity is uniform (which will be assumed herein).

This, in turn, shows that Eq. 1c is satisfied by a pressure which varies linearly with \( z \) and is characterized by a total pressure drop \( \Delta P(t) \) across the filter. Thus, Eqs. 1b,c can be written in the respective forms (taking Eq. 1d into account)

\[ e' = -M (q, e), \quad p = F (q, e) \quad \text{(4a,b)} \]

where a prime denotes time differentiation. These equations are identical to those associated with the normal model of filter cloths and sheets and can be solved to determine \( e(t), q(t), \) and \( P(t) \) when a relationship between \( p \) and \( q \) is given.

Once \( q(t) \) and \( p(t) \) are known, Eq. 1a can be solved to determine \( c(z(t)) \). A particularly simple solution results if the first term of Eq. 1a is neglected (quasistatic approximation). Then one obtains

\[ c = c_i (t) - M (t) / q (t) \quad \text{(5)} \]

where \( c_i = c(0,t) \) is the inlet concentration. The outlet concentration is denoted by \( c_o = c(L,t) \). In terms of \( c_i \) and \( c_o \), the collection efficiency \( E \) can be defined as \( E = 1 - c_p / c_i \). Thus from Eq. 5
Eq. 6 is written assuming that \( M \lambda (q_1) \) is \( L \) and that complete clogging has not occurred. It must be replaced by \( E = 1 \) during any time period in which \( M / (q_1) > 1 \) and by \( E = 0 \) if the filter becomes completely clogged.

**REPRESENTATIVE SOLUTIONS**

In this section some illustrative solutions are presented based on typical power law representations of the functions \( M \) and \( F \). While microscopic models can be used to estimate the constants appearing in these equations from filter and filtrate properties, it is believed that it would be most efficient to measure these constants directly for each filter/filtrate combination. This is the same philosophy inherent in the work on deep filtration reviewed in Chapter 2 of the book by Tien (1989).

It should be pointed out that the use of power law models is purely for the sake of illustration. It is not meant to imply that such expressions are optimal for the description of all filtration processes. In fact, one of the advantages of the present approach is that it allows the investigator to select any functional forms of \( M \) and \( F \) which represent a given process accurately. All such functional forms will contain material constants which must be determined by reference to experimental measurements. The number of constants in each case will depend on the level of sophistication of the model.

Consider first the power law expressions

\[
M = \alpha q (\varepsilon - \gamma)^m, \quad F = \beta q \left( \varepsilon - \gamma + \delta \right)^n
\]  

(7a,b)

where \( \alpha, \beta, \gamma, \delta, m \) and \( n \) (all positive) are constants. Eqs. 7a,b contain several well known models as special cases. Thus with \( \gamma = \delta = 0 \) and \( m = 0 \), \( n = 1 \), \( m = 0 \), \( n = 1 \) corresponds to the complete blocking model, \( m = 0, n \) corresponds to the partial blocking model, and \( m = 1 \), \( n = 0 \) corresponds to the intermediate blocking model. The behaviors of these three models are discussed in detail by Cheremisinoff and Azbel (1983). Eqs. 7a,b represent a logical extension and generalization of these models. Substituting Eqs. 7a,b into Eqs. 4a,b yields

\[
E = \alpha q (\varepsilon - \gamma)^m, \quad P = \beta q (\varepsilon - \gamma + \delta)^n
\]  

(8a,b)

For the case of constant volumetric flow rate of filtrate (same as constant superficial velocity, \( q \)) Eqs. 8a,b are easily solved. The results can be expressed conveniently in terms of the clogging time

\[
t_1 = \left( \varepsilon_1 - \gamma \right)^{-m} / \left( (1-m) \alpha q \right)
\]  

(9)

and the function

\[
G(\gamma) = \begin{cases} 
1 & (1-m) (\varepsilon_1 - \gamma)^{1-m} \alpha q t_1 = 1/m; m = 1; t > 0 \hfill \\
\exp(-\alpha q t) & m = 1; t < 0 \hfill \\
(1-1/m) \alpha q t_1 (\varepsilon_1 - \gamma)^{m=1-m} & m < 1; t > t_1 \hfill \\
0 & m < 1; t < t_1
\end{cases}
\]  

(10)

\[
E = \gamma' (\varepsilon_1 - \gamma) G_j
\]  

(11)

Substituting Eq. 11 into Eq. 8b yields

\[
P = \beta q (\varepsilon_1 - \gamma) G_j + \delta)^m
\]  

(12)

Substituting Eq. 11 into Eq. 7a and then using the result in Eq. 6 leads to

\[
E = \alpha L \left( \varepsilon_1 - \gamma \right) G_j m / \varepsilon_1
\]  

(13)

It can be seen from Eqs. 10 and 11 that the porosity decreases initially and then reaches a constant limit determined by the value of \( \gamma \). Thus the filter attains a saturated state and cannot remove additional particulates. Physically this can happen when all collection spaces are filled or when collection at some locations is balanced by entrainment at others. It can be seen from Eqs. 10 and 12 that the pressure drop increases initially and then reaches a corresponding saturation state dependent of the value of \( \delta \). Eq. 13 shows that saturation corresponds to zero collection efficiency. Behavior of the type discussed above is sometimes observed in filtration processes.

![Figure 1](image)

**Collection Efficiency vs. Time**

If \( \gamma' \) and \( \delta \) vanish, the saturation effects discussed above will be absent and complete clogging will occur at \( t = t_1 \) for \( m < 1 \). Figs. 1 and 2 present some typical collection efficiency
histories for γ = 0 and several values of m. It can be seen that
the shapes of the efficiency curves exhibit a significant
dependence on m. The value m = 0.5 (not shown) corresponds
to a straight line. In all cases the collection efficiency
decreases with time as normally observed in liquid filtration.
Turning Figs. 1 and 2 upside down would produce the
equivalent of the concentration versus time curves presented
by Adin and Rajagopalan (1989). The two most common
classes of filtration processes are those involving constant
flow rate (discussed above) and those involving constant
pressure drop. For this latter case, solving for q from Eq. 8b
and substituting the result into Eq. 8a yields the differential equation

\[ E' = -\alpha P (\gamma - \delta) (\gamma - \delta)^n / \beta \]  

(14)

Substituting Eq. 18 into Eq. 8b (with γ = δ = 0) leads to

\[ q = P (\varepsilon G_2)^n / \beta \]  

(19)

Substituting Eq. 18 into Eq. 7a (with υ = 0) and using

\[ E = \alpha L (\varepsilon G_2)^m / c_i \]  

(20)

Two examples of closed form solutions involving non-zero
values of υ and δ will be given. For m = 1, n = 1

\[ \delta = \gamma + (\varepsilon_1 - \gamma) G_3, \quad q = P \left( (\varepsilon_1 - \gamma) G_3 / \beta \right) \]

(21a, b, c)

where

\[ G_3(t) = \delta \exp (-\alpha \delta t / (\beta / (\delta + (\varepsilon_1 - \gamma) (1 - \exp (-\alpha \delta t / (\beta / (\beta)))))) \]

(22)

For m = 1, n = 0.5

\[ \delta = \gamma + (\varepsilon_1 - \gamma) G_4, \quad q = P \left( (\varepsilon_1 - \gamma) G_4 / \beta \right) \]

(23a, b)

\[ E = \alpha L (\varepsilon_1 - \gamma) G_4 / c_i \]

(23c)

where

\[ G_4(t) = \delta ((1 + S \exp (-\alpha \delta t / (\beta / (\beta)))) / (1 - S \exp (-\alpha \delta t / (\beta / (\beta))))^2 \right) \]

(24)

with

\[ S = ((\varepsilon_1 - \gamma) / (\delta + 1)) - 1 \] / (((\varepsilon_1 - \gamma) / (\delta + 1)) + 1) \]

(25)

It can be seen that both G3 and G4 have the value unity at t
= 0 and the value zero at t = ∞. Thus, the quantities γ, q and
E all exhibit saturation effects in both cases.
The mass exchange function M(q,t) given by Eq. 7a has the
property that δM<0. Since ε decreases during a filtration
process, the ability of the filter to collect particulates
decreases as constant flow rate filtration continues. Such
behavior is not always observed. The expressions

\[ M = \alpha q (1 + \gamma (\varepsilon_1 - \gamma)^m) \]  

(26)

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(with $\alpha$ and $m$ positive) allows the ability of the filter to collect material to either increase or decrease depending on whether $\gamma$ is respectively positive or negative.

As an example of the constant flow rate behavior predicted by Eq. 26, consider the simplest case of $m=1$. The solution of Eq. 41 can then be written in terms of the clogging time

$$t_3 = \ln \left( \frac{1 + \gamma \varepsilon_1}{(\alpha \gamma q)} \right)$$

(27)

as

$$c = \begin{cases} \frac{\varepsilon_1}{q}(\exp(\gamma t) - 1) / \gamma; & \gamma \geq 0, t > 0 \text{ or } \gamma > 0, t \leq t_3 \\ 0; & \gamma < 0, t > t_3 \end{cases}$$

(28)

As is apparent from Eq. 28, the porosity reaches zero in a finite time for $\gamma > 0$ and eventually reaches a saturation value $\varepsilon_f = \varepsilon_1 + 1/\gamma$ for $\gamma = 0$.

Substituting Eq. 28 into Eq. 26 (with $m=1$) and using the result in Eq. 6 yields

$$E = \alpha L \exp(\alpha t \gamma q) / c_1$$

(29)

For $\gamma < 0$ the collection efficiency decreases gradually to zero, for $\gamma > 0$ it increases at first and then drops suddenly to zero at $t = t_3$.

**Figure 3**

Collection Efficiency vs. Time

Figs. 3 and 4 show some typical dimensionless plots based on Eq. 29. Fig. 3 presents results for $\gamma > 0$. The collection efficiency is predicted to be an increasing function of time (as is typical of gas filtration) until clogging occurs at $t = t_3$, the clogging time decreases and the maximum efficiency increases with increasing $\gamma$. Fig. 4 depicts some representative efficiency curves for $\gamma < 0$. Here the efficiency can be seen to decrease with time (as is typical of liquid filtration).

**Figure 4**

Collection Efficiency vs. Time

The purpose of this section was to illustrate the proposed methodology, not to advocate specific forms of the functions $M$ and $F$. It is interesting to note, however, that in the attempts to correlate experimental data using Eqs. 11-13 have yielded favorable results.

CONCLUSION

In the present paper the usual theory of filtration by thin cloths and sheets was extended so as to allow for both a meaningful definition of collection efficiency and a greater choice of mass transfer and momentum transfer mechanisms. Some representative closed form solutions were then given for the variation of porosities, pressure drops, flow rates, and collection efficiencies with time. These solutions were illustrated by graphical presentation of typical collection efficiency histories.

**LITERATURE**