Effects of Physical Parameters on Natural Convection in a Solar Collector Filled with Nanofluid

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Solar energy is one of the best sources of renewable energy with minimal environmental impact. A numerical study has been conducted to investigate the natural convection inside a solar collector having a flat-plate cover and a sine-wave absorber. The water-alumina nanofluid is used as the working fluid inside the solar collector. The governing differential equations with boundary conditions are solved by the penalty finite element method using Galerkin’s weighted residual scheme. The effects of physical parameters on the natural convection heat transfer are simulated. These parameters include the number of wave $\lambda$ and non-dimensional amplitude $A$ of the sinusoidal corrugated absorber. Comprehensive average Nusselt number, average temperature, and mean velocity field for both nanofluid and base fluid within the collector are presented as functions of the parameters mentioned above. Comparison with previously published work is made and found to be in excellent agreement. The numerical results show that the highest heat transfer rate is observed for both the largest $\lambda$ and $A$. In addition, the design for enhancing the performance of the collector is determined by examining the above-mentioned results. © 2012 Wiley Periodicals, Inc. Heat Trans Asian Res, 42(1): 73–88, 2013; Published online 7 November 2012 in Wiley Online Library (wileyonlinelibrary.com/journal/htj). DOI 10.1002/htj.21026

Key words: natural convection, solar collector, finite element method, water-Al$_2$O$_3$ nanofluid

1. Introduction

Suspensions of nanoparticles (i.e., particles with diameters < 100 nm) in liquids, termed nanofluids, show remarkable thermal and optical property changes from the base liquid at low particle loadings. Recent studies also indicate that selected nanofluids may improve the efficiency of absorption in solar thermal collectors. To determine the effectiveness of nanofluids in solar applications, their ability to convert light energy to thermal energy must be known. That is, their absorption of the solar spectrum must be established. Direct absorption solar collectors have been proposed for a variety of applications such as water heating; however, the efficiency of these collectors is limited by the absorption properties of the working fluid, which is very poor for typical fluids used in solar collectors. It has been shown that mixing nanoparticles in a liquid (nanofluid) has a dramatic effect

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on the liquid’s thermophysical properties such as thermal conductivity. Nanoparticles also offer the potential of improving the radiative properties of liquids, leading to an increase in the efficiency of absorption in solar collectors.

Because of the desirable environmental and safety aspects it is widely believed that solar energy should be utilized instead of other alternative energy forms, even when the costs involved are slightly higher. The flat-plate solar collector is commonly used today for the collection of low-temperature solar thermal energy. Solar collectors are key elements in many applications, such as building heating systems, solar drying devices, etc. Solar energy has the greatest potential of all the sources of renewable energy especially when other sources in the country have been depleted. The fluids with solid-sized nanoparticles suspended in them are called “nanofluids.” The natural convection in enclosures continues to be a very active area of research during the past few decades. Applications of nanoparticles in the thermal field are to enhance heat transfer from solar collectors to storage tanks and to improve the efficiency of coolants in transformers.

Nanofluids, or suspensions of nanoparticles in liquids, have been studied for at least 15 years and have shown promise to enhance a wide range of liquid properties [1, 2]. Tyagi et al. [3] studied the predicted efficiency of a low-temperature nanofluid-based direct absorption solar collector. Nanofluid-based direct absorption in a solar collector was investigated by Otanicar et al. [4]. They demonstrated efficiency improvements of up to 5% in solar thermal collectors by utilizing nanofluids as the absorption mechanism. Their experimental and numerical results demonstrated an initial rapid increase in efficiency with volume fraction, followed by a leveling off in efficiency as the volume fraction continued to increase. A lot of information can be obtained from the book Radiative Heat Transfer, written by Modest [5].

Joudi et al. [6] numerically investigated the performance of a prism-shaped storage solar collector with a right triangular cross sectional area. A numerical experiment was performed for inclined solar collectors by Varol and Oztop [7]. Conventional analysis and design of solar collector is based on a one-dimensional conduction equation formulation [8]. The analysis has been substantially assisted by the derivation of plate-fin efficiency factors. The factors relate the design and operating conditions of the collector in a systematic manner that facilitates prediction of heat collection rates at the design stage. The one-dimensional analysis offers a desired accuracy required in a routine analysis even though a two-dimensional temperature distribution exists over the absorber plate of the collector. Therefore, for more accurate analysis at low mass flow rates, a two-dimensional temperature distribution must be considered. Various investigators have used two-dimensional conduction equations in their analysis with different boundary conditions. Stasiek [9] made experimental studies of heat transfer and fluid flow across corrugated and undulated heat exchanger surfaces. Piao et al. [10, 11] investigated experimentally natural, forced, and mixed convective heat transfer in a cross-corrugated channel solar air heater.

Noorshahi et al. [12] studied numerically the natural convection effect in a corrugated enclosure with mixed boundary conditions. Detailed experimental and numerical studies on the performance of the solar air heater were made by Gao [13]. There are so many methods introduced to increase the efficiency of the solar water heater [14–17]. But the novel approach is to introduce the nanofluids in a solar water heater instead of conventional heat transfer fluids (like water). The poor heat transfer properties of these conventional fluids compared to most solids are the primary obstacle.
to the high compactness and effectiveness of the system. The essential initiative is to seek the solid particles having a thermal conductivity several hundred times higher than those of conventional fluids. An innovative idea is to suspend ultrafine solid particles in the fluid to improve the thermal conductivity of the fluid [18]. These early studies, however, used suspensions of millimeter- or micrometer-sized particles, which, although they showed some enhancement, experienced problems such as poor suspension stability and hence channel clogging, which are particularly serious for systems using mini-sized and micro-sized particles.

The suspended metallic or nonmetallic nanoparticles change the transport properties and heat transfer characteristics of the base fluid. Stability and thermal conductivity characteristics of nano-fluids was performed by Hwang et al. [19]. In this study, they concluded that the thermal conductivity of ethylene glycol was increased by 30%. The absorptance of the collector surface for shortwave solar radiation depends on the nature and color of the coating and on the incident angle. Usually black color is used. Various color coatings had been proposed [20–22] mainly for aesthetic reasons. A low-cost mechanically manufactured selective solar absorber surface method has been proposed by Kontinen et al. [23]. Another category of collectors is the uncovered or unglazed solar collector [24]. These are usually low-cost units which can offer cost-effective solar thermal energy in applications such as water preheating for domestic or industrial use, heating of swimming pools [25], and space heating and air heating for industrial or agricultural applications. The principal requirement of the solar collector is a large contact area between the absorbing surface and the air. Various applications of solar air collectors are reported by Kolb et al. [26].

Very recently, Taylor et al. [27] analyzed nanofluid optical property characterization towards efficient direct absorption solar collectors. The authors concluded that nanofluids could be used to absorb sunlight with a negligible amount of viscosity and/or density increase. Rahman et al. [28] performed double-diffusive natural convection in a triangular solar collector where the effects of the thermal Rayleigh number and buoyancy ratio were presented by streamlines, isotherms, and isoconcentration as well as local and mean heat and mass transfer rates. Natural convection heat transfer in a nanofluid filled semi-annulus enclosure was performed by Soleimani et al. [29]. Mohammed et al. [30] studied the influence of nanofluids on parallel flow square microchannel heat exchanger performance where results revealed that as the nanoparticle volume fraction increased, the bulk average temperature of the cold fluid increased and the heat transfer rate decreased. Stoian et al. [31] made a comparative study of convective heat transfer in water and water-based magnetizable nanofluid for thermal applications. They concluded that the addition of nanoparticles in a usual heat transfer fluid as water led to a decrease in the heat transfer performance by natural convection.

In this paper, we investigate numerically the natural convection inside the solar collector having the flat-plate cover and wavelike absorber. The objective of this paper is to present flow and heat transfer used to harness solar energy.

**Nomenclature**

\[ A: \] dimensionless amplitude of wave
\[ A_c: \] area of cover plate, m²
\[ C_p: \] specific heat at constant pressure, kJ kg⁻¹ K⁻¹
\[ g: \] gravitational acceleration, m s⁻²
2. Problem Formulation

Figure 1 shows a schematic diagram of a solar collector. The fluid in the collector is a water-based nanofluid containing Al$_2$O$_3$ nanoparticles. The nanofluid is assumed incompressible and the flow is considered to be laminar. It is taken that the water and nanoparticles are in thermal equilibrium and no slip occurs between them. The solar collector is a metal box with a cover on top and a dark-colored wavelike absorber plate on the bottom. The top horizontal wall initially has constant temperature $T_h$, while the bottom sinusoidal wall is at temperature $T_c$, with $T_h > T_c$. The two vertical walls are considered adiabatic. The thermophysical properties of the nanofluid are taken from
Lin and Violi [32] and given in Table 1. The density of the nanofluid is approximated by the Boussinesq model. Only the steady-state case is considered.

The governing equations for laminar natural convection in a solar collector filled with water-alumina nanofluid in terms of the Navier–Stokes and energy equation (dimensional form) are given as:

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\(x\)-momentum equation:

\[
\rho_d \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_d \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}
\]

\(y\)-momentum equation:

\[
\rho_d \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_d \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \rho_d \beta_d (T - T_c) \tag{3}
\]

Energy equation:

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_d \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

Table 1. Thermophysical Properties of Fluid and Nanoparticles

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Fluid phase (Water)</th>
<th>Al₂O₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_p) (J/kgK)</td>
<td>417 le-6</td>
<td>850</td>
</tr>
<tr>
<td>(\rho) (kg/m³)</td>
<td>997.1</td>
<td>3900</td>
</tr>
<tr>
<td>(k) (W/mK)</td>
<td>0.6</td>
<td>46</td>
</tr>
<tr>
<td>(\beta) (1/K)</td>
<td>21x10⁻⁵</td>
<td>1.67x10⁻⁵</td>
</tr>
</tbody>
</table>
where $\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$ is the density,

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$$

is the heat capacitance,

$$\beta_{nf} = (1 - \phi)\beta_f + \phi\beta_s$$

is the thermal expansion coefficient,

$$\alpha_{nf} = k_{nf}$$

is the thermal diffusivity,

the dynamic viscosity of Brinkman model [33] is $\mu_{nf} = \mu_f(1 - \phi)^{-2.5}$.

and the thermal conductivity of the Maxwell-Garnett (MG) model [34] is

$$k_{nf} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$$

Radiation heat transfer by the top glass cover surface must account for the thermal radiation which can be absorbed, reflected, or transmitted. This decomposition can be expressed by

$$q_{\text{net}} = q_{\text{absorbed}} + q_{\text{transmitted}} + q_{\text{reflected}}$$

Outside the boundary layer, the amount of energy $q_{\text{reflected}}$ is neglected.

Thus, total energy of the glass cover plate becomes $q_{\text{net}} = q_{\text{absorbed}} + q_{\text{transmitted}}$.

Now the amount of transmitted energy is radiated from the cover plate to the bottom wavy absorber without any medium as

$$q_{\text{transmitted}} = q_r = \varepsilon\sigma A_c \left( T_w^4 - T_c^4 \right)$$

Here, $\varepsilon$ is emissivity of the glass cover plate, $\sigma$ is the Stefan–Boltzmann constant $5.670400 \times 10^{-8}$ Js$^{-1}$m$^{-2}$K$^{-4}$, and $T_w$ is the variable temperature of the top wall. Again, the amount of absorbed energy is transferred from cover plate to bottom absorber by natural convection where the medium is nanofluid as

$$q_{\text{absorbed}} = q_c = hA_c \left( T_w - T_i \right)$$

So total energy gained or lost by the cover plate is $q_{\text{net}} = hA_c(T_w - T_i) + \varepsilon\sigma A_c(T_w^4 - T_c^4)$.

The boundary conditions are:

at all solid boundaries $u = v = 0$

at the top cover plate $q = hA_c(T_w - T_i) + \varepsilon\sigma A_c(T_w^4 - T_c^4)$

at the vertical walls $\partial T/\partial x = 0$

at the bottom wavy absorber $T = T_c$

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables:
Then the non-dimensional governing equations are

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

(5)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + Pr \frac{v_{nf}}{v_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  

(6)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + Pr \frac{v_{nf}}{v_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \frac{(1-\phi) \rho_f \beta_f + \phi \rho_s \beta_s}{\rho_{nf} \beta_f} \theta
\]  

(7)

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]  

(8)

where \( Pr = \nu_f/\alpha_f \) is the Prandtl number and \( Ra = g \beta_f L^3 (T_h - T_c)/\nu_f \alpha_f \) is the Rayleigh number.

The corresponding boundary conditions take the following form:

at all solid boundaries \( U = V = 0 \)

at the vertical walls \( \partial \theta/\partial X = 0 \)

at the bottom wavy absorber \( \theta = 0 \)

The shape of the bottom wavy absorber profile is assumed to mimic the following pattern \( Y = A [1 - \cos(2\lambda \pi X)] \) where \( A \) is the dimensionless amplitude of the wavy surface and \( \lambda \) is the number of undulations.

2.1 Average Nusselt number

The average Nusselt number \((Nu)\) is expected to depend on a number of factors such as thermal conductivity, heat capacitance, viscosity, flow structure of nanofluids, volume fraction, dimensions, and fractal distributions of nanoparticles. The local variation of the convective Nusselt number of the fluid at the top cover plate is \( Nu_c = -k_{nf}/k_f \partial T/\partial y \).

The non-dimensional form of local convective heat transfer is \( \bar{Nu}_c = -k_{nf}/k_f \partial \theta/\partial Y \).

By integrating the local Nusselt number over the top heated surface, the average convective heat transfer along the heated wall of the collector is used by Saleh et al. [35] as \( Nu_c = \int \bar{Nu}_c \, dX \).

The radiated heat transfer rate is expressed as \( Nu_r = \int q_r \, dX \).

The average Nusselt number is \( Nu = Nu_c + Nu_r \).
The mean bulk temperature and average subdomain velocity of the fluid inside the collector may be written as \( \theta_{av} = \frac{\int \theta \, dV}{V} \) and \( \omega_{av} = \frac{\int \omega \, dV}{V} \), where \( V \) is the volume of the collector.

3. Numerical Implementation

The Galerkin finite element method [36, 37] is used to solve the non-dimensional governing equations along with boundary conditions for the considered problem. The equation of continuity has been used as a constraint due to mass conservation and this restriction may be used to find the pressure distribution. The penalty finite element method [38] is used to solve Eqs. (6) to (8), where the pressure \( P \) is eliminated by a penalty constraint \( \xi \), and the incompressibility criteria given by Eq. (8) which can be expressed as

\[
P = -\xi \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \tag{9}
\]

The continuity equation is automatically fulfilled for large values of \( \xi \). Then the velocity components \( (U, V) \) and temperature \( (\theta) \) are expanded using a basis set \( \{\Phi\}_{k=1}^{N} \) as

\[
U \approx \sum_{k=1}^{N} U_k \Phi_k (X,Y), \quad V \approx \sum_{k=1}^{N} V_k \Phi_k (X,Y) \quad \text{and} \quad \theta \approx \sum_{k=1}^{N} \theta_k \Phi_k (X,Y) \tag{10}
\]

The Galerkin finite element technique yields the subsequent nonlinear residual equations for Eqs. (6), (7), and (8) respectively at nodes of the internal domain \( \Omega \):

\[
R_{U}^{(1)} = \sum_{k=1}^{N} U_k \int_{\Omega} \left[ \left( \sum_{k=1}^{N} U_k \Phi_k \right) \frac{\partial \Phi_k}{\partial X} + \left( \sum_{k=1}^{N} V_k \Phi_k \right) \frac{\partial \Phi_k}{\partial Y} \right] \Phi_k \, dX \, dY - \xi \frac{\rho_f}{\rho_0} \sum_{k=1}^{N} U_k \int_{\Omega} \left[ \frac{\partial \Phi_k}{\partial X} \frac{\partial U_k}{\partial X} + \frac{\partial \Phi_k}{\partial Y} \frac{\partial U_k}{\partial Y} \right] \, dX \, dY - \left( \frac{Pr}{\nu_f} \sum_{k=1}^{N} V_k \int_{\Omega} \left[ \frac{\partial \Phi_k}{\partial X} \frac{\partial V_k}{\partial X} + \frac{\partial \Phi_k}{\partial Y} \frac{\partial V_k}{\partial Y} \right] \, dX \, dY \right)
\]

\[
R_{V}^{(2)} = \sum_{k=1}^{N} V_k \int_{\Omega} \left[ \left( \sum_{k=1}^{N} U_k \Phi_k \right) \frac{\partial \Phi_k}{\partial X} + \left( \sum_{k=1}^{N} V_k \Phi_k \right) \frac{\partial \Phi_k}{\partial Y} \right] \Phi_k \, dX \, dY - \xi \frac{\rho_f}{\rho_0} \sum_{k=1}^{N} V_k \int_{\Omega} \left[ \frac{\partial \Phi_k}{\partial X} \frac{\partial V_k}{\partial X} + \frac{\partial \Phi_k}{\partial Y} \frac{\partial V_k}{\partial Y} \right] \, dX \, dY - \left( \frac{Pr}{\nu_f} \sum_{k=1}^{N} U_k \int_{\Omega} \left[ \frac{\partial \Phi_k}{\partial X} \frac{\partial U_k}{\partial X} + \frac{\partial \Phi_k}{\partial Y} \frac{\partial U_k}{\partial Y} \right] \, dX \, dY \right) - Ra \left( \frac{1-\phi}{\rho_f \beta_f} + \phi \rho_f \beta_f \right) \int_{\Omega} \left( \sum_{k=1}^{N} \theta_k \Phi_k \right) \, dX \, dY \tag{11}
\]

\[
R_{\theta}^{(3)} = \sum_{k=1}^{N} \theta_k \int_{\Omega} \left[ \left( \sum_{k=1}^{N} U_k \Phi_k \right) \frac{\partial \Phi_k}{\partial X} + \left( \sum_{k=1}^{N} V_k \Phi_k \right) \frac{\partial \Phi_k}{\partial Y} \right] \Phi_k \, dX \, dY - \left( \frac{1}{Pr} \sum_{k=1}^{N} \theta_k \left[ \frac{\partial \Phi_k}{\partial X} \frac{\partial U_k}{\partial X} + \frac{\partial \Phi_k}{\partial Y} \frac{\partial U_k}{\partial Y} \right] \, dX \, dY \right) \tag{12}
\]
Three-point Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations (11), (12), and (13) are solved using the Newton–Raphson method to determine the coefficients of the expansions in Eq. (10). The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion \( \varepsilon \) such that \( |\Psi_{n+1} - \Psi_n| \leq 10^{-4} \), where \( n \) is the number of iteration and \( \Psi \) is a function of \( U, V, \) and \( \theta \).

### 3.1 Mesh generation

In the finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volume, etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Figure 2 displays the finite element mesh of the present physical domain.

### 3.2 Grid-independent test

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for \( Ra = 10^4, \Lambda = 0.075, \lambda = 3.5, \phi = 5\%, \) and \( Pr = 6 \) in a solar collector. In the present work, we examine five different non-uniform grid systems with the following number of elements within the resolution field: 2969, 5130, 6916, 9057, and 11,426. The numerical scheme is carried out for a highly precise key in the average convective and radiated Nusselt numbers, namely, \( Nu_c \) and \( Nu_r \) for the aforesaid elements, to develop an understanding of the grid fineness as shown in Table 2. In addition, Fig. 3 shows the convergence of average Nusselt numbers (convective and radiative) with grid refinement. The scale of the average Nusselt and Sherwood numbers for 9057 elements shows a little difference with the results obtained for the other elements. Hence, the non-uniform grid system of 9057 elements is preferred for the computation.

### 3.3 Code validation

The present numerical solution is validated by comparing the current code results for heat transfer-temperature difference profile at \( Pr = 0.73, Gr = 10^4 \) with the graphical representation of Gao et al. [39] which was reported for heat transfer augmentation inside a channel between the flat-plate cover and sine-wave absorber of a cross-corrugated solar air heater. Figure 4 demonstrates the above...
stated comparison. As shown in Fig. 4, the numerical solutions (present work and Gao et al. [39]) are in good agreement.

4. Results and Discussion

In this section, numerical results of streamlines and isotherms for various values of physical parameters such as amplitude of wave $A$ and number of wave $\lambda$ with an Al$_2$O$_3$/water nanofluid in a solar collector are displayed. The considered values of $A$ and $\lambda$ are $A = (0, 0.025, 0.05, \text{ and } 0.075)$, $\lambda = (0, 1.5, 2.5, \text{ and } 3.5)$ while the solid volume fraction $\phi = 5\%$, the emissivity $\varepsilon = 0.9$, the Rayleigh number $Ra = 10^4$, and Prandtl number $Pr = 6$. In addition, the values of the average Nusselt number both for convection and radiation as well as mean bulk temperature and average subdomain velocity profile are shown graphically.

<table>
<thead>
<tr>
<th>Nodes (elements)</th>
<th>$Nu_c$</th>
<th>$Nu_r$</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6224 (2969)</td>
<td>6.82945</td>
<td>4.32945</td>
<td>226.265</td>
</tr>
<tr>
<td>10982 (5130)</td>
<td>7.98176</td>
<td>5.38176</td>
<td>292.594</td>
</tr>
<tr>
<td>13538 (6916)</td>
<td>8.8701</td>
<td>5.88701</td>
<td>388.157</td>
</tr>
<tr>
<td>20295 (9057)</td>
<td>9.275492</td>
<td>6.004909</td>
<td>421.328</td>
</tr>
<tr>
<td>27524 (11426)</td>
<td>9.285492</td>
<td>6.014909</td>
<td>627.375</td>
</tr>
</tbody>
</table>

Table 2. Grid Sensitivity Check at $Pr = 6$, $A = 0.075$, $\lambda = 3.5$, $\phi = 5\%$, and $Ra = 10^4$
The effect of wave amplitude $A$ on the thermal and flow fields are presented in Figs. 5(a) and 5(b) while $\lambda = 3.5$. The strength of the flow circulation and the thermal current activities is much more activated with escalating $A$. Isotherms are almost similar to the active parts for both the nanofluid and base fluid. The temperature lines inside the solar collector take a wavy pattern for increasing $A$. But initially ($A = 0$), i.e., for the flat plate solar collector, they are horizontal due to the physical changes of the current geometry. Due to rising values of wave amplitude, the temperature distributions become distorted, resulting in an increase in the overall heat transfer. This result can be attributed to the performance of the corrugated surface. It is worth noting that as the wave amplitude increases, the thickness of the thermal boundary layer near the top cover plate enhances, which indicates steep temperature gradients and hence an increase in the overall heat transfer from the transparent cover plate to the wavelike absorber. On the other hand, for both types of fluids, a primary recirculation cell occupying the whole solar collector is found at the lowest value of the wave amplitude ($A = 0$). This cell rotates in a clockwise direction indicating that the fluid filling the solar collector is moving up along both the left insulated wall and the top heated wall, down along the insulated right wall, and horizontally to the left along the cool base wavy absorber of the solar collector. Escalating $A$ to 0.025, the primary cell divides and makes six vortices. In each wave, the right and left vortices rotate in a counter-clockwise and clockwise direction, respectively. The size of these cells becomes larger for clear water than the water-alumina nanofluid. This happens due to the greater velocity of base fluid rather than the nanofluid. For further increasing $A$, velocity of both types of fluids increases and thus the size of the created eddies becomes larger. As well, two tiny eddies appear near the left and right vertical walls in this case. In addition, more perturbation is observed in the streamlines at $A = 0.075$ because of rising wave amplitude.

![Fig. 5. Effect of $A$ on (a) isotherms and (b) streamlines at $\lambda = 3.5$ (solid lines for nanofluid and dashed lines for base fluid). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com/journal/htj.]](image)

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Figures 6(a) and 6(b) expose the heat transfer and fluid flow for various numbers of wave \( \lambda \) (= 0–3.5). In this figure we observe that as the number of wave enhances from 0 to 3.5, the isothermal contours tend to be affected considerably. In addition, these lines corresponding to \( \lambda = 0 \) (absence of wavy surface) become less bent whereas finally (\( \lambda = 3.5 \)) the lines take a sinusoidal wavelike form. At \( \lambda = 0 \), the isothermal lines almost coincide for both nanofluid and clear water. The rising number of waves leads to the formation of the thermal boundary layer at the heated top cover plate. However, the increase in the thermal gradients at the upper horizontal wall is much higher for the considered nanofluid than for the clear water. This means that a higher heat transfer rate is predicted by the nanofluid than the base fluid (water). Figure 6(b) shows that the fluid flow covers the entire collector in the absence of \( \lambda \), forming a big rotating eddy. The streamlines have a major change due to rising \( \lambda \). At \( \lambda = 1.5 \), there are two vortices created near the wavy part (middle of the bottom wall), as well as two small eddies which appear near the left and right top corner of the solar collector. Similarly, the number of eddy increases from two to four and six due to growing values of the number of the wave (\( \lambda \)) from 1.5 to 2.5 and 3.5, respectively. But the size of these vortices becomes smaller with a rising \( \lambda \).

The average Nusselt (convective and radiative) numbers, average temperature (\( \theta_{av} \)), and mean subdomain velocity (\( \omega_{av} \)) profile along with the wave amplitude (\( A \)) for both types of fluids are depicted in Figs. 7(i) to 7(iii). It is seen from Fig. 7(i) that \( N_{uc} \) and \( N_{ur} \) enhance gradually due to the performance of wave amplitude. The rate of heat transfer for the water-alumina nanofluid is found to be more effective than the clear water due to higher thermal conductivity of the solid nanoparticles. The rate of convective heat transfer enhances by 19% and 12% for the nanofluid and base fluid, respectively, whereas this rate for radiation is 6% with the variation of \( A \) from 0 to 0.075. It is well
known that the heat transfer rate is always higher for convection than radiation and is justified by the current investigation. Consequently Fig. 7(ii) shows that $(\theta_{av})$ falls sequentially for all values of $A$. Figure 7(iii) describes the enhancing phenomena in the $\omega_{av}-A$ profile for getting a smaller region to move. Here base fluid has higher mean velocity than the nanofluid.

Fig. 7. Effect of $A$ on (i) $Nu$, (ii) $\theta_{av}$, and (iii) $\omega_{av}$ at $\lambda = 3.5$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com/journal/htj.]

Fig. 8. Effect of $\lambda$ on (i) $Nu$, (ii) $\theta_{av}$, and (iii) $\omega_{av}$ at $A = 0.075$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com/journal/htj.]
Figures 8(i) to 8(iii) display the $N_{u_1}$, $N_{u_2}$, $\theta_{av}$, and $\omega_{av}$ for the effect of number of wave $\lambda$. An increasing $\lambda$ enhances the average Nusselt number for both convection and radiation. From Fig. 8(i), this is found with the increasing values of $\lambda$ from 0 to 3.5. On the other hand, $\theta_{av}$ devalues due to the variation of $\lambda$ as the average temperature of fluid decreases with a falling working area of the collector. Note that $\omega_{av}$ has notable changes with different values of the number of the wave.

5. Conclusion

The influences of physical parameters, wave amplitude, and number of the waves on the natural convection boundary layer flow inside a solar collector with water-$Al_2O_3$ nanofluid were analyzed. Various wave amplitudes and the number of the waves have been considered for the flow and temperature fields as well as the convective and radiative heat transfer rates, mean bulk temperature of the fluids, and average velocity field in the collector while $\phi$, $Ra$, $Pr$, and $\varepsilon$ are fixed at 5%, $10^4$, 6, and 0.9, respectively. The results of the numerical analysis lead to the following conclusions:

- The structure of the fluid streamlines and isotherms within the solar collector is found to significantly depend upon $A$ and $\lambda$.
- The $Al_2O_3$ nanoparticles with the highest wave amplitude and number of waves are established to be most effective in enhancing performance of heat transfer rate for the base fluid.
- Average heat transfer obtained is higher for convection than radiation.
- Mean temperature diminishes for both fluids with rising parameters.
- Average velocity field increases due to growing wave amplitude and number of waves.

Overall, the analysis also defines the operating range where an water-$Al_2O_3$ nanofluid can be considered effective in determining the level of heat transfer augmentation.

Literature Cited