Mixed Convection in Non-Newtonian Fluids along a Vertical Plate in Porous Media with Constant Surface Heat Flux

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***Abstract-* A non-similar boundary layer analysis is presented for the problem of mixed convection in power-law type non-Newtonian fluids along a vertical plate embedded in saturated porous medium. The boundary condition of uniform surface heat flux distribution is assumed. Mixed convection covers the heat transfer regime in which forced and free convection mechanisms are of a comparable magnitude, and as such both are considered in the analysis. Numerical results are presented for the details of the velocity and temperature fields. A discussion is provided for the effects of viscosity index and the surface mass transfer on the surface heat transfer rate.**

Keywords- Mixed Convection; Porous Media; Non-Newtonian Fluids

Nomenclature

f	dimensionless stream function;
g	acceleration due to gravity [m s^{-2}];
h	heat transfer coefficient [W m^{-2} K^{-1}];
k	thermal conductivity[W m^{-1} K^{-1}];
K	permeability for the porous medium;
L	plate length [m];
n	viscosity index;
Nu	Nusselt number;
Pe	Peclet number
q_w	wall heat flux[W m^{-2}];
Ra	Rayleigh number;
T	Temperature [K];
u,v	velocity components in x and y directions [m s^{-1}];
$U\infty$	free stream velocity [m s^{-1}];
x,y	axial and normal co-ordinates[m];
α	thermal diffusivity of porous medium [m^{2}s^{-1}];
β	volumetric coefficient of thermal expansion[K^{-1}];
η	similarity variable;
θ	dimensionless temperature;
ν	kinematic viscosity[m^{2}s^{-1}];
ξ	mass transfer parameter;
ρ	density of fluid[kg m^{-3}];
μ	consistency index for viscosity[kg m^{-1}s^{-1}];
τ_w	wall shear stress [N m^{-2}];
ψ	stream function;

Subscripts

w	wall conditions;
∞	free stream conditions;

I. INTRODUCTION

Combined forced and free convection flows, or mixed convection flows, arise in many transport processes in engineering devices in nature. Examples of mixed convection processes can be found in connection with heat exchangers, hot-wire anemometers, nuclear reactors, electronic devices, atmospheric boundary-layer flows, solar collectors, energy storage and heat rejection systems, etc.

Cheng and Minkowycz [1] presented similarity solutions for free convective heat transfer from a vertical plate in a fluid-saturated porous medium. Gorla and co-workers [2-4] solved the non-similar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. The mixed convection from surfaces embedded in porous media was studied by Minkowycz et al. [5] and Ranganathan and Viskanta [6]. Hsieh et al. [7] presented non-similar solutions for mixed convection in porous media. All these studies were concerned with Newtonian fluid flows. A number of industrially important fluids display non-Newtonian behavior and exhibit a non-linear relationship between shear stress and shear rate.

Chen and Chen [8] presented similarity solutions for natural convection of a non-Newtonian fluid over vertical surfaces in porous media. Nakayama and Koyama [9] studied the natural convection of a non-Newtonian fluid over non-isothermal body of arbitrary shape in a porous medium. Yang and Wang [10] studied the natural convection heat transfer of non-Newtonian power-law fluids with yield stress over axisymmetric and two-dimensional bodies of arbitrary shape embedded in a fluid-saturated porous medium. Rastogi and Poulikakos [11] considered the double diffusion from a plate in a porous medium saturated with a non-Newtonian power law fluid. Getachew et al. [12] examined the double-diffusive natural convection in a rectangular porous cavity saturated with a non-Newtonian power law fluid. Jumah and Mujumdar [13] studied the free convection heat and mass transfer of non-Newtonian power law fluids with yield stress over a vertical plate in saturated porous media subjected to constant wall temperature and concentration. Jumah and Mujumdar [14] studied the natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress over a vertical plate in saturated porous media subjected to variable wall temperature and concentration. Cheng [15] studied the natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes.

Mehta and Rao [16, 17] investigated buoyancy-induced flow of non-Newtonian fluids over a nonisothermal horizontal plate embedded in a porous medium. Gorla and coworkers [18-23] studied the mixed convection in non-Newtonian fluids along horizontal and vertical plates in porous media. Mansour and Gorla [24] studied the mixed convection–radiation interaction in power-law fluids along a non-isothermal wedge embedded in a porous medium. Jumah and Mujumdar considered the free convection heat and mass transfer of non-Newtonian power-law fluids with yield stress from a vertical flat plate in saturated porous media. Chamkha and Al-Humoud [25] studied the mixed convection heat and mass transfer of non-Newtonian fluids from a permeable surface embedded in a porous medium under uniform surface temperature and concentration of species.

The present work has been undertaken in order to analyze the mixed convection from a vertical plate in non-Newtonian fluid saturated porous media with uniform surface heat flux. The governing equations are first transformed into a dimensionless form and the resulting non-similar set of equations is solved by a finite difference method. Numerical results are presented for some representative values of the viscosity index.

II. ANALYSIS

Let us consider the mixed convection from a permeable vertical plate embedded in a non-Newtonian fluid-saturated porous medium, in the presence of surface injection or suction at a uniform velocity V_0. The co-ordinate system and flow model are shown in figure 1. We consider the Darcy model assuming low velocity and porosity. The governing equations under the Boussinesq and boundary layer approximations are given by,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u^n = U_\infty^n + \frac{K}{\mu}\left[\rho g \beta (T - T_\infty)\right], \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

In the above equations, T is the temperature of the wall, n is the viscosity index, ρ is the density, K is the permeability of the porous medium, β is the volumetric coefficient of thermal expansion, μ is the viscosity, and α is the equivalent thermal diffusivity of the porous medium.

The appropriate Boundary conditions are given by

$$y = 0:\ v = v_0,\ q_w = -k\frac{\partial T}{\partial y}$$

$$y = \infty:\ u = u_\infty,\ T = T_\infty, \tag{4}$$

The analysis is performed for the buoyancy assisting flow direction. Therefore, for an upward forced flow, we have $T_w > T_\infty$ and for downward flow, $T_w < T_\infty$. A stream function ψ defined such that,

$$u = \frac{\partial \psi}{\partial y}$$

and

$$v = -\frac{\partial \psi}{\partial x};$$

Proceeding with the analysis, we define

$$\eta = \frac{y}{x}\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)} + Ra_x^{\left(\frac{n}{2n+1}\right)}\right]$$

$$\xi = \frac{v_0 x}{\alpha}\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)} + Ra_x^{\left(\frac{n}{2n+1}\right)}\right]^{-1}$$

$$\psi = \alpha\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)} + Ra_x^{\left(\frac{n}{2n+1}\right)}\right]\cdot f(\xi,\eta)$$

$$\theta = \frac{(T - T_\infty)\left(Pe_x^{\left(\frac{n+1}{2n+1}\right)} + Ra_x^{\left(\frac{n}{2n+1}\right)}\right)}{\frac{q_w x}{\kappa}}$$

$$\chi = \left[1 + \frac{Ra_x^{\left(\frac{n}{2n+1}\right)}}{Pe_x^{\left(\frac{n+1}{2n+1}\right)}}\right]^{-1}, \tag{5}$$

$$Pe_x = \frac{U_\infty x}{\alpha},\ Ra_x = \frac{x}{\alpha}\left[\frac{\rho K g \beta q_w x}{\mu\kappa}\right]^{\frac{1}{n}}$$

Upon substituting Expression (5) into Equations (2) and (3) we have:

$$n\cdot(f')^{n-1}\cdot f'' = (1-\chi)^{2n+1}\cdot\theta', \tag{6}$$

$$\theta'' + \left(\frac{n+1}{2n+1}\right)\cdot f\,\theta' - \left(\frac{n}{2n+1}\right)\cdot[f'\cdot\theta] = \left(\frac{n}{2n+1}\right)\cdot$$

$$\xi \cdot \left[f' \cdot \frac{\partial \theta}{\partial \xi} - \theta' \cdot \frac{\partial f}{\partial \xi} \right], \quad (7)$$

The transformed boundary conditions are given by

$$\eta = 0: \ f(\xi,0) + \xi \cdot \frac{\partial f}{\partial \xi}(\xi,0) = -2\xi, \quad \theta'(\xi, 0) = -1$$

$$\eta \to \infty: \ f'(\xi,\infty) = Pe_x^{-\left(\frac{1}{2n+1}\right)} \chi^2, \quad \theta(\xi, \infty) = 0. \quad (8)$$

The primes in the previous equations denote partial differentiation with respect to η only. We note that $\chi = 0$ corresponds to pure natural convection whereas $\chi = 1$ corresponds to pure forced convection. ξ is positive for injection and negative for suction. In practical applications, it is usually the surface characteristics such as friction factor and Nusselt number that are of importance.

Defining the local Nusselt number $Nu_x = hx/k$, where the heat transfer coefficient $h = q_w/[T_w(x) - T_\infty]$,

we have

$$Nux = \frac{\left[Pe_x^{\left(\frac{n+1}{2n+1}\right)} + Ra_x^{\left(\frac{n}{2n+1}\right)} \right]}{\theta(\xi,0)}, \quad (9)$$

III. NUMERICAL SCHEME

The numerical scheme to solve Equations (6) and (7) adopted here is based on a combination of the following concepts:

(a) The boundary conditions for $\eta = \infty$ are replaced by

$$f'(\xi,\eta_{\max}) = Pe_x^{-\left(\frac{1}{2n+1}\right)} \chi^2, \quad \theta(\xi, \eta_{\max}) = 0$$

where η_{max} is a sufficiently large value of η at which the boundary Conditions (8) are satisfied. Here, we have set $\eta_{max} = 25$.

(b) The two dimensional domain of interest (ξ, η) is discretized with an equispaced mesh in the ξ-direction and another equispaced mesh in the η-direction.

(c) The partial derivatives with respect to η are valued by the second order difference approximation.

(d) Two iteration loops based on the successive substitution are used because of the nonlinearity of the equations.

(e) In each inner iteration loop, the value of ξ is fixed while each of the Equations (6) and (7) is solved as a linear second order boundary value problem of ODE on the η-domain. The inner iteration is continued until the nonlinear solution converges for the fixed value of ξ.

(f) In the outer iteration loop, the value of ξ is advanced. The derivatives with respect to ξ are updated after every outer iteration step.

In the inner iteration step, the finite difference approximation for Equations (6) and (7) is solved as a boundary value problem. We consider Equation (6) first. By defining f=Ø, Equation (6) may be written in the form

$$a_1\varphi'' + b_1\varphi' + c_1\varphi = s_1, \quad (11)$$

Where $a_1 = n \cdot (\varphi')^{n-1}$

$b_1 = C_1 = 0$

$$S_1 = (1-\chi)^{2n+1} \cdot \theta', \quad (12)$$

The coefficients a_1, b_1, C_1 and the source term in Equation (11) in the inner iteration step are evaluated by using the solution from the previous iteration step. Equation (11) is then transformed to a finite difference equation by applying the central difference approximations to the first and second derivatives. The finite difference equations form a tridiagonal system and can be solved by the tridiagonal solution scheme.

Equation (7) is also written as a second-order boundary value problem similar to Equation (12), namely:

$$a_2\theta'' + b_2\theta' + c_2\theta = s_2 \quad (13)$$

Where $a_2 = 1$

$$b2 = \left(\frac{n+1}{2n+1}\right) \cdot \varphi$$

$$c_2 = -\left(\frac{n}{2n+1}\right)\varphi'$$

$$S2 = \left(\frac{n}{2n+1}\right) \cdot \xi \cdot \left[\varphi' \cdot \frac{\partial \theta}{\partial \xi} - \theta' \cdot \frac{\partial \varphi}{\partial \xi} \right]. \quad (14)$$

The numerical results are affected by the number of mesh points in both directions. To obtain accurate results, a mesh sensitivity study was performed. After some trials, in the η-direction 190 mesh points were chosen whereas in the ξ-direction, 41 mesh points were used.

IV. RESULTS AND DISCUSSION

The velocity and temperature profiles are displayed in Figures 1-6 for the various values of the mass transfer parameter, ξ. The momentum and thermal boundary layer thicknesses increase as ξ increases in the case of injection and decrease with increasing suction. The velocity and temperature profiles tend to become box type as n and ξ approach a value of 2. The velocity in the boundary layer increases with suction parameter.

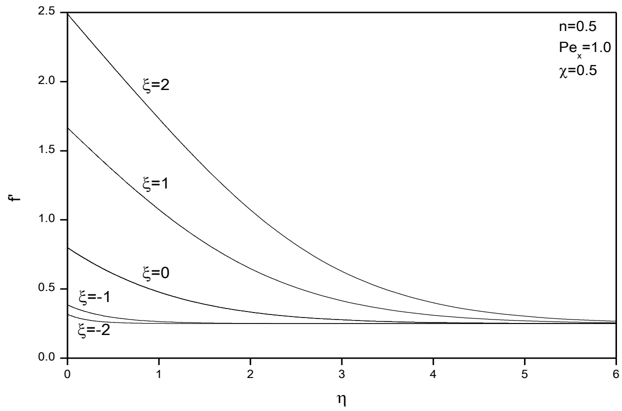

Fig. 1 Velocity profiles for different values of ξ (n=0.5)

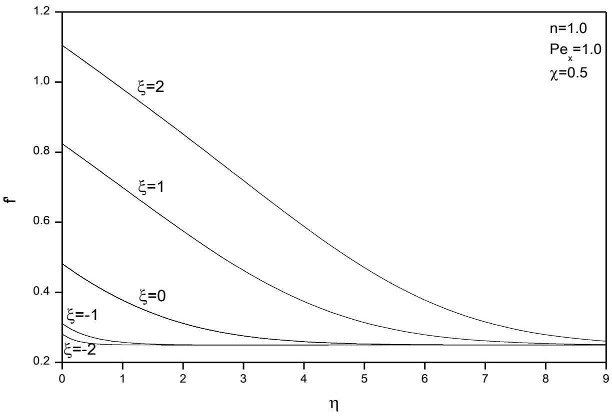

Fig. 2 Velocity profiles for different values of ξ (n=1.0)

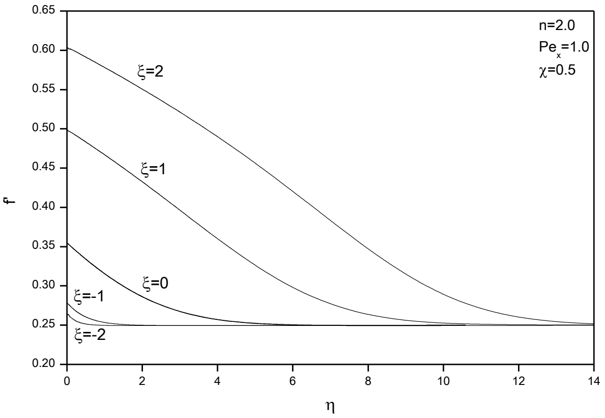

Fig. 3 Velocity profiles for different values of ξ (n=2.0)

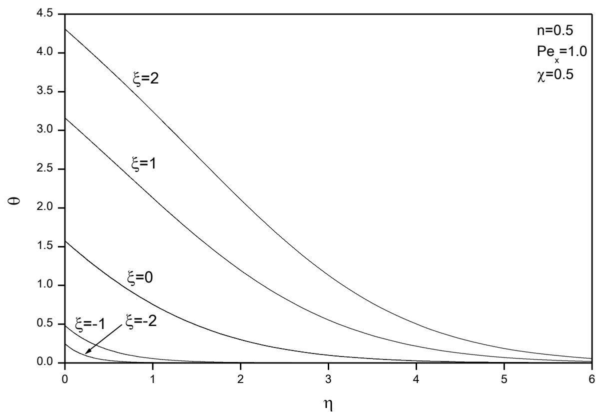

Fig. 4 Temperature profiles for different values of ξ (n=0.5)

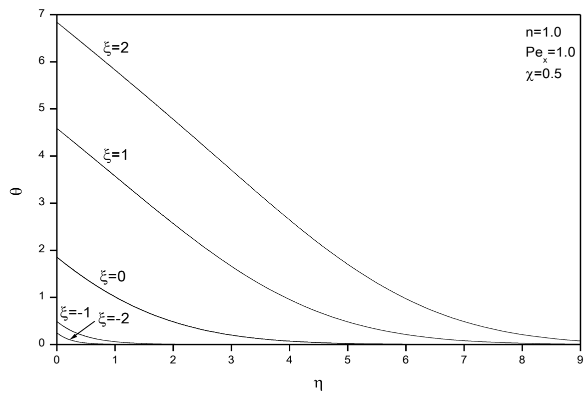

Fig. 5 Temperature profiles for different values of ξ (n=1.0)

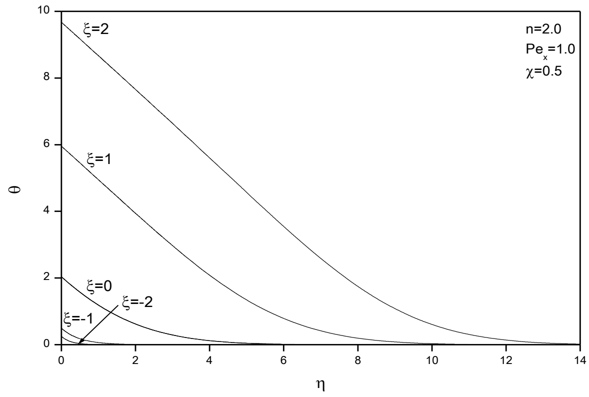

Fig. 6 Temperature profiles for different values of ξ (n=2.0)

Figures 7-11 display the variation of Nusselt number with χ for the cases of suction and injection. Figure 7 shows that increasing values of suction parameter ξ results in augmented surface heat transfer rate. This expected because the increased gravitational force would enhance heat transfer rates. As the viscosity index n increases, we notice that the Nusselt number decreases as shown in Figure 8. Therefore, non-Newtonian fluids display heat transfer reducing characteristics. This could be utilized in practical applications where a selection of non-Newtonian fluids would have to be made for effective heat transfer. Figures 9-11 display the variation in the Nusselt number for three different values of Pex as 1.0, 100 and 10000. As Pex increases, the heat transfer rate decreases. As the Peclet number increases, the forced convection dominates and therefore, a reduction in heat transfer rates is expected.

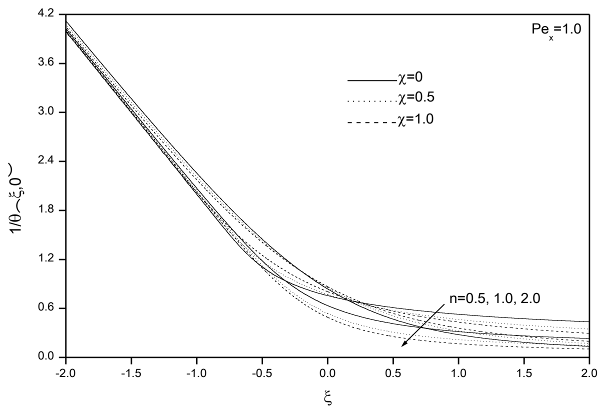

Fig. 7 Variation of local Nusselt number with ξ for different values of n and χ

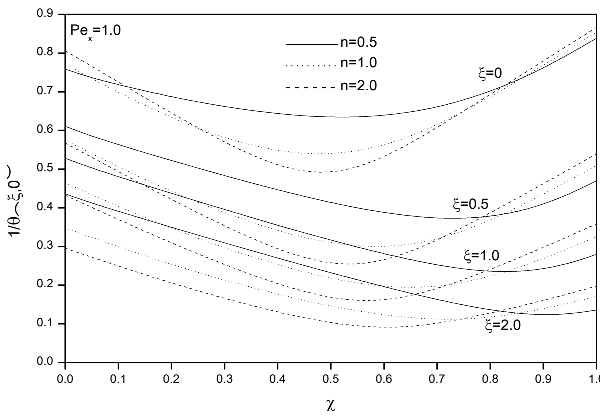

Fig. 8 Effects of n and χ on local Nusselt number for different values of ξ

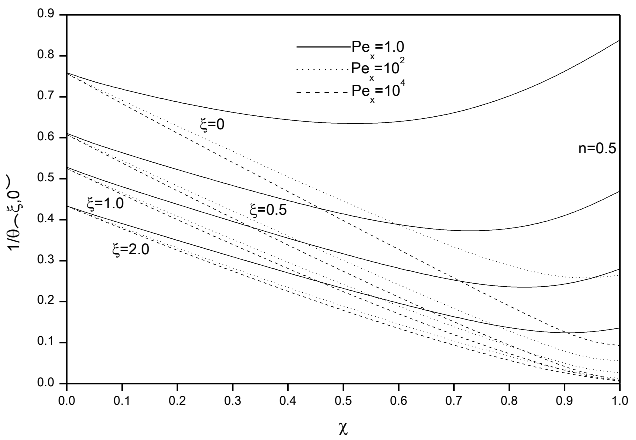

Fig. 9 Effects of Pex and χ on local Nusselt number for different values of ξ and n=0.5

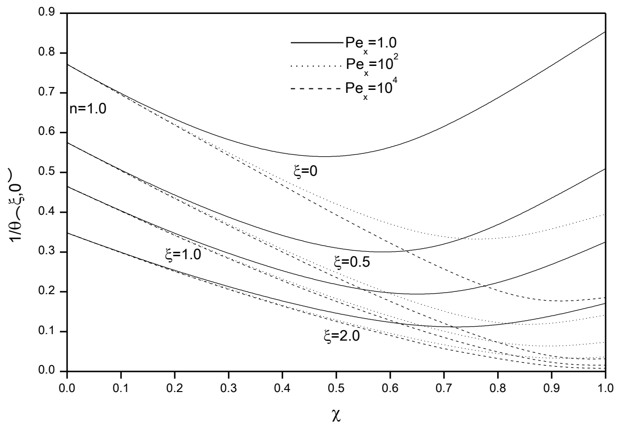

Fig. 10 Effects of Pex and χ on local Nusselt number for different values of ξ and n=1.0

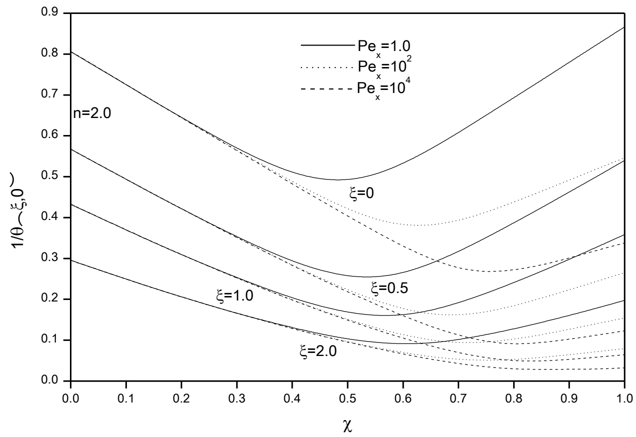

Fig. 11 Effects of Pex and χ on local Nusselt number for different values of ξ and n=2.0

V. CONCLUDING REMARKS

In this paper, we have presented a boundary layer analysis for the problem of mixed convection in non-Newtonian fluids along vertical plate embedded in a porous medium with constant surface heat flux boundary condition. Numerical results are presented for the velocity and temperature profiles as well as Nusselt number variation with the combined convection parameter χ. We examined the influence of suction and injection surface mass transfer and the viscosity index on the surface heat transfer rate. Suction increases the heat transfer whereas the injection decreases heat transfer rate. The combined convection mode promotes higher heat transfer rates when compared to forced convection alone.

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