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Unsteady Heat and Mass Transfer by MHD Mixed Convection Flow over an Impulsively Stretched Vertical Surface with Chemical Reaction and Soret and Dufour Effects

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This work considers unsteady, laminar, and coupled heat and mass transfer by MHD mixed convective boundary-layer flow of an electrically conducting fluid over an impulsively stretched vertical surface in an unbounded quiescent fluid with aiding external flow in the presence of a transverse magnetic field, homogeneous chemical reaction, and Soret and Dufour effects. The stretching velocity and surface temperature and concentration are assumed to vary linearly with the distance along the surface. The flow is impulsively set into motion and both the temperature and concentration at the surface are also suddenly changed from those of the ambient fluid. The governing partial differential equations are transformed into a set of non-similar equations and solved numerically by an efficient implicit, iterative, finite-difference method. A parametric study illustrating the influence of various physical parameters is performed. Numerical results for the steady-state velocity, temperature, and concentration profiles as well as the time histories of the skin-friction coefficient, local Nusselt number, and local Sherwood number are presented graphically and discussed.

Keywords Chemical reaction: Dufour and soret effects; Heat and mass transfer; Magnetohydrodynamics; Unsteady boundary layer flow

Introduction

The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of a magnetic field on the boundary-layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers due to its...
application in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extraction. By the application of a magnetic field, hydromagnetic techniques are used for the purification of molten metals from nonmetallic inclusions. Therefore, the type of problem considered in this work is very useful to polymer technology and metallurgy. The effect of a magnetic field on the flow over an unsteady stretching surface with or without heat and mass transfer was studied by a number of researchers. Chamkha (1998) studied unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching surface immersed in a porous medium. Chamkha (2000) also investigated transient hydromagnetic three-dimensional natural convection from an inclined stretching permeable surface. Xu and Liao (2005) investigated unsteady MHD flows of a non-Newtonian fluid over an impulsively stretching flat sheet. El-Kabeir et al. (2007) analyzed unsteady MHD combined convection over a moving vertical sheet in a fluid-saturated porous medium with uniform surface heat flux. Xu et al. (2007) analyzed unsteady three-dimensional MHD flow and heat transfer in the boundary layer over an impulsively stretching sheet analytically in the form of series solution. El-Kabeir et al. (2008) discussed unsteady MHD three-dimensional flow by natural convection from an inclined stretching surface embedded in a saturated porous medium. Modather et al. (2009) reported on MHD heat and mass transfer oscillatory flow of a micropolar fluid over an infinite moving permeable plate in a saturated porous medium. Kumari and Nath (2010) considered unsteady MHD mixed convection flow over an impulsively stretched permeable vertical surface. Abbas et al. (2010) included the effects of magnetic field and heat transfer on a stretching sheet in a rotating fluid.

In all of the above studies, the Soret and Dufour effects were assumed to be negligible. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials may be of a more intricate nature. Coupled heat and mass transfer finds applications in a variety of engineering problems such as the migration of moisture through the air contained in fibrous insulation and grain storage installations, filtration technology, chemical catalytic reactors and processes, spreading of chemical pollutants into the soil, and diffusion of medicine in blood. An energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is termed the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the Soret or thermal-diffusion effect. Such effects are significant when density differences exist in the flow regime. For example, when species are introduced at a surface in a fluid domain with a different (lower) density than the surrounding fluid, both Soret and Dufour effects can become influential. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in fluid binary systems that are often encountered in chemical process engineering.

Kafoussias and Williams (1995) considered boundary layer flows in the presence of Soret and Dufour effects associated with thermal diffusion and diffusion thermo for mixed forced-natural convection. Alam et al. (2006a) theoretically studied the problem of steady two-dimensional free convection and mass transfer flow past a continuously moving semi-infinite vertical porous plate in a porous medium including the effects of Soret and Dufour. Weaver and Viskanta (1991) pointed out that when the differences of the temperature and the concentration are large or when the difference of the molecular mass of the two elements in a binary mixture is great,
the coupled interaction is significant. Soret and Dufour effects were found by Anghel et al. (2000) to appreciably influence the flow field in a free convection boundary layer over a vertical surface embedded in a porous medium. Alam et al. (2006b) reported the effects of Dufour and Soret on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium numerically. Li et al. (2006) used an implicit finite-volume method to investigate Soret and Dufour effects in a strongly endothermic chemically reacting flow in a porous medium. Postelnicu (2007) discussed the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces embedded in fluid-saturated porous media considering Soret and Dufour effects. Alam et al. (2007) studied the diffusion-thermo and thermal-diffusion effects on free convective heat and mass transfer flow in a porous medium with time-dependent temperature and concentration. Chamkha and Ben-Nakh (2008) studied the Dufour and Soret effects on heat and mass transfer by mixed convection from a vertical permeable plate embedded in porous media in the presence of thermal radiation and magnetic field. Afify (2009) examined the effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface. Bég et al. (2009) analyzed free convection MHD heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects. El-Kabeir et al. (2010b) studied Soret and Dufour effects on heat and mass transfer by non-Darcy natural convection from a permeable sphere embedded in a high porosity medium. El-Kabeir et al. (2010a) investigated coupled heat and mass transfer on MHD stagnation-point flow of a power-law fluid towards a stretching surface with radiation, chemical reaction, and Soret and Dufour effects. Pal and Mondal (2011) studied the effects of Soret, Dufour, chemical reaction, and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet. Chamkha and Aly (2011) considered heat and mass transfer in stagnation-point flow of a polar fluid towards a stretching surface in porous media in the presence of Soret, Dufour, and chemical reaction effects.

There are many practical applications of these effects in industrial processes. There are still a relatively small number of published articles focused on this issue. Therefore, the objective of the present article is to investigate the unsteady simultaneous heat and mass transfer of an electrically conducting fluid flow over an impulsively stretched vertical surface in an unbounded quiescent fluid with assisting external laminar flow in the presence of a transverse magnetic field, chemical reaction, and Soret and Dufour effects. The effects of the governing parameters on the velocity, temperature, and concentration profiles as well as the variation of the rate of heat and mass transfer for the whole transient from initial state to final steady state are presented graphically and analyzed.

**Problem Formulation**

Consider unsteady, laminar, heat, and mass transfer by MHD mixed convection boundary layer flow of an electrically conducting fluid over a heated vertical linearly stretched sheet with assisting external laminar flow in the presence of a chemical reaction and Soret and Dufour effects. A uniform magnetic field is applied in the transverse direction $y$ normal to the plate. It is assumed that the wall is impulsively stretched with a velocity $U_e(x) = ax$, which is proportional to the distance along the sheet surface (see Figure 1). The sheet surface is maintained at a variable temperature
\[ T_w(x) = T_\infty + (T_0 - T_\infty)(x/L)^n \] and a variable concentration \[ C_w(x) = C_\infty + (C_0 - C_\infty)(x/L)^n \] such that \( n \) is a constant. Far from the sheet surface, the free stream is kept at a constant temperature \( T_\infty \) and a constant concentration \( C_\infty \). Initially \((t < 0)\), the temperature \( T_\infty \) and concentration \( C_\infty \) of the ambient fluid-saturated porous medium are quiescent. At \( t = 0 \), the fluid is impulsively started in motion with the velocity \( U_e(x) \) and both the temperature and concentration at the sheet are suddenly changed to constant values \( T_w > T_\infty \) and \( C_w = C_\infty \), respectively. Further, the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. In addition, the Hall effect and the electric field are assumed negligible. The small magnetic Reynolds number assumption uncouples the Navier-Stokes equations from Maxwell’s equations. All physical properties are assumed constant except the density in the buoyancy force term. By invoking all of the boundary layer and Boussinesq approximations, the governing equations for this investigation can be written as (see Ishak et al., 2006, and Kumari and Nath, 2010):

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u
\]
\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_v c_p} \frac{\partial^2 C}{\partial y^2} \]

where \( t, x, \) and \( y \) represent time, tangential distance, and transverse or normal distance, respectively; \( u, v, T, \) and \( C \) are the fluid tangential velocity, normal velocity, temperature, and concentration, respectively; \( g, \rho, \nu, \alpha, \) and \( D_m \) are the acceleration due to gravity, fluid density, kinematic viscosity, thermal diffusivity, and mass diffusivity, respectively; \( \sigma, B_o, K, \beta_T, \) and \( \beta_c \) are the fluid electrical conductivity, magnetic induction, dimensional chemical reaction parameter, thermal expansion coefficient, and concentration expansion coefficient, respectively; and \( c_p, c_s, \) and \( k_T \) are the specific heat at constant pressure, concentration susceptibility, and thermal diffusion ratio, respectively.

The corresponding initial and boundary conditions for this problem can be written as:

\[
\begin{align*}
t < 0 & : u(x,y,t) = 0, v(x,y,t) = 0, T(x,y,t) = T_\infty, C(x,y,t) = C_\infty \\
t > 0 & : u(x,0,t) = U_e(x) = ax, v(x,0,t) = 0, T(x,0,t) = T_w(x) \\
C(x,0,t) &= C_w(x), u(x,\infty,t) = 0, T(x,\infty,t) = T_\infty, C(x,\infty,t) = C_\infty
\end{align*}
\]

where \( a \) is a constant and \( v_w \) is the transpiration velocity of fluid through the surface of the plate or the permeability of the porous surface where its sign indicates suction or withdrawal of fluid (<0), blowing or injection of fluid (>0), and (=0) is the case when the sheet surface is impermeable.

It is convenient to nondimensionalize and transform Equations (1)–(4) by using

\[
\psi = (av)^{1/2} \zeta^{1/2} \xi, \quad \eta = (a/\nu)^{1/2} \zeta^{-1/2} y, \quad \zeta = 1 - e^{-t'},
\]

\[
t' = at, \quad \theta(\zeta, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\zeta, \eta) = \frac{C - C_\infty}{C_w - C_\infty},
\]

\[
v_w = v_0 \zeta^{1/2}, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}
\]

Substituting Equation (6) into Equations (1)–(4) yields:

\[
f''' + \frac{1}{2} \eta (1 - \zeta) f'' + \zeta (f f'' - f'^2 - Ha f' + \lambda (\theta + N \phi)) = \zeta (1 - \zeta) \frac{\partial f'}{\partial \zeta}
\]

\[
\frac{1}{Pr} \theta'' + \frac{1}{2} \eta (1 - \zeta) \theta' + \zeta (f \theta' - nf' \theta) + Du \phi'' = \zeta (1 - \zeta) \frac{\partial \theta}{\partial \zeta}
\]

\[
\frac{1}{Sc} \phi'' + \frac{1}{2} \eta (1 - \zeta) \phi' + \zeta (f \phi' - nf' \phi) + Sr \theta'' - \gamma \zeta \phi = \zeta (1 - \zeta) \frac{\partial \phi}{\partial \zeta}
\]

where Equation (1) is identically satisfied. In Equations (7)–(9), a prime indicates differentiation with respect to \( \eta \) and the parameters.
The transformed initial and boundary conditions become:

\[
\begin{align*}
    f(\xi, 0) &= f_w, f'(\xi, 0) = 1, \theta(\xi, 0) = 1, \phi(\xi, 0) = 1, \\
    f'(\xi, \infty) &= 0, \theta(\xi, \infty) = \phi(\xi, \infty) = 0
\end{align*}
\]

where \( f_w = -v_0/(\alpha \nu)^{1/2} \) is the suction/injection parameter.

Of special significance for this type of flow and heat transfer situation are the local skin-friction coefficient, local Nusselt number, and local Sherwood number. These physical parameters can be defined in dimensionless form as:

\[
\begin{align*}
    C_{f_x} &= -\frac{\mu(\partial u/\partial y)_{y=0}}{\rho U_e^2} = -\text{Re}_x^{-1/2}z^{-1/2}f''(\xi, 0) \\
    N_{ux} &= -\frac{x(\partial T/\partial y)_{y=0}}{(T_w - T_\infty)} = -\text{Re}_x^{-1/2}z^{-1/2}0'(\xi, 0) \\
    S_{hx} &= -\frac{x(\partial C/\partial y)_{y=0}}{(C_w - C_\infty)} = -\text{Re}_x^{-1/2}z^{-1/2}0'(\xi, 0)
\end{align*}
\]

**Numerical Method**

The initial-value problem represented by Equations (7)–(9) is nonlinear and possesses no analytical solution. Therefore, a numerical solution was sought for this problem. The standard implicit, iterative, finite-difference method discussed by Blottner (1970) has proven to be adequate and accurate for this type of problem and, therefore, it was chosen for the solution of Equations (7)–(9) subject to Equation (11). The computational domain was divided into 196 by 196 nodes in the \( \xi \) and \( \eta \) directions, respectively. Since the changes in the dependent variables are large in the immediate vicinity of the plate while these changes decrease greatly as the distance away from the plate increases, variable step sizes in the \( \eta \) direction were used. For the same reason, variable step sizes in the \( \xi \) direction were also employed. The initial step sizes employed were \( \Delta \eta_j = 0.001 \) and \( \Delta \xi_j = 0.001 \) and the growth factors were \( K_\eta = 1.03 \) and \( K_\xi = 1.03 \) such that \( \Delta \eta_n = K_\eta \Delta \eta_{n-1} \) and \( \Delta \xi_m = K_\xi \Delta \xi_{m-1} \). The...
convergence criterion used was based on the relative difference between the current and the previous iterations, which was set to $10^{-5}$ in the present work. These values were found to give accurate grid-independent results as verified by the comparison mentioned below. For more details on the numerical procedure, the reader is advised to read the article by Blottner (1970).

In order to access the accuracy of the numerical results, a comparison with previously published work for the case of steady state flow ($n = 1$) in the absence of the magnetic field, buoyancy force, and suction/injection effects ($Ha = N = f_w = 0$) with uniform wall temperature ($n = 1$) was performed. This comparison is presented in Table I. It is obvious from this table that excellent agreement between the results exists. This favourable comparison lends confidence to the graphical results to be reported in the next section.

### Results and Discussion

Figures 2–13 represent typical numerical results based on the solution of Equations (7)–(9). These results were obtained to illustrate the influences of the Hartmann number, Dufour number, Soret number, mixed convection parameter, suction/injection parameter, dimensionless chemical reaction parameter, and the wall temperature and concentration exponent on the profiles of the fluid tangential velocity and temperature and the solute concentration as well as the transient developments of the local skin-friction coefficient $C_{fx}$, local Nusselt number $Nu_x$, and the local Sherwood number $Sh_x$. It should be mentioned that in all the results, the conditions are intended for an electrically conducting fluid such as metal ammonia suspension ($Pr = 0.78$) polluted by water vapor ($Sc = 0.6$), which represents physically buoyant

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gases diffusing in a magnetoaerodynamic boundary layer convection flow with thermal-diffusion and diffusion-thermo effects present at a temperature of 25°C and one atmosphere pressure. The corresponding buoyancy force parameter (ratio of the buoyancy force due to mass diffusion to the buoyancy force due to the thermal diffusion) $N$ takes the values 0.5 or 1.0 for low concentration.

Figures 2(a)–(c) show typical unsteady-state fluid tangential velocity $f'$, temperature $\theta$, and concentration $\phi$ for various values of the magnetic Hartmann number $Ha$ and $\gamma$. Figures 2. Effects of $Ha$ and $\gamma$ on the (a) velocity, (b) temperature, and (c) concentration profiles.

**Figure 2.** Effects of $Ha$ and $\gamma$ on the (a) velocity, (b) temperature, and (c) concentration profiles.

**Figure 3.** Effects of $Ha$ and $\gamma$ on the local skin-friction coefficient.
$Ha$ and the chemical reaction parameter $\gamma$, respectively. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of the flow. This has a tendency to reduce the fluid tangential velocity at the expense of increasing its temperature and the solute concentration. This is clearly seen from the decreases in the profiles of $f'$ and increases in the profiles of $\theta$ and $\phi$ as $Ha$ increases, shown in Figures 2(a)–(c), respectively. Moreover, it can be seen that both the fluid velocity and solute concentration profiles decrease (which stands for severe destructive reactant) as the value of the chemical reaction parameter $\gamma$ increases, while an opposite effect is found for the fluid temperature in which it increases as $\gamma$ increases. This is expected since the increase of the diffusing species generation $\gamma$ causes the concentration boundary layer to become slightly thinner and the concentration of the diffusing species to decrease. This decrease in the concentration of the diffusing species reduces the mass diffusion, consequently reducing the fluid velocity and increasing its temperature.
Figures 3–5 illustrate the transient development of the local skin-friction coefficient $C_{fx}$, local Nusselt number $Nu_x$ (heat transfer rate), and the local Sherwood number $Sh_x$ (mass transfer rate) for different values of $Ha$ and $\gamma$, respectively. As mentioned before, increases in $Ha$ cause respective decreases in $f'$ and increases in $h$ and $\phi$. This results in increasing the slope of the tangential velocity and decreasing the slopes of the temperature and concentration. This has the direct effect of increasing $C_{fx}$ and decreasing both $Nu_x$ and $Sh_x$ due to increases in $Ha$, as depicted in Figures 3–5, respectively. Further, it can be seen that as $\gamma$ increases, both the local skin-friction coefficient and the local Sherwood number increase, while the opposite effect is found for the local Nusselt number. This is because as $\gamma$ increases, the concentration difference between the sheet surface and the fluid decreases, causing the

Figure 6. Effects of $\lambda$ and $n$ on the (a) velocity, (b) temperature, and (c) concentration profiles.

Figure 7. Effects of $\lambda$ and $n$ on the local skin-friction coefficient.
rate of mass transfer at the sheet surface to increase. On the other hand, the rate of heat transfer decreases as a result of the increase in fluid temperature. In addition, it is observed that, in general, both the local Nusselt number and the local Sherwood number increase as the dimensionless time $\xi$ increases, while an opposite effect is found for the local skin-friction coefficient.

Figures 6(a)–(c) depict the influence of increasing the mixed convection parameter $\lambda$ and the wall temperature and concentration exponent $n$ on the behavior of the unsteady profiles of velocity, temperature, and concentration, respectively. It can be seen that increasing the value of the mixed convection parameter $\lambda$ has a tendency to accelerate the flow. This, in turn, produces increases in the fluid velocity profiles while the temperature and concentration profiles decrease. The faster speed of the flow velocity is easily seen to take the heat and species away, leading to stabilization and reduction in the growth of the thermal and diffusion boundary layers along the vertical wall. These behaviors are clearly shown in Figures 6(a)–(c). Moreover, as $\lambda$ increases, the hydrodynamic boundary layer increases while the thermal

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**Figure 8.** Effects of $\lambda$ and $n$ on the heat transfer rate.

**Figure 9.** Effects of $\lambda$ and $n$ on the mass transfer rate.
and solutal boundary layers decrease. Also, for a given value of $\lambda$, an increase in the exponent $n$ tends to decelerate the flow around the stretching surface with reductions in the temperature and concentration profiles, as depicted in Figures 6(a)–(c).

Figures 7–9 present the effects of the mixed convection parameter $\lambda$, the wall temperature and concentration exponent $n$ on the time histories of the local skin-friction coefficient $C_f$, local Nusselt number $N_U$ (heat transfer rate), and the local Sherwood number $S_h$ (mass transfer rate), respectively. As indicated before, increasing the mixed convection parameter $\lambda$ causes increases in the velocity profiles and decreases in the temperature and concentration profiles. This causes the negative wall slopes of velocity to decrease, while the negative wall slopes of temperature and concentration profiles to increase.

![Figure 10](image1.png)

Figure 10. Effects of $Du$, $Sr$, and $f_w$ on the (a) velocity, (b) temperature, and (c) concentration profiles.

![Figure 11](image2.png)

Figure 11. Effects of $Du$, $Sr$, and $f_w$ on the local skin-friction coefficient.
concentration increase. Moreover, for \( \lambda > 0 \) (assisting flow), which is considered in this problem, there is a favorable pressure gradient due to the buoyancy forces, which results in flow acceleration, and, consequently, there is a smaller skin-friction coefficient than in the nonbuoyant case (\( \lambda \equiv 0 \)). This yields enhancement in the local Nusselt and Sherwood numbers and reduction in the local skin-friction coefficient. Moreover, it is seen that both the local Nusselt number \( N_{\text{ux}} \) and the local Sherwood number \( S_{\text{hx}} \) are enhanced by increasing the value of \( n \). This can be explained from Figures 6(a) and 6(b), where it is clearly seen that the negative wall slopes of these profiles became steep and increased as \( n \) increased.

Figures 10(a)–(c) present the effects of the suction/injection parameter \( f_w \) on the velocity, temperature, and concentration profiles for various values of the Dufour number \( Du \) and Soret number \( Sr \), respectively. For fixed values of \( Du \) and \( Sr \), imposition of wall fluid suction (\( f_w < 0 \)) tends to decelerate the flow around the sheet with reduced temperature and concentration profiles. On the other hand, imposition of

![Figure 12. Effects of \( Du \), \( Sr \), and \( f_w \) on the heat transfer rate.](image)
fluid injection or blowing at the sheet surface ($f_w > 0$) produces the opposite behavior, namely, an increase in the flow velocity and increases in the temperature and concentration, as depicted in Figures 10(a)–(c). The definition of Soret number characterizes the ratio of temperature difference to concentration, whereas the Dufour number is the opposite. Hence, an increasing Soret number stands for a larger temperature difference and precipitous gradient, while the Dufour number symbolizes the same meaning in mass transfer. Therefore, from these figures, it can be observed that the concentration profiles increase with increasing value of $Sr$ (or decreasing $Du$), whereas the fluid velocity and temperature decrease as $Sr$ increases (or $Du$ decreases). We note that this behavior is a direct consequence of the Soret effect, which produces a mass flux from lower to higher solute concentration driven by the temperature gradient.

Finally, Figures 11–13 illustrate the effects of the Dufour number $Du$, Soret number $Sr$, and the suction/injection parameter $f_w$ on the local skin-friction coefficient $C_{fx}$, local Nusselt number $Nu_x$ (heat transfer rate), and local Sherwood number $Sh_x$ (mass transfer rate), respectively. As the Dufour number increases, both the skin-friction coefficient and Nusselt number decrease, while the Sherwood number increases. This is because either increase in concentration difference or decrease in temperature difference leads to an increase in the value of $Du$, resulting in trends similar to the above observation. Similarly, either a decrease in concentration difference or an increase in temperature difference leads to an increase in the value of the Soret number $Sr$. Therefore, increasing the parameter $Sr$ causes increases in the skin-friction coefficient and the local Nusselt number while it produces decreases in the local Sherwood number. This is because of the smaller $Sr$ and the larger $Du$, which produces stronger diffusion-thermo but weaker thermal-diffusion effects. However, both $Nu_x$ and $Sh_x$ are enhanced as the suction/injection parameter $f_w$ increases. Also, the negative wall slopes of the velocity profiles, which correspond to the skin-friction coefficient $C_{fx}$, became steep and slightly increased as $f_w$ increased.

**Conclusion**

The problem of unsteady, laminar, and coupled heat and mass transfer by MHD mixed convective boundary-layer flow of an electrically conducting fluid over an impulsively stretched vertical surface in an unbounded quiescent fluid with aiding external flow in the presence of a transverse magnetic field, chemical reaction, and Soret and Dufour effects was formulated. The surface of the stretching sheet was maintained at a variable temperature and concentration. It was also assumed to be permeable so as to allow for fluid wall suction or injection. The obtained non-similar differential equations were solved numerically by an efficient, implicit finite-difference method. From the above numerical investigation the following conclusions could be drawn:

1. It was found that the local skin-friction coefficient increased as either the Hartmann number, chemical reaction parameter, suction/injection parameter, or the wall temperature and concentration exponent increased, while it decreased as the mixed convection parameter increased.
2. The local Nusselt number was increased as either the mixed convection parameter, suction/injection parameter, or the wall temperature and concentration
exponent increased, while it decreased as either the Hartmann number or the chemical reaction parameter increased.

3. The local Sherwood number increased as either the mixed convection parameter, suction/injection parameter, chemical reaction parameter, the wall temperature or concentration exponent increased, while the opposite behavior was observed as the Hartmann number increased.

4. As the Dufour number increased or the Soret number decreased, both the skin-friction coefficient and the local Nusselt number were reduced while the Sherwood number was enhanced.

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Nomenclature

- \( a \): constant defined by Equation (5)
- \( B_0 \): magnetic induction
- \( C \): concentration
- \( C_{fx} \): local skin-friction coefficient
- \( C_w \): concentration at the vertical surface
- \( C_\infty \): ambient concentration attained as \( y \) tends to infinity
- \( c_p \): specific heat at constant pressure
- \( c_s \): concentration susceptibility
- \( D_m \): mass diffusivity
- \( Du \): Dufour number
- \( f \): dimensionless stream function
- \( f_w \): suction/injection parameter
- \( g \): gravitational acceleration vector
- \( G \): local Grashof number
- \( Ha \): Hartmann number
- \( K \): dimensional chemical reaction
- \( k_T \): thermal diffusion ratio
- \( L \): characteristic length
- \( n \): wall temperature and concentration exponent
- \( Nu_x \): local Nusselt number
- \( Pr \): Prandtl number
- \( Re_x \): local Reynolds number
- \( Sc \): Schmidt number
- \( Sh_x \): local Sherwood number
- \( Sr \): Soret number
- \( t \): time
- \( T \): temperature
- \( T_m \): mean fluid temperature
- \( T_w \): temperature at vertical plate
- \( T_\infty \): ambient temperature attained as \( y \) tends to infinity
- \( U_e \): stretching velocity
\( u, v \) velocity components
\( v_w \) transpiration velocity of fluid through the surface
\((x, y)\) Cartesian coordinates

**Greek letters**
- \( \alpha \) thermal diffusivity
- \( \beta_c \) concentration expansion coefficient
- \( \beta_T \) thermal expansion coefficient
- \( \gamma \) dimensionless chemical reaction parameter
- \( \theta \) dimensionless temperature
- \( \lambda \) mixed convection parameter
- \( \nu \) kinematic viscosity
- \( \rho \) fluid density
- \( \sigma \) fluid electrical conductivity
- \( \phi \) dimensionless concentration
- \( \psi \) stream function

**Subscripts**
- \( w \) conditions at the wall
- \( \infty \) conditions in the free stream

**References**


