Heat and Mass Transfer by Mixed Convection from a Vertical Slender Cylinder with Chemical Reaction and Soret and Dufour Effects

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This work is focused on the study of heat and mass transfer by mixed convection over a vertical slender cylinder in the presence of chemical reaction and thermal-diffusion and diffusion-thermo effects. The resulting equations have the property whereby they reduce to various special cases previously considered in the literature. An adequate implicit, tri-diagonal finite-difference scheme is employed for the numerical solution of the obtained equations. Various comparisons with previously published work are performed and the results are found to be in excellent agreement. Representative results for the local skin-friction coefficient, local Nusselt number, and the local Sherwood number illustrating the influence of the surface transverse curvature parameter, Richardson number, concentration to thermal buoyancy ratio, Schmidt number, chemical reaction, and the Dufour and Soret numbers are presented and discussed. © 2013 Wiley Periodicals, Inc. Heat Trans Asian Res, 42(7): 618–629, 2013; Published online 20 April 2013 in Wiley Online Library (wileyonlinelibrary.com/journal/htj). DOI 10.1002/htj.21045

Key words: heat and mass transfer, mixed convection, chemical reaction, Soret number, Dufour number, vertical slender cylinder

1. Introduction

The analysis of mixed convection boundary-layer flow over a vertical cylinder embedded in viscous fluids has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors, and so on. Several authors have studied the problem of mixed convection about different surface geometries. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials may be of a more intricate nature. An energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is termed the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the Soret or thermal-diffusion effect. Such effects are significant when density differences exist in the flow regime. For example, when species are introduced at a surface in a fluid...
domain, with a different (lower) density than the surrounding fluid, both Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects can become influential. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in fluid binary systems, often encountered in chemical process engineering. Tewfik and Yang [1] experimentally investigated helium mass transfer via a porous horizontal cylinder and also confirmed the major influence of Soret and Dufour phenomena on transpiration-cooled boundary layer flows. An analytical study of convection along a horizontal cylinder for a helium–air system was reported subsequently by Sparrow et al. [2]. Sparrow et al. [3] further described their experimental investigations using hydrogen, helium, carbon dioxide, and Freon-12, injected via a horizontal porous cylinder, indicating that for injected gases with a lower density than air (i.e., helium and hydrogen), Soret effects on the flow field were considerable.

A further study of Dufour and Soret effects on convection heat and mass transfer past a vertical surface was later communicated by Zeh and Gill [4]. It has, therefore, been highlighted that Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering. Mac Gowan and Evans [5] studied the heat and mass transfer in binary liquid mixtures using molecular dynamic algorithms to elucidate more exactly the cross-coupling between Soret and Dufour effects. Dursunkaya and Worek [6] have studied Soret/Dufour effects on both unsteady and steady natural convection from vertical surfaces. Bergeron et al. [7] numerically studied the Marangoni convective flow in binary mixtures with Soret effects. Seddeek [8] studied thermal-diffusion and diffusion-thermo effects on mixed convection flow over an accelerating surface with a heat source with suction/blowing for the case of variable viscosity. Both free and forced convection boundary layer flows with Soret and Dufour effects have been addressed by Abreu et al. [9]. Chamkha and Ben-Nakhi [10] analyzed the MHD mixed convection flow under the radiation interaction along a vertical permeable surface immersed in a porous medium in the presence of Soret and Dufour effects. Recently, Bég et al. [11] used the local non-similarity method with a shooting procedure to analyze mixed convective heat and mass transfer from an inclined plate with Soret/Dufour effects with applications in solar energy collector systems. Bhargava et al. [12] also studied oscillating hydromagnetic heat and mass transfer with Soret and Dufour effects. Tsai and Huang [13] studied heat and mass transfer for Soret and Dufour effects on Hiemenz flow through porous medium onto a stretching surface. El-Kabeir et al. [14] presented a discussion on Soret and Dufour effects on heat and mass transfer by non-Darcy natural convection from a permeable sphere embedded in a high porosity medium with chemically-reactive species. El-Kabeir et al. [15] discussed heat and mass transfer by MHD stagnation-point flow of a power-law fluid towards a stretching surface with radiation, chemical reaction, and Soret and Dufour effects. Bég et al. [16] analyzed free convection heat and mass transfer from an isothermal sphere to a micropolar regime with Soret/Dufour effects. El-Kabeir [17] studied the Soret and Dufour effects on heat and mass transfer by mixed convection over a vertical permeable surface saturated porous medium with chemical reaction and temperature-dependent viscosity. He [18] has also studied the Soret and Dufour effects on heat and mass transfer due to a stretching cylinder saturated porous medium with chemically-reactive species.

In the present article, we analyze interactive effects of the chemical reaction and the Soret/Dufour parameters on mixed convection flow over a vertical slender cylinder. The cylinder surface is maintained with uniform surface temperature and uniform surface concentration, and the order of chemical reaction in this work is taken as a first order. The boundary-layer equations
governing the flow and heat and mass transfer are reduced to non-similarity equations which are solved using the implicit, iterative, finite-difference method. Numerical results for the local skin-friction coefficient and local heat and mass transfer parameters are presented and discussed.

2. Mathematical Formulation

Consider steady, incompressible, laminar, two-dimensional heat and mass transfer by a mixed convection boundary-layer flow over a permeable vertical slender cylinder of length \( L \) and outer radius \( r_0 (L \gg r_0) \) in the presence of chemical reaction, thermal-diffusion, and diffusion-thermo effects. The physical model and coordinate system are shown in Fig. 1. The gravitational acceleration, \( g \), acts in the downward direction. The velocity at a distance remote from the cylinder is given by \( u_\infty \) and the cylinder surface is maintained at a constant temperature \( T_w \) and a constant concentration \( C_w \), and the ambient temperature and concentration far away from the surface of the cone \( T_\infty \) and \( C_\infty \) are assumed to be uniform. For \( T_w \gg T_\infty \) and \( C_w \gg C_\infty \) an upward flow is induced as a result of the thermal and concentration buoyancy effects. A first-order homogeneous chemical reaction is assumed to take place in the flow and fluid suction or injection is imposed at the cylinder surface. Following Sparrow et al. [2], we assume that the Dufour effect may be described by a second-order concentration derivative with respect to the transverse coordinate in the energy conservation equation, and the Soret effect by a second-order temperature derivative in the concentration equation. Under these assumptions, along with Boussinesq’s approximation, the governing equations for the continuity, momentum, energy, and species diffusion in a laminar incompressible boundary-layer flow can be written as follows (see Chang [19] and Aydin and Kaya [20]):

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{v}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty)
\]
The corresponding boundary conditions for this problem can be written as

\[ r = r_0, u = 0, v = v_w(x), T = T_w, C = C_w \] (5a)
\[ r \to \infty, u \to u_\infty, T \to T_\infty, C \to C_\infty \] (5b)

where \( u \) and \( v \) are the velocity components in the \( x \)- and \( r \)-direction, respectively. \( T \) and \( C \) are the temperature and species concentration, respectively, where \( g, \nu, u_\infty, D, \beta_T, \) and \( \beta_C \) are the gravitational acceleration, kinematic viscosity, velocity at a distance remote from the cylinder, mass diffusivity, coefficient of thermal expansion, and the coefficient of concentration of expansion, respectively. \( K, C_p, T_m, K_T, \) and \( C_s \) are the dimensional chemical reaction parameter, specific heat at constant pressure, mean fluid temperature, thermal diffusion ratio, and the concentration susceptibility, respectively. Here \( v_w \) is the uniform surface suction (\(<0\)) or blowing (\(>0\)) velocity. The last term on the right-hand side of each energy equation (3) and diffusion equation (4) signifies the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively.

It is convenient to non-dimensionalize and transform Eqs. (1) through (4) by using

\[ \xi = \frac{4}{r_0} \left( \frac{v_w}{u_\infty} \right)^{1/2}, \eta = \left[ \frac{r^2 - r_0^2}{4r_0} \right]^{1/2} \left( \frac{u_\infty}{v_x} \right)^{1/2}, \psi = r_0 (nu_x x)^{1/2} f (\xi, \eta), \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \] (6)

Substituting Eqs. (6) into Eqs. (1) through (4) yields:

\[ (1 + \xi \eta)^f'' + \xi f'' + f'' + Ri \xi^2 (\theta + N \phi) = \xi \left( f' \frac{\partial f'}{\partial \xi} - f \frac{\partial f}{\partial \xi} \right) \] (7)

\[ \frac{1}{Pr} (1 + \xi \eta) \theta'' + \xi \theta' + f \theta' + Du \left[ (1 + \xi \eta) \phi'' + \xi \phi' \right] = \xi \left( f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \] (8)

\[ \frac{1}{Sc} (1 + \xi \eta) \phi'' + \xi \phi' + f \phi' + Sr \left[ (1 + \xi \eta) \theta'' + \xi \theta' \right] - \gamma \xi^2 \phi = \xi \left( f' \frac{\partial \phi}{\partial \xi} - \phi' \frac{\partial f}{\partial \xi} \right) \] (9)

where Eq. (1) is identically satisfied. In Eqs. (7)–(9), a prime indicates differentiation with respect to \( \eta \) and the parameters

\[ Ri = \frac{Gr}{Re}, Re = \frac{u_\infty r_0}{\nu}, Gr = \frac{g \beta_T (T_w - T_\infty)}{2 \nu^2}, N = \frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}, Du = \frac{D_m k_T (C_w - C_\infty)}{c_p \nu (T_w - T_\infty)} \]
are the Richardson number, Reynolds number, Grashof number, ratio of the buoyancy forces due to the temperature and concentration, Dufour number, Soret number, dimensionless chemical reaction parameter, Schmidt number, Prandtl number, and the suction/injection parameter, respectively. It should be noted that $f_w < 0$ for $v_w > 0$ corresponds to the case of injection and $f_w > 0$ for $v_w < 0$ corresponds to the case of suction.

The transformed boundary conditions become:

$$
\eta = 0; f' = 0, f + \frac{\xi}{\partial f/\partial \xi} = -f_w, \theta = 1, \phi = 1
$$

$$
\eta \to \infty; f' = 2, \theta = 0, \phi = 0
$$

Of special significance for this type of flow and heat transfer situation are the local skin-friction coefficient, local Nusselt number, and the local Sherwood number. These physical parameters can be defined in dimensionless form as

$$
2C_f \cdot Re_{\xi}^{1/2} = f''(\xi, 0)
$$

$$
2Nu_x \cdot Re_{\xi}^{-1/2} = -\theta'(\xi, 0)
$$

$$
2Sh_x \cdot Re_{\xi}^{-1/2} = -\phi'(\xi, 0)
$$

3. Numerical Method

The non-similar equations (10) through (12) are nonlinear and possess no analytical solution and must be solved numerically. The efficient, iterative, tri-diagonal implicit finite-difference method discussed by Blottner [21] has proven to be adequate for the solution of such equations. The equations are linearized and then discretized using three-point central difference quotients with variable step sizes in the $\eta$ direction and using two-point backward difference formulas in the $\xi$ direction with a constant step size. The resulting equations form a tri-diagonal system of algebraic equations that can be solved by the well-known Thomas algorithm (see Blottner [21]). The solution process starts at $\xi = 0$ where Eqs. (7) through (9) are solved and then marches forward using the solution at the previous line of constant $\xi$ until it reaches the desired value of $\xi$. Due to the nonlinearities of the equations, an iterative solution with successive over or under relaxation techniques is required. The convergence criterion required that the maximum absolute error between two successive iterations be $10^{-6}$. The computational domain was made of 196 grids in the $\eta$ direction and 201 grids in the $\xi$ direction. A starting step size of 0.001 in the $\eta$ direction with an increase of 1.038 times the previous step size and a constant step size in the $\xi$ direction of 0.01 were found to give very accurate results. The maximum value of $\eta (\eta_{\infty})$ which represented the ambient conditions was assumed to be 38. The step sizes employed were arrived at after performing numerical experimentations to assess grid independence and ensure accuracy of the results. The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by Chang [19] and Aydin.
and Kaya [20] for the case of forced convection flow about an impermeable vertical cylinder 
\( Ri = f_w = 0 \) in the absence of concentration buoyancy, chemical reaction, and Soret/Dufour effects 
\( N = \gamma = Sr = Du = 0 \). Table 1 presents the results of these various comparisons. It can be seen 
from this table that excellent agreement between the results exists. This favorable comparison 
lends confidence in the numerical results to be reported in the next section.

4. Results and Discussion

In this section, a representative set of graphical results for the local skin-friction coefficient 
\( f''(\xi, 0) \), local Nusselt number \(-\theta'(\xi, 0)\) (rate of heat transfer), and the local Sherwood number 
\(-\phi'(\xi, 0)\) (rate of mass transfer) is presented and discussed for various parametric conditions. These 
conditions are intended at a temperature of 25 °C and 1 atmospheric pressure. The values of the 
Schmidt number \( Sc \) are chosen to represent the most common diffusing chemical species which are 
of interest and the values of the Dufour and Soret numbers are chosen in such a way that their product 
is constant provided that the mean temperature \( T_m \) is kept constant as well.

Figures 2 to 4 present the effects of the Dufour number \( Du \), Soret number \( Sr \), and the buoyancy ratio \( N \) on the local skin-friction coefficient \( f''(\xi, 0) \), local Nusselt number \(-\theta'(\xi, 0)\), and the local

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Table 1. Comparison of Values of \( f''(\xi, 0) \) and \(-\theta'(\xi, 0)\) for Various Values of \( \xi \) with 
\( f_w = N = Ri = Sr = Du = 0 \), and \( \gamma = 0 \)

<table>
<thead>
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<td>3.0</td>
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<tr>
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<td>3.217243</td>
<td>1.4925</td>
<td>1.4918</td>
<td>1.489833</td>
</tr>
</tbody>
</table>

Fig. 2. Effects of \( N \), \( Sr \), and \( Du \) on the local skin-friction coefficient.
Fig. 3. Effect of $N$, $Sr$, and $Du$ on the local Nusselt number.

Fig. 4. Effect of $N$, $Sr$, and $Du$ on the local Sherwood number.

Fig. 5. Effects of $f_w$ and $\gamma$ on the local skin-friction coefficient.
Sherwood number \( \phi'(\xi, 0) \), respectively. It is predicted that for \( N = 0 \) the flow around the cylinder surface is induced only by the thermal buoyancy effect due to the temperature gradient. However, for \( N > 0 \) the flow around the cylinder is caused by both temperature and concentration gradients. In this case, the flow will be aided by the concentration buoyancy effects. Therefore, as \( N \) increases the local skin-friction coefficient, local Nusselt number, and the local Sherwood number all increase. On the other hand, increasing the value of \( Sr \) (or decreasing \( Du \)) causes increases in the fluid temperature and decreases in the solute concentration. Since the Soret effect appears in the concentration equation (9), the changes in the local Sherwood number that are brought about by changing \( Sr \) are much more significant than those in the local Nusselt number. These behaviors are evident from Figs. 3 and 4.

Figures 5 to 7 show the effects of the suction/injection parameter \( f_w \) and the chemical reaction parameter \( \gamma \) on the local skin-friction coefficient \( f''(\xi, 0) \), local Nusselt number \( -\theta'(\xi, 0) \), and the local Sherwood number \( \phi'(\xi, 0) \), respectively. It can be seen that, as the suction/injection parameter \( f_w \) increases, the local skin-friction coefficient \( f''(\xi, 0) \), local Nusselt number \( -\theta'(\xi, 0) \), and the local
Sherwood number $-\phi'(\xi, 0)$ decrease. Also, as the chemical reaction parameter $\gamma$ increases, the local Sherwood number increases, while the opposite effect is found for both the local skin-friction coefficient and the local Nusselt number. This is because as $\gamma$ increases, the concentration difference between the cylinder surface and the fluid decreases and consequently the rate of mass transfer at the cylinder surface must increase, while both the skin-friction coefficient and the rate of heat transfer decrease.

Finally, Figs. 8 to 10 show the effects of the Richardson number $Ri$ and the surface curvature parameter $\xi$ on the local skin-friction coefficient $f''(\xi, 0)$, local Nusselt number $-\theta'(\xi, 0)$, and the local Sherwood number $-\phi'(\xi, 0)$, respectively, for three values of the Schmidt number $Sc$ [reciprocal of different gases like hydrogen ($Sc = 0.22$), oxygen ($Sc = 0.6$), and water vapor ($Sc = 0.66$)]. It is observed from these figures that as $Sc$ increases, both the skin-friction coefficient and the local Nusselt number decrease while the opposite trend is observed for the local Sherwood number. These results are expected because increases in the values of Schmidt number $Sc$ cause the temperature profile and its
boundary layer thickness to increase while the concentration profile and its boundary layer decrease. On other hand, the Richardson number $R_i$ represents a measure of the effect of the buoyancy in comparison with that of the inertia of the external forced or free stream flow on the heat and fluid flow. Outside the mixed convection region, either the pure forced convection or the free convection analysis can be used to describe accurately the flow or the temperature field. Forced convection is the dominant mode of transport when $R_i = 0$, whereas free convection is the dominant mode when $R_i \to \infty$. Buoyancy forces can enhance the surface heat transfer rate when they assist the forced convection. However, it is found that increasing the Richardson number $R_i$ or the surface curvature parameter $\xi$ leads to increases in local skin-friction coefficient, local Nusselt number, and local Sherwood number.

5. Conclusions

The effect of chemical reaction on coupled heat and mass transfer by mixed convection boundary-layer flow of a chemically-reacting fluid over a vertical slender cylinder with constant surface temperature and species concentration in the presence of Dufour and Soret effects was investigated. Comparisons with previously published work were performed and the results are found to be in excellent agreement. Numerical results for the local skin-friction coefficient, local Nusselt number, and local Sherwood number were reported graphically. It was found that the local skin-friction coefficient, local Nusselt number, and the local Sherwood number increased due to increases in either the Richardson number, buoyancy ratio or the surface curvature parameter, whereas they decreased as the suction/injection parameter was increased. However, increases in the values of the chemical reaction parameter and the Schmidt number produced decreases in both the local skin-friction coefficient and the local Nusselt number and increases in the local Sherwood number. Finally, both the local skin-friction coefficient and the local Sherwood number were enhanced, whereas the local Nusselt number was reduced with increasing values of the Dufour number and decreasing values of the Soret number. It is hoped that the present work will serve as a motivation for future experimental work which seems to be lacking at the present time.
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