The Taylor dispersion of a solute for a composite porous medium between two parallel plates is studied analytically. The fluids in both the regions are incompressible and the transport properties are assumed to be constant. The closed-form solutions are obtained in both fluid regions of the channel. The results are presented graphically for various values of porosity, pressure gradient, height of the channel, viscosity, and diffusivity on the concentration. The volumetric flow rate in the channel for variation of viscosity, height, and porous parameter is also found. The validity of the results obtained for composite porous media is verified by comparison with the available porous medium results in the literature for one fluid model, and good agreement is found.

KEY WORDS: Taylor dispersion, two-fluid model, horizontal channel, porous media

1. INTRODUCTION

Studies on natural convection in porous enclosures have important applications in engineering and environmental research. Heat exchangers, underground spread of pollutants, environmental control, grain storage, food processing, material processing, geothermal systems, oil extraction, optimal design of furnaces, and packed-bed catalytic reactors are just a few examples of applications of this subject of study. Nield and Bejan (2006) give an excellent summary of the subject.

Dispersion in composite channels is a subject of intensive investigation. This is due to the rapid development of technology and numerous modern thermal applications relevant to this area, such as cooling of microelectronic devices. A new concept for a two-layer heat sink with countercurrent flow arrangement for cooling of the electronic components was studied by Vafai and Zhu (1999). In realistic situations, however, the fluid system oftentimes consists of two (and possibly more) separate, immiscible liquids, a layer of one liquid overlying a layer of another liquid. The problem formulation now contains additional dynamical ingredients such as the interfacial stresses and deformation of the interface shape.

Alzami and Vafai (2001) analyzed fluid flow and heat transfer interfacial conditions between a porous medium and a fluid layer. Kuznetsov (2000) gives detailed introduction to the applications and analytical studies of forced convection in partly filled porous configurations. The first exact solution for the fluid flow in the interface region was presented in Vafai and Kim (1990). In this study, the shear stresses in the fluid and the porous medium were taken to be equal at the interface region. Vafai and Thiagaraja (1987) analytically studied the fluid flow and heat transfer for three types of interfaces, namely, the interface between two different porous media, the interface separating a porous medium from a fluid region, and the interface between a porous medium from a fluid region and the interface between a porous medium and an impermeable medium. Continuity of shear stress and heat flux were taken into account in their study while employing the Forchheimer Darcy equation in their analysis.

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The linear encroachment in a two-immiscible-fluid system in a porous medium was studied by Vafai and Alzami (2003). Malashetty and Umavathi (1997), Malashetty et al. (2001, 2004, 2005), and Umavathi et al. (2005, 2007) studied two fluid models in horizontal and inclined channels. Recently, Yang and Vafai (2011) analyzed the phenomenon of heat flux bifurcation inside a porous medium by considering the convective heat transfer process within a channel partially filled with a porous medium under local thermal nonequilibrium conditions. The fundamental aspects of transport through biomaterials for a vast range of biorelated studies were discussed by Shafahi and Vafai (2011). They considered the interfacial interactions in the biodegradation process and found that the porosity alteration as a result of biofilm formation within the carbon bed has a noticeable effect on the removable efficiency.

Generally, the mixing of miscible fluids as they flow through porous media is referred to as hydrodynamical dispersion. With respect to the importance of dispersion processes, studies in water quality management, and pollution control, the dispersion has been referred to as a hydraulic mixing process by which the waste concentrations are attenuated while the waste pollutants are being transported downstream. There are large occasions when waste pollutant from industrial plants, urban areas, and other operations reach a natural water course. An ever-increasing pressure on the waste-assimilating capacity of our water resources does make it more difficult to ensure that concentrations of various contaminants will remain below the limits detected by a variety of competing uses. Interest in dispersion in porous media has also resulted from seawater intrusions into coastal aquifer seepage from canals and streams into and through aquifers and the deliberate release of herbicides into canals to kill weeds.

The problem of dispersion in flow through packed beds is of central importance to chemical engineers, and it is treated in some detail in textbooks on chemical reaction engineering, at least since the pioneering text of Levenspiel (1962). Hydrologists and geophysicists have had a natural interest in the topic for nearly a hundred years now (Slichter, 1905), with good coverage on the subject offered in books devoted to the study of flow through porous media (see, e.g., Bear, 1972; Scheidegger, 1974). This variety of fields of application has propelled the search for experimental data, with a marked peak of activity in the 1950s and 1960s. The vast majority of data on dispersion are either for air or water near ambient temperature, and some data also have been published for the flow of viscous liquids (Blackwell, 1962). At a “macroscopic” level, the quantitative treatment of dispersion is currently based on the use of an equation analogous to Fick’s law, with a dispersion coefficient used instead of a molecular diffusion coefficient. The longitudinal (or axial) dispersion coefficient $D_L$ measures dispersion in the direction of flow, whereas dispersion perpendicular to the direction of flow is related to the transverse dispersion coefficient $D_T$. Consideration was given only to the effect of fluid properties on $D_L$. [The effect of fluid properties on $D_T$ was reported elsewhere by Delgado and Guedes de Carvalho (2001).]
In his pioneering work, Taylor (1953) used an equation analogous to the equation
\[
\frac{\partial (\varphi \bar{c})}{\partial t} + \frac{\partial (\varphi \bar{v}_x \bar{c})}{\partial x} - \frac{\partial}{\partial x} \left( \varphi D + \varphi D_m (1 - G) \frac{\partial \bar{c}}{\partial x} \right) = 0
\]

to study the dispersion of a soluble substance in a slow-moving fluid in a small-diameter tube, and he primarily focused on modeling the molecular diffusion coefficient using concentration profiles along a tube for a large time. Following that work, Gill and Sankarasubramanian (1970) developed an exact solution for the local concentration for the fully developed laminar flow in a tube for all time. Their work shows that the time-dependent dimensionless dispersion coefficient approaches an asymptotic value for a larger time, proving that Taylor’s analysis is adequate for steady-state diffusion through tubes. Even though the preceding analyses are primarily concerned with the dispersive flow in small-diameter tubes, because a porous medium can be modeled as a pack of tubes, we could expect similar insights from the advection–dispersion models derived for porous media flow. Verwoerd and Kulasiri (2001) researched the effects of variable flow velocity on contaminant dispersion in porous flow. The effect of homogeneous and heterogeneous reactions on the dispersion of a solute in a porous medium in a magnetohydrodynamic laminar flow between two parallel plates was studied by Narasimha Murthy and Ali (1975). The flow and heat transfers in biological tissues were analyzed by Khaled and Vafai (2003). Pertinent works were reviewed to show how transport theories in porous media advance the progress in biology. The main concepts studied were transport in porous media using mass diffusion and different convective flow models such as the Darcy and the Brinkman models.

The literature on hydrodynamic dispersion in a porous medium is very sparse despite its versatile applications in many branches of science, engineering, and technology. This dispersion in a porous medium is the macroscopic outcome of the actual movements of the individual solute particles through the pores and the various physical and chemical phenomena that take place within the pores. Keeping in view the various applications of composite porous medium, the objective of this work is to analyze dispersion of a solute for a composite porous medium using Taylor’s (1953) model.

2. MATHEMATICAL FORMULATION

Consider the laminar flow of two immiscible fluids filled with different porous matrices with different permeabilities between two parallel plates distant \((h_1 + h_2)\) apart, taking the \(x\)-axis along the midsection of the channel and the \(y\)-axis perpendicular to the walls as shown in Fig. 1. Region 1 \((-h_1 \leq y \leq 0)\) is filled with a fluid-saturated porous medium of density \(\rho_1\), viscosity \(\mu_1\), and permeability \(k_1\) under a uniform pressure gradient \(dp_1/dx_1\), and region 2 \((0 \leq y \leq h_2)\) is filled with another fluid saturated with a porous medium of density \(\rho_2\), viscosity \(\mu_2\), and permeability \(k_2\) under a uniform pressure gradient \(dp_2/dx_2\).

It is assumed that the fluids are incompressible and the flow is steady, laminar, fully developed, and that fluid properties are constant. The transport properties of both the fluids are assumed to be constant. The flow in both regions is assumed to be driven by a common constant pressure gradient.

Under these assumptions, the governing equations of motion for incompressible fluids are as follows:

Region 1:
\[
\begin{align*}
\frac{d^2 u_1}{dy_1^2} - \frac{1}{k_1} u_1 &= \frac{1}{\mu_1} \frac{dp_1}{dx_1} \tag{1}
\end{align*}
\]

Region 2:
\[
\begin{align*}
\frac{d^2 u_2}{dy_2^2} - \frac{1}{k_2} u_2 &= \frac{1}{\mu_2} \frac{dp_2}{dx_2} \tag{2}
\end{align*}
\]

where \(u_i\) is the \(x\)-component of fluid velocity and \(p_i\) is the pressure. The subscripts 1 and 2 denote the values for region 1 and region 2, respectively.

The boundary conditions on velocity are no-slip conditions requiring that the velocity vanish at the walls. In addition, continuity of velocity and shear stress at the interface is assumed. With these assumptions, the boundary and interface conditions on velocity become

\[
u_1 = 0 \quad \text{at} \quad \eta_1 = -1
\]

**FIG. 1:** Physical model and the coordinate system
\[ u_2 = 0 \quad \text{at} \quad \eta_2 = 1 \]
\[ u_1 = u_2 \quad \text{at} \quad \eta_1 = \eta_2 = 0 \]
\[
\frac{\mu_1}{h_1} \frac{du_1}{d\eta_1} = \frac{\mu_2}{h_2} \frac{du_2}{d\eta_2} \quad \text{at} \quad \eta_1 = \eta_2 = 0
\]

3. SOLUTIONS

Solutions of Eqs. (1) and (2) using boundary and interface conditions (3) become

\[ u_1 = l_1 \cosh \sigma_1 \eta_1 + l_2 \sinh \sigma_1 \eta_1 - \frac{P_1 h_1^2}{\sigma_1^2} \]

\[ u_2 = l_3 \cosh \sigma_2 \eta_2 + l_4 \sinh \sigma_2 \eta_2 - \frac{P_2 h_2^2}{\sigma_2^2} \]

where \( \sigma_1 = h_1/\sqrt{k_1} \), \( \sigma_2 = h_2/\sqrt{k_2} \), \( P_1 = (1/\mu_1) \times (dp_1/dx_1) \), \( P_2 = (1/\mu_2)(dp_2/dx_2) \).

The average velocity in region 1 is defined by

\[ \bar{u}_1 = \int_{-1}^{0} u_1 \ d\eta_1 \]

Using the solution of \( u_1 \) from Eq. (4), the average velocity becomes

\[ \bar{u}_1 = l_1 \sinh \sigma_1 + l_2 (1 - \cosh \sigma_1) - \frac{P_1 h_1^2}{\sigma_1^2} \]

The average velocity in region 2 is defined by

\[ \bar{u}_2 = \int_{0}^{1} u_2 \ d\eta_2 \]

Using the solution of \( u_2 \) from Eq. (5), the preceding equation becomes

\[ \bar{u}_2 = l_3 \sinh \sigma_2 + l_4 (1 - \cosh \sigma_2) - \frac{P_2 h_2^2}{\sigma_2^2} \]

We assume that a solute diffuses in the absence of a first-order irreversible chemical reaction in the liquid under isothermal conditions. The equation for the concentration \( C_1 \) of the solute for the first region satisfies

\[ \frac{\partial C_1}{\partial t_1} + u_1 \frac{\partial C_1}{\partial x_1} = D_1 \left( \frac{\partial^2 C_1}{\partial x_1^2} + \frac{\partial^2 C_1}{\partial y_1^2} \right) \]

Similarly, the equation for the concentration \( C_2 \) of the solute for the second region satisfies

\[ \frac{\partial C_2}{\partial t_2} + u_2 \frac{\partial C_2}{\partial x_2} = D_2 \left( \frac{\partial^2 C_2}{\partial x_2^2} + \frac{\partial^2 C_2}{\partial y_2^2} \right) \]

in which \( D_1 \) and \( D_2 \) are the molecular diffusion coefficients (assumed constants) for the first and second region, respectively. We now assume that the longitudinal diffusion is much less than the transverse diffusion, that is,

\[ \frac{\partial^2 C_1}{\partial x_1^2} \ll \frac{\partial^2 C_1}{\partial y_1^2} \quad \text{and} \quad \frac{\partial^2 C_2}{\partial x_2^2} \ll \frac{\partial^2 C_2}{\partial y_2^2} \]

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

\[ u_{1x} = u_1 - \bar{u}_1 = l_1 \cosh \sigma_1 \eta_1 + l_2 \sinh \sigma_1 \eta_1 + l_5 \]

for region 1 and

\[ u_{2x} = u_2 - \bar{u}_2 = l_3 \cosh \sigma_2 \eta_2 + l_4 \sinh \sigma_2 \eta_2 + l_6 \]

for region 2.

Introducing the dimensionless quantities

\[ \theta_1 = \frac{t_1}{\bar{t}_1}, \quad \xi_1 = \frac{x_1 - \bar{u}_1 \bar{t}_1}{\bar{L}_1}, \quad \eta_1 = \frac{y_1}{h_1}, \]

\[ \theta_2 = \frac{t_2}{\bar{t}_2}, \quad \xi_2 = \frac{x_2 - \bar{u}_2 t_2}{\bar{L}_2}, \quad \eta_2 = \frac{y_2}{h_2} \]

Equations (8) and (9) using Eqs. (10) and (11) and using the nondimensional quantities as defined in Eq. (12) become

Region 1:

\[ \frac{1}{\bar{t}_1} \frac{\partial C_1}{\partial \theta_1} + u_{1x} \frac{\partial C_1}{\partial \xi_1} = \frac{D_1 \frac{\partial^2 C_1}{\partial \xi_1^2}}{h_1^2} \]

Region 2:

\[ \frac{1}{\bar{t}_2} \frac{\partial C_2}{\partial \theta_2} + u_{2x} \frac{\partial C_2}{\partial \xi_2} = \frac{D_2 \frac{\partial^2 C_2}{\partial \xi_2^2}}{h_2^2} \]

where \( \bar{L}_1 \) and \( \bar{L}_2 \) are the typical lengths along the flow direction for the first and second regions, respectively. Following Taylor (1953), we now assume that the partial equilibrium is established in any cross section of the channel so that the variations of \( C_1 \) with \( \eta_1 \) and \( C_2 \) with \( \eta_2 \) from Eqs. (13) and (14) become

Region 1:

\[ \frac{\partial^2 C_1}{\partial \eta_1^2} = \frac{h_1^2}{D_1 \bar{L}_1} u_{1x} \frac{\partial C_1}{\partial \xi_1} \]

Region 2:

\[ \frac{\partial^2 C_2}{\partial \eta_2^2} = \frac{h_2^2}{D_2 \bar{L}_2} u_{2x} \frac{\partial C_2}{\partial \xi_2} \]
To solve Eqs. (15) and (16), we use the following three types of boundary conditions: The first one connected with an insulated type of boundary conditions, namely,

\[
\frac{\partial C_1}{\partial \eta_1} = 0 \text{ at } \eta_1 = -1 \quad \text{and} \quad \frac{\partial C_2}{\partial \eta_2} = 0 \text{ at } \eta_2 = 1 \quad (17)
\]

which expresses the fact that the walls of the channel are impermeable. However, in many biological problems, the condition at the upper wall is conducting and the lower is insulating. In other words,

\[
\frac{\partial C_1}{\partial \eta_1} = 0 \text{ at } \eta_1 = -1 \quad \text{and} \quad C_2 = 1 \text{ at } \eta_2 = 1 \quad (18)
\]

where the former condition represents the impermeable and the latter the permeable. Since the fluids in both the regions is filled with porous matrix, we can also consider the condition that the upper wall is insulating and the lower wall is conducting. That is,

\[
C_1 = 1 \text{ at } \eta_1 = -1 \quad \text{and} \quad \frac{\partial C_2}{\partial \eta_2} = 0 \text{ at } \eta_2 = 1 \quad (19)
\]

### 3.1 Case 1: Concentration Distribution with Impermeable Wall Conditions

Equations (15) and (16) are solved exactly for \(C_1\) and \(C_2\) using boundary conditions as defined in Eq. (17), which are given by

Region 1:

\[
C_1 = \frac{h_1^3}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1} \left( \frac{l_1 \cosh \sigma_1 \eta_1}{\sigma_1^2} + \frac{l_2 \sinh \sigma_1 \eta_1}{\sigma_1^2} \right) + l_5 \eta_1
\]

\[
+ \left( \frac{l_5 \eta_1^2}{2} \right) + l_7 \eta_1
\]

(20)

Region 2:

\[
C_2 = \frac{h_2^3}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \left( \frac{l_3 \cosh \sigma_2 \eta_2}{\sigma_2^2} + \frac{l_4 \sinh \sigma_2 \eta_2}{\sigma_2^2} \right) + l_9 \eta_2
\]

\[
+ \left( \frac{l_9 \eta_2^2}{2} \right) + l_9 \eta_2
\]

(21)

where integrating constants are determined using the entry condition.

The volumetric rates at which the solute is transported across a section of the channel of unit breadth \(Q_1\) (region 1) and \(Q_2\) (region 2) using Eqs. (10), (11) and (20), (21), respectively, are given by

\[
Q_1 = \int_{-1}^{0} h_1 C_1 u_{1x} \, d \eta_1 = \frac{h_1^3}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1} \left( \frac{l_5^2}{2\sigma_1^4} \right)
\]

\[
\times \left[ 1 + \frac{\sinh 2\sigma_1}{2\sigma_1} \right] + l_5 l_2 \left[ \frac{1 - \cosh 2\sigma_1}{2\sigma_1} \right] + l_5 l_1
\]

\[
\times \left( \frac{\sinh \sigma_1}{\sigma_1^2} - \cosh \sigma_1 \right) + \frac{l_5^2}{6} \right] + h_1 l_7 l_1 \left( \frac{\cosh \sigma_1 - 1}{\sigma_1^2} \right)
\]

\[
\times \left( \frac{\sinh \sigma_1}{\sigma_1} \right) + h_1 l_7 l_2 \left( \frac{\cosh \sigma_1 - \sinh \sigma_1}{\sigma_1^2} - \frac{l_5 l_7}{2} \right)
\]

(22)

\[
Q_2 = \int_{0}^{1} h_2 C_2 u_{2x} \, d \eta_2 = \frac{h_2^3}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \left( \frac{l_5^2}{2\sigma_2^4} \right)
\]

\[
\times \left[ 1 + \frac{\sinh 2\sigma_2}{2\sigma_2} \right] + l_5 l_4 \left( \frac{\cosh 2\sigma_2 - 1}{2\sigma_2} \right) + \frac{l_5^2}{2\sigma_2^2}
\]

\[
\times \left[ 1 - \frac{\sinh 2\sigma_2}{2\sigma_2} \right] + l_5 l_6 \left( \frac{\cosh 2\sigma_2 - 1}{2\sigma_2} \right) + \frac{l_5^2}{2\sigma_2^2}
\]

\[
\times \left( \frac{\sinh \sigma_2}{\sigma_2^2} - \cosh \sigma_2 \right) + \frac{2 \sinh \sigma_2}{\sigma_2^2}
\]

\[
+ \frac{l_5^2}{6} \right] + h_2 l_9 l_3 \left( \frac{1 - \cosh \sigma_2}{\sigma_2^2} + \frac{\sinh \sigma_2}{\sigma_2^2} \right) + h_2 l_9 l_4
\]

\[
\times \left( \frac{\cos \sigma_2}{\sigma_2^2} + \frac{\sin \sigma_2}{\sigma_2^2} \right) + \frac{h_2 l_9 l_9}{2}
\]

(23)

### 3.2 Case 2: Concentration Distribution with Upper Wall Conducting and Lower Wall Insulating Condition

To find the exact solutions of Eqs. (15) and (16), we require two more interface conditions along with boundary conditions defined as in Eq. (18), which are defined as

\[
C_1 = C_2; \quad D_1 \frac{\partial C_1}{\partial y_1} = D_2 \frac{\partial C_2}{\partial y_2} \text{ at } y_1 = y_2 = 0 \quad (24)
\]

The solutions of Eqs. (15) and (16) using boundary and interface conditions given in Eqs. (18) and (24) are Region 1:

\[
C_1 = a_1 \cos \sigma_1 \eta_1 + a_2 \sin \sigma_1 \eta_1 + a_3 \eta_1^2 + l_7 \eta_1 + l_8
\]

(25)
3.3 Case 3: Concentration Distribution with Upper Wall Insulating and Lower Wall Conducting

The solutions of Eqs. (15) and (16) using boundary and interface conditions given in Eqs. (19) and (24) are

Region 1:

\[ C_1 = a_1 \cosh \sigma_1 n_1 + a_2 \sinh \sigma_1 n_1 + a_3 n_1^2 + l_1 n_1 + l_8 \]  

Region 2:

\[ C_2 = a_4 \cosh \sigma_2 n_2 + a_5 \sinh \sigma_2 n_2 + a_6 n_2^2 + l_9 n_2 + l_{10} \]

The volumetric rates at which the solute is transported across a section of the channel of unit breadth \( Q_1 \) (region 1) and \( Q_2 \) (region 2) are

\[
Q_1 = \int_{-1}^{0} h_1 C_1 u_{1x} \, d\eta_1 = \frac{h_1 a_1 l_1}{2} \left( 1 + \sinh 2 \sigma_1 \right) \\
+ \frac{h_1 (a_1 l_2 + a_2 l_1)}{2} \left( 1 - \cosh 2 \sigma_1 \right) + (h_1 a_1 l_5 + h_1 l_8 l_1) \\
\times \frac{\sinh \sigma_1}{\sigma_1} + \frac{h_1 a_2 l_2}{2} \left( \frac{\sinh 2 \sigma_1}{\sigma_1} - 1 \right) + h_1 a_2 l_5 \\
\times \left( \frac{1 - \cosh \sigma_1}{\sigma_1} \right) + h_1 a_3 l_1 \left( \frac{\sinh \sigma_1}{\sigma_1} - \frac{2 \cosh \sigma_1}{\sigma_1^2} \right) + h_1 a_3 l_2 \\
+ \frac{2 \sinh \sigma_1}{\sigma_1^3} \right) + h_1 l_7 l_2 \left( \frac{\cosh \sigma_1}{\sigma_1} - \frac{\sinh \sigma_1}{\sigma_1^2} \right) + h_1 a_3 l_2 \\
\times \left[ - \cosh \sigma_1 + \frac{2 \sinh \sigma_1}{\sigma_1^2} - \frac{2 (\cosh \sigma_1 - 1)}{\sigma_1^3} \right] \\
+ \frac{h_1 a_3 l_5}{3} + h_1 l_7 l_1 \left[ \frac{\sinh \sigma_1}{\sigma_1} - \left( \cosh \sigma_1 - 1 \right) \right] \\
- \frac{h_1 l_7 l_5}{2} + h_1 l_7 l_2 \left( 1 - \cosh \sigma_1 \right) + h_1 l_8 l_5 \quad (27)
\]

\[
Q_2 = \int_{0}^{1} h_2 C_2 u_{2x} \, d\eta_2 = \frac{h_2 a_4 l_2}{2} \left( 1 + \sinh 2 \sigma_2 \right) \\
+ \frac{h_2 (a_4 l_4 + a_5 l_3)}{2} \left( \cosh 2 \sigma_2 - 1 \right) + (h_2 a_4 l_6) \\
+ h_2 a_6 l_3 \left( \frac{\sinh \sigma_2}{\sigma_2} + \frac{h_2 a_6 l_4}{2} \left( 1 - \cosh 2 \sigma_2 \right) + h_2 a_6 l_3 \right) \\
\times \left( \frac{\sinh \sigma_2}{\sigma_2} - \frac{2 \cosh \sigma_2}{\sigma_2^2} + \frac{2 \sinh \sigma_2}{\sigma_2^3} \right) + (h_2 a_6 l_6) \\
+ h_2 l_{10} l_4 \left( 1 - \cosh \sigma_2 \right) + h_2 a_6 l_4 \left( \frac{\cosh \sigma_2}{\sigma_2} \right) \\
- \frac{2 \sinh \sigma_2}{\sigma_2^2} + \frac{2 (\cosh \sigma_2 - 1)}{\sigma_2^3} \right) + h_2 l_6 l_6 + h_2 l_{10} l_6 \\
+ \frac{h_2 a_6 l_6}{3} + h_2 l_8 l_3 \left[ \frac{\sinh \sigma_2}{\sigma_2} + (1 - \cosh \sigma_2) \right] \\
+ h_2 l_8 l_4 \left( \frac{\cosh \sigma_2}{\sigma_2} - \frac{\sinh \sigma_2}{\sigma_2^2} \right) \quad (28)
\]

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For insulating-insulating boundary conditions, the effects of various parameters of the flow are shown graphically in Figs. 2–8. The effect of porous parameter $\sigma (\sigma_1 = \sigma_2 = \sigma)$ of region 1 and region 2 on the velocity is shown in Fig. 2. It is observed that the velocity decreases in both the regions with the increase of porous parameter $\sigma$. The dragging effect in both the regions is the same as we are considering $\sigma_1 = \sigma_2 = \sigma$. This graph is drawn explicitly to understand the interface conditions. The effect of the pressure gradients $P_1$ and $P_2$ on the concentration is shown in Fig. 3. As the pressure gradient increases, the solute concentration increases in magnitude in both the regions. Since the boundary conditions are insulating-insulating, the concentration profiles are symmetric.

Varying the height of the porous matrix $h_1$ of region 1, the concentration decreases in both the regions but is more influential in region 1 compared to region 2, as shown in Fig. 4(a). For variations of $h_1$, results show the variation of concentration in both the regions due to the coupling effect. A similar effect is observed varying the height of permeable fluid in region 2, as seen in Fig. 4(b). That is varying $h_2$, concentration decreases in both the regions, but its effect is more significant in region 2 compared to region 1. Figure 5 displays the effect of viscosity on the

![FIG. 2: Velocity profiles for different values of porous parameter $\sigma$](image)
FIG. 3: Concentration profiles for different pressure gradients $P_1$ and $P_2$

FIG. 4: Concentration profiles for different values of height ratios: (a) height ratio $h_1$ and (b) height ratio $h_2$
FIG. 5: Concentration profiles for different values of viscosity: (a) viscosity $\mu_1$ and (b) viscosity $\mu_2$

FIG. 6: Concentration profiles for different values of diffusivity: (a) diffusivity $D_1$ and (b) diffusivity $D_2$
FIG. 7: Concentration profiles for different values of $\sigma$

FIG. 8: Volumetric rate $Q$ versus viscosity $\mu_1 = \mu_2$
solute concentration. As the viscosity of permeable fluid in region 1 increases, concentration decreases in region 1 and increases in region 2, considering equal permeabilities, as shown in Fig. 5(a). A similar effect is observed for varying viscosity of the fluid in region 2, that is, the solute concentration decreases in region 2 and increases in region 1 for varying the viscosity of the fluid in region 2, considering the same porous matrix in both the regions, as seen in Fig. 5(b). Keeping the diffusivity coefficient of region 2 fixed and varying the diffusivity coefficient $D_1$ of region 1, the solute concentration decreases in region 1 and remains constant in region 2, as shown in Fig. 6(a). Figure 6(b) displays that the solute concentration decreases in region 2 and remains constant in region 1 for variations of diffusivity coefficient $D_2$.

Considering the equal permeabilities in both the regions, the effect of porous parameter $\sigma$ is to decrease the solute concentration in both the regions, as shown in Fig. 7. It is interesting to note that there is no effect of $\sigma$ at the interface on the concentration profiles. Here also the concentration profiles are symmetric. The volumetric flow rate for different values of viscosity ($\mu_1 = \mu_2 = \mu$) is shown in Fig. 8. Volumetric flow rate increases in magnitude for values of $\mu$ approximately up to 0.4, and then there is no change in flow rate for values of $\mu = 0.4$ and onward.

Figures 9–18 display the effect of various parameters on the flow for conducting-insulating and insulating-conducting wall conditions. The effect of pressure gradient on solute concentration is to increase for $(P_1 = P_2 =) P > 0$ and decrease for $(P_1 = P_2 =) P < 0$ in both the regions, as seen in Fig. 9 for conducting-insulating wall boundaries. A similar effect is observed for insulating-conducting wall boundaries, as seen in Fig. 14. The effect of porous parameter $\sigma$ ($\sigma_1 = \sigma_2 = \sigma$) is to decrease the solute concentration in both the regions for conducting-insulating wall boundaries, as shown in Fig. 10. The concentration profiles for variations of $\sigma$ are symmetric. A similar result is observed for insulating-conducting wall boundaries, as seen in Fig. 15.

Varying the height of the fluid region 1 decreases the solute concentration in region 1 and increases it in region 2, as shown in Fig. 11(a) for conducting-insulating wall boundaries, and its effect is very significant near the lower boundary. For insulating-conducting wall boundaries, the effect of $h_1$ is to increase the solute concentration in both the regions, and its effect is more significant at the upper wall [Fig. 16(a)]. For conducting-insulating wall bound-

![FIG. 9: Concentration profiles for different pressure gradients $P_1$ and $P_2$](image-url)
FIG. 10: Concentration profiles for different values of porous parameter $\sigma$

FIG. 11: Concentration profiles for different values of height: (a) height $h_1$ and (b) height $h_2$
FIG. 12: Concentration profiles for different values of viscosity: (a) viscosity $\mu_1$ and (b) viscosity $\mu_2$

FIG. 13: Concentration profiles for different values of diffusivity: (a) diffusivity $D_1$ and (b) diffusivity $D_2$
FIG. 14: Concentration profiles for different values of $P_1$ and $P_2$

FIG. 15: Concentration profiles for different values of porous parameter $\sigma$
FIG. 16: Concentration profiles for different values of height: (a) height $h_1$ and (b) height $h_2$

FIG. 17: Concentration profiles for different values of viscosity: (a) viscosity $\mu_1$ and (b) viscosity $\mu_2$
FIG. 18: Concentration profiles for different values of diffusivity: (a) diffusivity $D_1$ and (b) diffusivity $D_2$

aries, the effect of height of the fluid $h_2$ in region 2 is to increase the solute concentration in both the regions, and its effect is significant at the lower plate [Fig. 11(b)]. Figure 16b shows that increasing $h_2$, the concentration increases in region 1 and decreases in region 2, and its effect is more significant at the upper plate for insulating-conducting wall boundaries. Figure 12 shows the effect of variations of viscosities of the fluids on concentration for conducting-insulating wall boundaries. Increasing viscosity of the fluid in region 1 decreases the solute concentration in both the regions, as seen in Fig. 12(a). A similar effect is observed for variations of viscosity of the fluid in region 2, as seen in Fig. 12(b). Concentration profiles are symmetric in both the regions for variations of $\mu_1$ and $\mu_2$. It is also observed that the concentration profiles for variations of $\mu_1$ and $\mu_2$ are similar for insulating-conducting wall boundaries, as seen in Fig. 17.

Fixing the diffusivity coefficient $D_2$ of region 2, the variations of diffusivity coefficient $D_1$ are to increase the solute concentration in region 1 without any variations in region 2. The effect of $D_1$ is influential near the lower boundary, as seen in Fig. 13(a) (conducting-insulating). For insulating-conducting wall boundaries, variation of diffusion coefficient $D_1$ is to decrease the concentration in both the regions. Its effect is more significant at the upper plate, as seen in Fig. 18(a). Keeping the diffusivity coefficient $D_1$ of region 1 fixed, the variations of diffusivity coefficient $D_2$ of region 2 decrease the solute concentration in both the regions. The variations of $D_2$ are more effective at the lower plate, as seen in Fig. 13(b) (conducting-insulating). Figure 18(b) depicts that the solute concentration increases for increasing values of diffusivity coefficient $D_2$ in region 2 and is invariant for region 1. Its effect is more significant near the boundary of the upper plate for insulating-conducting wall boundaries.

The effect of variations of viscosity $\mu$ ($\mu_1 = \mu_2 = \mu$) on flow rate for conducting-insulating and insulating-conducting wall boundaries is similar to insulating-insulating wall boundaries and is not shown graphically.

The effect of porous parameter $\sigma$ on the flow rate is shown in Table 1. As porous parameter $\sigma$ increases, flow rate increases in magnitude in both the regions. To validate the results obtained for the two-fluid model, results are compared with Rudraiah and Ng (2007) for a one-fluid model. An excellent agreement is observed for the values of volumetric flow rate obtained by Rudraiah and Ng (2007) for the one-fluid model.
TABLE 1: Height $h_1 = h_2$ versus volumetric rate $Q$ porous-porous case when varying porous parameters with respect to width

<table>
<thead>
<tr>
<th>Height $h_1 = h_2$</th>
<th>Porous parameter</th>
<th>Volumetric rate</th>
<th>One-fluid model</th>
<th>Two-fluid model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$Q$</td>
<td>$Q_1$</td>
<td>$Q_2$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>$-0.000006$</td>
<td>$-0.000003$</td>
<td>$-0.000003$</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>$-0.000088$</td>
<td>$-0.000044$</td>
<td>$-0.000044$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>$-0.000540$</td>
<td>$-0.000270$</td>
<td>$-0.000270$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>$-0.002030$</td>
<td>$-0.001015$</td>
<td>$-0.001015$</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>$-0.005596$</td>
<td>$-0.002798$</td>
<td>$-0.002798$</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
<td>$-0.012487$</td>
<td>$-0.006243$</td>
<td>$-0.006243$</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>$-0.023938$</td>
<td>$-0.011969$</td>
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</tr>
<tr>
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<td>1.8</td>
<td>$-0.041000$</td>
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</tr>
<tr>
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<td>2.0</td>
<td>$-0.064447$</td>
<td>$-0.032223$</td>
<td>$-0.032223$</td>
</tr>
</tbody>
</table>

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REFERENCES


Umavathi, J. C., Chamkha, A. J., Mateen, A., and Al-


**APPENDIX**

### A1. Case 1: Insulating-Insulating Conditions

\[ P_1 = \frac{1}{\mu_1} \frac{dp_1}{dx_1}; \quad P_2 = \frac{1}{\mu_2} \frac{dp_2}{dx_2}; \quad m = \frac{\mu_1}{\mu_2}; \]

\[ h = \frac{h_2}{h_1}; \quad \sigma = \frac{\sigma_1}{\sigma_2}; \]

\[ l_1 = \left[ \frac{P_1 h_1^2}{\sigma_1^2} (m\sigma) \sinh \sigma_2 + \frac{P_2 h_2^2}{\sigma_2^2} \sinh \sigma_1 \right] \]

\[ l_2 = l_1 \coth \sigma_1 - \frac{P_1 h_1^2}{\sigma_1^2} \frac{1}{\sinh \sigma_1}; \]

\[ l_3 = l_1 + \frac{P_2 h_2^2}{\sigma_2^2} - \frac{P_1 h_1^2}{\sigma_1^2}; \]

\[ l_4 = m h_1 \sigma_1; \quad l_5 = -l_1 \sinh \sigma_1 - \frac{l_2}{\sigma_1} (1 - \cosh \sigma_1); \]

\[ l_6 = -\frac{l_2 \cosh \sigma_2}{\sigma_2} - \frac{l_4 (\cosh \sigma_2 - 1)}{\sigma_2}; \]

\[ l_7 = -\frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1} \left( \frac{l_1 \sinh \sigma_1}{\sigma_1} - \frac{l_2 \cosh \sigma_1}{\sigma_1} + l_5 \right); \]

\[ l_9 = -\frac{h_2^2}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \left( \frac{l_3 \sinh \sigma_2}{\sigma_2} + \frac{l_4 \cosh \sigma_2}{\sigma_2} + l_6 \right). \]

### A2. Case 2: Conducting-Insulating Conditions

\[ P_1 = \frac{1}{\mu_1} \frac{dp_1}{dx_1}; \quad P_2 = \frac{1}{\mu_2} \frac{dp_2}{dx_2}; \]

\[ l_1 = \left[ \frac{P_1 h_1^2}{\sigma_1^2} (m\sigma) \sinh \sigma_2 + \frac{P_2 h_2^2}{\sigma_2^2} \sinh \sigma_1 \right] \]

\[ l_2 = l_1 \coth \sigma_1 - \frac{P_1 h_1^2}{\sigma_1^2}; \quad l_3 = l_1 + \frac{P_2 h_2^2}{\sigma_2^2} - \frac{P_1 h_1^2}{\sigma_1^2}; \]

\[ l_4 = m h_1 \sigma_1; \quad l_5 = -l_1 \sinh \sigma_1 - \frac{l_2}{\sigma_1} (1 - \cosh \sigma_1); \]

\[ a_1 = \frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1}; \quad a_2 = \frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1}; \]

\[ l_7 = a_1 \sinh \sigma_1 - a_2 \sinh \sigma_1 \cosh \sigma_1 + 2a_3; \]

\[ l_8 = a_4 + l_{10} - a_1; \]

\[ l_9 = \frac{D_1 h_1}{D_2} [a_1 \sigma_1 + l_7] - a_5 \sigma_2; \]

\[ l_{10} = 1 - a_4 \cosh \sigma_2 - a_5 \sinh \sigma_2 - a_6 - l_9 \]

### A3. Case 3: Insulating-Conducting Conditions

\[ P_1 = \frac{1}{\mu_1} \frac{dp_1}{dx_1}; \quad P_2 = \frac{1}{\mu_2} \frac{dp_2}{dx_2}; \]

\[ l_1 = \left[ \frac{P_1 h_1^2}{\sigma_1^2} (m\sigma) \sinh \sigma_2 + \frac{P_2 h_2^2}{\sigma_2^2} \sinh \sigma_1 \right] \]

\[ (m\sigma. \cosh \sigma_1 \sinh \sigma_2 + \sinh \sigma_1 \cosh \sigma_2). \]

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\[
\begin{align*}
l_2 &= l_1 \coth \sigma_1 - \frac{P_1 h_1^2}{\sigma_1^2} \frac{1}{\sinh \sigma_1}; \\
l_3 &= l_1 + \frac{P_2 h_2^2}{\sigma_2^2} - \frac{P_1 h_1^2}{\sigma_1^2}; \\
l_4 &= m h \sigma_2; \\
l_5 &= -l_1 \sinh \sigma_1 - l_2 (1 - \cosh \sigma_1); \\
l_6 &= -l_4 \sinh \sigma_2 - l_4 (\cosh \sigma_2 - 1); \\
a_1 &= \frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1} l_1; \\
a_2 &= \frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_1} l_2; \\
a_3 &= \frac{h_1^2}{D_1 L_1} \frac{\partial C_1}{\partial \xi_2} \frac{l_5}{2}; \\
a_4 &= \frac{h_1^2}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \frac{l_3}{2}; \\
a_5 &= \frac{h_2^2}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \frac{l_4}{2}; \\
a_6 &= \frac{h_2^2}{D_2 L_2} \frac{\partial C_2}{\partial \xi_2} \frac{l_6}{2}; \\
l_9 &= -a_4 \sinh \sigma_2 - a_5 \cosh \sigma_2 - 2a_6; \\
l_{10} &= a_1 - a_4 + l_8; \\
l_7 &= \frac{D_2}{D_1 h} [a_5 \sigma_2 + l_9] - a_2 \sigma_1; \\
l_8 &= 1 - a_1 \cosh \sigma_1 + a_2 \sinh \sigma_1 - a_3 + l_7.
\end{align*}
\]