An analysis on free convection in an odd-shaped cavity filled with nanofluid

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A study of natural convective flow, heat transfer and entropy generation in an odd-shaped geometry is presented here. The geometry considered is a combination of the horizontal and vertical enclosure shapes. The cavity is filled with Cu–water nanofluid. The numerical study focuses specifically on the effect of natural convection parameter and solid volume fraction of nanoparticle on the average Nusselt number, total entropy generation and Bejan number. Also isotherms, stream function and entropy generation due to heat transfer are presented for various Rayleigh number and solid volume fraction. The governing equations are solved by using penalty finite element method with Galerkins weighted residual technique. The results reveal that increasing Rayleigh number causes increase of the average Nusselt number as well as the heat transfer term of entropy generation and decrease of the viscous term. The proper choice of Rayleigh number could be able to maximize heat transfer rate simultaneously minimizing entropy generation.

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1. Introduction

Natural convection is the main heat transfer mechanism used in numerous applications. Heat transfer within enclosures has many engineering applications such as solar collectors, thermal storage systems, and cooling of electrical and mechanical components. Therefore, it is important to understand the thermal behavior of such systems when the natural convection is the dominant mode of heat transfer. The low thermal conductivity of conventional heat transfer fluids, commonly water, has restricted designers. Fluids containing nanosized solid particles offer a possible solution to conquer this problem. The nanofluid has greater effective thermal conductivity than pure base fluid. Most of the available literature on this topic concerns regular geometries such as rectangular or square enclosures, while actual applications demand the consideration of irregular shapes. In particular, for applications involving the cooling of electronic equipment, solar collectors, ingot castings, thermal hydraulic analysis, etc., it is often necessary to consider geometries as the present one.

Many researchers studied heat transfer in cavities filled with nanofluid. Ho et al. [1] studied numerically the effects of dynamic viscosity and thermal conductivity of nanofluid in a square enclosure filled with Al2O3–water nanofluid. Enhancements in the thermal conductivity and dynamic viscosity estimated from the two adopted formulas led to the enhanced or mitigated heat transfer. Parvin et al. [2] studied the natural convection heat transfer in an enclosure with a heated body filled with nanofluid. The results showed that it was possible to achieve higher cooling performance by adding nanoparticles into pure water. Abu-Nada and Chamkha [3] performed a numerical study of natural convection heat transfer in a differentially heated enclosure filled with CuO–EG–water nanofluid. The results were compared with Brinkman model and MG models for nanofluid viscosity and thermal conductivity. Either enhancement or decline was reported for the average Nusselt number as the volume fraction of nanoparticles increased. Lin and Violi [4] analyzed numerically the natural convection heat transfer and fluid flow in a cavity with differentially heated walls containing Al2O3–water nanofluid. Their results showed the enhancement in the heat transfer due to the presence of nanoparticles, which increased the effective thermal diffusivity, and lessen the Prandtl number. Natural convection of SiO2–water nanofluid using two different models has been studied by Jahanshahi et al. [5]. In the first model they have employed a set of experimental data for thermal conductivity of nanofluid and in the second model they have calculated the thermal conductivity from the equation proposed by Hamilton and Crosser [6]. Their results showed an enhancement in thermal conductivity due to the adding of nanoparticles at both models. To estimate the thermal conductivity of nanofluids, many models have been developed. Patel et al. [7] have improved the model proposed by Hemanth et al. [8]. Their model is capable to estimate the thermal conductivity of nanofluids for volume fraction between 1% and 8% and particle size
results have indicated that the total entropy generation in a steady state increases linearly with the aspect ratio and the irreversibility coefficient, and exponentially with the Rayleigh number. Furthermore, the entropy generation due to the viscous effects increases with increasing the Rayleigh number. Singh et al. [19] analyzed theoretically the entropy generation in a tube containing alumina–water nanofluid for different tube diameters. They showed that there was an optimum diameter at which the entropy generation rate was minimum for both laminar and turbulent flow. Some investigation has been done on the natural convection around an obstacle in a cavity. Another study has been done by Sheikhzadeh et al. [20]. They have investigated the effects of Prandtl number on the steady magneto-convection around a centrally located adiabatic body inside a square cavity. Many researchers have studied the entropy generation due to natural convection in square or rectangular cavities [21–26]. The problem of entropy generation in square or wavy-wall cavities filled with nanofluids has attracted significant attention in recent years. Shahi et al. [27] investigated the entropy generation induced by natural convection heat transfer in a square cavity containing Cu–water nanofluid and a protruding heat source. The results showed that the Nusselt number increased and the entropy generation reduced as the nanoparticle volume fraction was increased. In addition, it was shown that the heat transfer performance could be maximized and the entropy generation minimized by positioning the heat source on the lower cavity wall.

As discussed above, the literature contains many investigations into the heat transfer performance and entropy generation rate of natural convection in cavities in regular shapes. Accordingly, the present study performs a numerical investigation into the natural convection heat transfer characteristics and entropy generation rate within an odd cavity containing Cu–water nanofluid. The simulations focus specifically on the effects of the nanoparticle volume fraction and the Rayleigh number on the flow streamlines, isotherm distribution, entropy generation, mean Nusselt number, total entropy generation and Bejan number.

2. Problem formulation

Natural convection in Cu–water nanofluid in the annular space between a hot inner body and its enclosure is considered in Fig. 1. The horizontal/vertical length of the outer and inner body is \( L \) and \( L_1 \) respectively. W is the width of the enclosure. The inner wall temperature is held constant at \( T_h \) and the outer wall temperature is maintained at the temperature \( T_c \) while the other two sides are insulated.

![Fig. 1. Geometry of the problem.](image-url)
3. Mathematical formulation

In the present problem, it is considered that the flow is steady, two-dimensional, laminar, incompressible and there is no viscous dissipation. The gravitational force acts in the vertically downward direction and radiation effect is neglected. The governing equations under Boussinesq approximation are as follows

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

(2)

\[
\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta_{nf} \rho_{nf} (T - T_c)
\]

(3)

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(4)

where, \( \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p \) is the density, \( \rho_C_{nf} = (1 - \phi)(\rho_C_f + \phi\rho_C_p) \) is the heat capacitance, \( \beta_{nf} = (1 - \phi)\beta_f + \phi\beta_p \) is the thermal expansion coefficient, \( \alpha_{nf} = k_{nf}(\rho C_{nf}) \) is the thermal diffusivity, \( \mu_{nf} = \mu_f (1 - \phi)^{-2.5} \) is the dynamic viscosity and \( k_{nf} = k_f \frac{1 + k_p}{k_f + k_p} \) is the thermal conductivity of the nanofluid.

The boundary conditions are

- At the inner walls \( T = T_h \)
- At the outer walls \( T = T_c \)
- At the rest of the surfaces \( \frac{\partial u}{\partial n} = 0 \)
- At all solid boundaries \( u = v = 0 \)

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{ul}{\alpha_f}, \quad V = \frac{vl}{\alpha_f}, \quad \nu = \frac{pl^2}{\rho_f \alpha_f^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}
\]

After substitution of the above variables into Eqs. (1) to (4), we get the following non-dimensional equations as

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(5)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial p}{\partial X} + \frac{\nu_U}{\nu_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

(6)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial p}{\partial Y} + \frac{\nu_U}{\nu_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \left( 1 - \phi \right) \beta_f \beta \theta
\]

(7)

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \alpha_{nf} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

(8)

where \( Pr = \frac{\nu_f}{\nu} \) is the Prandtl number and \( Ra = \frac{\left( g\beta_f \left( T_h - T_c \right) \right)^2}{\nu \alpha_f} \) is the Rayleigh number.

The corresponding boundary conditions then take the following form

- At the inner walls \( \theta = 1 \)
- At the outer walls \( \theta = 0 \)
- At other surfaces \( \frac{\partial \theta}{\partial n} = 0 \)
- At all solid boundaries \( U = V = 0 \)

The average Nusselt number at the heated surface of the enclosure may be expressed as

\[
Nu = \frac{1}{S} \int_0^S \left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial N} dN
\]
Fig. 4. Effect of $Ra$ on (a) isotherms (b) streamlines and (c) entropy due to heat transfer.
Fig. 5. Effect of $\phi$ on (a) isotherms (b) streamlines and (c) entropy due to heat transfer.
where \( \frac{S_{\text{gen}}}{S_{\text{gen}}^{\text{h}}} = \frac{1}{2} \sqrt{\left(\frac{S_{\text{gen}}}{S_{\text{gen}}^{\text{h}}} \right)^2 + \left(\frac{S_{\text{gen}}}{S_{\text{gen}}^{\text{v}}} \right)^2} \) and \( S, N \) are the non-dimensional length and coordinate along the heated surface respectively.

The entropy generation in the flow field is caused by the non-equilibrium flow imposed by boundary conditions. In the convection process, the entropy generation is due to the irreversibility caused by the heat transfer phenomena and fluid flow friction. According to Bejan [15], the dimensional local entropy generation, \( s_{\text{gen}} \), can be expressed by:

\[
s_{\text{gen}} = k_{\text{eff}} \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\mu_{\text{eff}}}{T_0} \left( \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right) \]

(9)

where \( T_0 = \frac{T_c + T_f}{2} \).

In Eq. (9), the first term represents the dimensional entropy generation due to heat transfer \( (s_{\text{gen}}^{\text{h}}) \), while the second term represents the dimensional entropy generation due to viscous dissipation \( (s_{\text{gen}}^{\text{v}}) \). By using dimensionless parameters, the expression of the non-dimensional entropy generation, \( S_{\text{gen}} \), can be written by:

\[
S_{\text{gen}} = \frac{S_{\text{gen}}^{\text{h}}}{k_f (T_h - T_c)^2} \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 + \mu_{\text{eff}} \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \]

\[
= \frac{S_{\text{gen}}^{\text{h}}}{S_{\text{gen}}^{\text{v}}} + S_{\text{gen}}^{\text{v}} \]

(10)

where, \( S_{\text{gen}}^{\text{h}} \) and \( S_{\text{gen}}^{\text{v}} \) are the dimensionless entropy generation for heat transfer and viscous effect respectively. In Eq. (10), \( \chi \) is the irreversibility factor which represents the ratio of the viscous entropy generation to thermal entropy generation. It is given as:

\[
\chi = \frac{T_0 \mu_f}{k_f (T_h - T_c)^2} \left( \frac{U_j}{L} \right)^2
\]

The Bejan number, \( Be \), defined as the ratio between the entropy generation due to heat transfer by the total entropy generation, is expressed as

\[
Be = \frac{S_{\text{gen}}^{\text{h}}}{S_{\text{gen}}}
\]

It is known that the heat transfer irreversibility is dominant when \( Be \) approaches 1. When \( Be \) becomes much smaller than 1/2 the irreversibility due to the viscous effects dominates the processes and if \( Be = 1/2 \) the entropy generation due to the viscous effects and the heat transfer effects are equal.

4. Numerical implementation

The Galerkin’s finite element method is used by Dechaumphai [28] to solve the non-dimensional governing equations along with boundary conditions for the considered problem. The equation of continuity has been used as a constraint such that mass conservation and this restriction may be used to find the pressure distribution. The finite element method of Reddy [29] is used to solve Eqs. (6)–(8), where the pressure \( P \) is eliminated by a constraint. The continuity equation is automatically fulfilled for large values of this constraint. Then the velocity components \( (U, V) \) and temperature \( (\theta) \) are expanded by using a basis set. The Galerkin’s finite element technique yields the subsequent non-linear residual equations. Three point Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved by using the Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion such that \( |\psi_{n+1} - \psi_{n}| \leq 10^{-4} \), where \( n \) is the number of iteration and \( \psi \) is a function of \( U, V \) and \( \theta \).

4.1. Mesh generation

In the finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volume, etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a

Fig. 6. Average Nusselt number for different (i) \( Ra \) with \( \phi = 5\% \) and (ii) \( \phi \) with \( Ra = 10^4 \).
collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Fig. 2 displays the finite element mesh of the present physical domain.

4.2. Thermo-physical properties

The thermo-physical properties of the nanofluid are taken from Ogut [30] and given in Table 1.

4.3. Grid independent test

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for $Ra = 10^4$ and $Pr = 6.6$ in the considered geometry. In the present work, we examine five different non-uniform grid systems with the following number of elements within the resolution field: 282, 572, 964, 2288 and 6794. The numerical scheme is carried out for highly precise key in the average Nusselt number for water–Cu nanofluid ($\phi = 5\%$) as well as base fluid ($\phi = 0\%$) for the aforesaid elements to develop an understanding of the grid fineness as shown in Fig. 3. The scale of the average Nusselt numbers for nanofluid and clear water for 2288 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 2288 elements is preferred for the computation.

5. Results and discussion

In this section, numerical results of streamlines, isotherms and entropy generation due to heat transfer for different values of Rayleigh number ($Ra$) and solid volume fraction ($\phi$) of Cu/water nanofluid in an odd type cavity are displayed. The considered values of $Ra$ and $\phi$ are $Ra = (10^3, 10^4, 10^5$ and $10^6)$ and $\phi = (0\%, 1\%, 3\%$ and $5\%)$ while the Prandtl number $Pr = 6.6$. In addition, the values of the average Nusselt number, entropy generation due to heat transfer term, entropy generation due to viscous term, total entropy generation and Bejan number are shown graphically.

![Fig. 7.](image-url)
5.1. Effect of Rayleigh number

The isothermal lines, streamlines and entropy generation due to heat transfer irreversibility are displayed in Fig. 4(a)–(c) for the effect of Rayleigh number with $\phi = 5\%$. As regards the isotherms, these are almost parallel to isothermal walls indicating the predominance of heat conduction at a low $Ra$ of $10^3$. Moreover, heat transfer within the cavity occurs primarily as a result of conduction and consequently the isotherms basically follow the geometry profile of the cavity. It is noted that for high $Ra$, at the inner body, a high flux region develops owing to the clustering of isotherms. At higher $Ra$ the isothermal patterns indicate that the conduction is dominated, while convection effects are significant. When convective effects are important, alternate spots of high and low fluxes are observed. The alternate spots of high and low heat fluxes are observed more clearly, especially in the horizontal part with the increase of $Ra$ from $10^2$ to $10^6$. The isotherms are more densely packed near the hot wall corner. This increases the heat transfer coefficient.

Fig. 4(b) indicates a mixed flow structure between that of a vertical and a horizontal enclosure. At lower values of the Rayleigh number, the buoyancy effect is weak, and thus no significant perturbation of the fluid flow occurs. It is seen that the vertical vortex is predominant and it fills a significant part of the horizontal extension also. The penetration of the vertical vortex into the horizontal chamber is more, for smaller $Ra$. For a Rayleigh number of $10^5$, it is observed that the flow pattern in the horizontal part also exerts a significant influence. The vertical flow is still able to enter partly into the horizontal portion. Within the horizontal extension, the typical cellular convection pattern is observed. The features seen at $Ra = 10^5$ are further accentuated for the Rayleigh number of $10^6$. The cellular pattern of the horizontal part becomes predominant. In this case three vortices are observed; one big vortex in the vertical portion as in the above cases and two vortices in the horizontal part.

The local entropy generation is the result mainly of heat transfer irreversibility. In Fig. 4(c), it can be seen that at a lower Rayleigh number, smaller local entropy generation occurs in the cavity. At higher
values of the Rayleigh number, the thermally-induced buoyancy effect is more intense, in other words, a higher temperature gradient exists in cavity walls, and thus a greater local entropy generation occurs as a result.

5.2. Effect of solid volume fraction

Fig. 5(a)–(c) shows the temperature, flow field and entropy generation due to heat transfer irreversibility for the effect of φ with \( Ra = 10^4 \). As the volume fraction of nanoparticles enhances from 0% to 5%, the isotherm contours tend to get affected considerably. In addition, these lines corresponding to \( \phi = 5\% \) become less bended. The isotherms are crowded around the active location on the heated surface of the enclosure for clear water (\( \phi = 0\% \)). A significant twisting of the isotherms occurs for smaller values of solid volume fraction which indicates the convection dominancy. Rising \( \phi \) leads to deformation of the thermal boundary layers at the heated surface and isothermal lines become parallel to the isothermal walls. That is heat transfer occurs mainly by conduction which is due to higher thermal conductivity of nanoparticles.

In Fig. 5(b), we observe that in the absence of \( \phi \) that is, the case of base fluid, the fluid flow covers the entire cavity with two rotating cells: vertical and horizontal. The streamlines have slight change due to raising the values of \( \phi \) from 0% to 5%. The core of the vortices becomes slightly smaller. The penetration of vertical vortex into the horizontal extension is more higher values of \( \phi \). The strength of the vortices in the streamlines becomes lower with the increase of the solid volume fraction because of higher concentration of nanoparticles which slower the fluid movement also.

Fig. 5(c) shows the contours of local entropy generation for the variation of \( \phi \) from 0% to 5%.

It can be seen from the figures that at lower values of \( \phi \), smaller local entropy generation occurs in the cavity. Increment of \( \phi \) causes slightly higher temperature gradient that leads to higher entropy generation. That is adding more nanoparticle to the water increased the entropy.

5.3. Nusselt number variation

The average Nusselt number \( Nu \) is obtained by integrating the local Nusselt number over the wall length and averaging it. The \( Nu \) variation along the hot wall for various values of Rayleigh numbers and solid volume fraction are shown in Fig. 6(i)–(ii). It is seen from the figures that \( Nu \) enhances sharply for increasing the values of \( Ra \) from \( 10^3 \) to \( 10^6 \) for both nanofluid and clear water. This is due to the fact that greater buoyancy effect causes high temperature as well as density gradient which augmented the rate of heat transfer. As in Fig. 6(ii), \( Nu \) also increases for greater values of \( \phi \) because nanofluid has higher thermal conductivity than the base fluid. The rate of increment of \( Nu \) becomes lower for upper values of \( \phi \) that says enhancement of heat transfer is possible up to a certain limit by adding nanoparticle to the pure water.

5.4. Entropy variation

In Fig. 7(i)–(iii), different forms of entropy generation are shown for the effects of \( Ra \). It should be noted that in Fig. 7(i), the entropy generation due to heat transfer increases by increasing \( Ra \) because increasing buoyancy force causes high temperature gradient. However, Fig. 7 (ii) tells that the increase of \( Ra \) causes the reduction of entropy generation due to viscous effects. This is due to the increase of buoyancy effects which induces the flow intensity thus causing reduction of shear effects and viscosity. Variation of total entropy generation is shown in Fig. 7(iii). As observed, the effect of \( Ra \) in decreasing the total entropy generation is dominant for \( Ra \) less than \( 10^6 \), where viscous effects are dominant. However, the effect of \( Ra \) is more pronounced in nanofluid than the base fluid in all the forms of entropy.

Various types of entropy generation are shown for the variation of \( \phi \) in Fig. 8(i)–(iii). It is clearly seen from the figures that, all forms of the entropy generation increases due to increase of the solid volume fraction of the nanofluid because the addition of more nanoparticles causes higher temperature gradient as well as density which increases the shear forces.

5.5. Bejan number variation

A better understanding of the effects of \( Ra \) and \( \phi \) on entropy generation is obtained by studying the variation of Bejan number \( Be \) as shown in Fig. 9(i)–(ii). It is noticed that at low \( Ra \), the viscous effect is pronounced, while at higher \( Ra \), \( Be \) converges to one for all of the cases either pure or nanofluid. The addition of nanoparticles to water increases the viscosity. However, at the same time the thermal

![Fig. 9. Bejan number for different (i) Ra with \( \phi = 5\% \) and (ii) \( \phi \) with \( Ra = 10^6 \).](image-url)
conductivity is also increased. The results obtained show that the addition of nanoparticles increases both terms of entropy generation almost with the same rate. The entropy due to viscous effect is dominant for $Ra = 10^5$ and $Ra = 10^4$ while the entropy due to heat transfer is dominant for $Ra = 10^3$ and more.

5.6. Comparison

The average Nusselt number values for the present study are compared to that of Nithiarasu et al. [31] for the same geometry. The comparison is shown in Table 2. The present results for $\phi = 0\%$ are in good agreement with Nithiarasu et al. [31].

6. Conclusions

The laminar natural convection and entropy generation in a nanofluid filled complex cavity with a horizontal and a vertical portion are studied. The cavity is filled with either water or Cu–water nanofluid. The effects on fluid flow, heat transfer and entropy generation at various Rayleigh numbers and solid volume fractions are investigated. The results show that using the nanofluid, generally leads to lowering the flow strength whereas increases the Nusselt number, entropy generation and the Bejan number. By increasing the Rayleigh number, the Nusselt number and Bejan number increase. The total entropy generation is found minimum at $Ra = 10^5$. A general study of the variation of $Nu$ and the total entropy generation with change of parameters involved shows that if enhanced heat transfer rate with the lowest entropy generation is required in the considered geometry the choice of $Ra$ and $\phi$ is important.

Table 2
Comparison of average Nusselt numbers for the present study with Nithiarasu et al. [31].

<table>
<thead>
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<th>$Ra$</th>
<th>Nithiarasu et al. [31]</th>
<th>Present work for $\phi = 0%$</th>
<th>Present work for $\phi = 5%$</th>
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References