CHEMICAL REACTION EFFECTS ON UNSTEADY MAGNETOHYDRODYNAMIC FREE CONVECTIVE FLOW IN A ROTATING POROUS MEDIUM WITH MASS TRANSFER

by

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An investigation of unsteady magnetohydrodynamic free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium, bounded by an infinite vertical porous surface, in a rotating system is presented. The porous plane surface and the porous medium are assumed to rotate in a solid body rotation. The vertical surface is subjected to uniform constant suction perpendicular to it and the temperature at this surface fluctuates in time about a non-zero constant mean. Analytical expressions for the velocity, temperature and concentration fields are obtained using the perturbation technique. The effects of rotation parameter, permeability parameter, Hartmann number, and frequency parameter on the flow characteristics are discussed. It is observed that the primary velocity component decreases with the increase in either of the rotation parameter, the permeability parameter, or the Hartmann number. It is also noted that the primary skin friction increases whenever there is an increase in the Grashof number or the modified Grashof number. It is clear that the heat transfer coefficient in terms of the Nusselt number decreases in the case of both air and water when there is an increase in the Hartmann number. It is observed that the magnitude of the secondary velocity profiles increases whenever there is an increase in either of the Grashof number or the modified Grashof number for mass transfer or the permeability of the porous media. Concentration profiles decreases with an increase in the Schmidt number.

Key words: *mass transfer, free convection, porous medium, magnetohydrodynamic, rotation, heat transfer, unsteady, primary and secondary velocity components*

Introduction

The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic flow problems with mass transfer is of interest in power engineering and metallurgy. Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems, *etc.* Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Mag-

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netohydrodynamic (MHD) flows have attracted the attention of a large number of scholars due to their diverse applications. In astrophysics and geophysics, they are applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, *etc.* In engineering, MHD flows find their application in MHD pumps, MHD bearings, *etc.* Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomenon of mass transfer is also very common in the theory of stellar structure and observable effects are detectable, at least on the solar surface.

Malathy and Srinivas [1] investigated the pulsating flow of a hydromagnetic fluid between two permeable beds. Singh [2] analyzed the influence of a moving magnetic field on 3-D Couette flow. Das *et al.* [3] discussed mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source.

Muthucumaraswamy and Ganesh [4] studied unsteady flow of an incompressible fluid past an impulsively started vertical plate with heat and mass transfer. Acharya *et al.* [5] discussed magnetic field effects on free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Chaudhary and Jain [6] analyzed combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. Dinarvand and Rashidi [7] studied a reliable treatment of homotopy analysis method for 2-D viscous flow in a rectangular domain bounded by two moving porous walls. Muthuraj and Srinivas [8] discussed heat transfer effects on MHD oscillatory flow in an asymmetric wavy channel. Muthucumaraswamy *et al.* [9] analyzed chemical reaction effects on infinite vertical plate with uniform heat flux and variable mass diffusion. Singh and Verma [10] discussed heat transfer effects in a 3-D flow through a porous medium with a periodic permeability. Gersten and Gross [11] analyzed flow and heat transfer effects along a plane wall with periodic suction. Singh [12] discussed the effect of injection/suction parameter on 3-D Couette flow with transpiration cooling. Gupta and Johari [13] studied the effect of MHD incompressible flow past a highly porous medium which was bounded by a vertical infinite porous plate. Singh *et al.* [14] analyzed the heat transfer effects on 3-D fluctuating flow through a porous medium with a variable permeability. Rashidi and Sadri [15] analyzed the solution of the laminar viscous flow in a semi-porous channel in the presence of a uniform magnetic field by using the differential transform method. Rashidi and Erfani [16] discussed a new analytical study of MHD stagnation-point flow in porous media with heat transfer.

Jain and Gupta [17] discussed free convection effects on 3-D Couette flow with transpiration cooling. Singh and Prakesh [18] analyzed the MHD effects on 3-D Couette flow with transpiration cooling. Singh *et al.* [19] studied the effects of permeability and rotation parameters on oscillatory Couette flow through a porous medium in a rotating system. Raptis and Perdikis [20] discussed the effect of permeability on oscillatory and free convection flow through a porous medium. Chemical reactions can be classified as either heterogeneous or homogenous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production.

In the studies mentioned above, unsteady free convective flow with heat and mass transfer effects in a rotating porous medium have not been discussed while such flows are very important in geophysical and astrophysical problems. Therefore, the objective of the present paper is to analyze the effects of permeability variation, mass transfer and chemical reaction on flow of a viscous incompressible fluid past an infinite vertical porous surface in a rotating system, when the temperature of the surface varies with time about a non-zero constant mean and the temperature at the free stream is constant.

Formulation of the problem

Consider unsteady flow of a viscous incompressible fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite porous surface in a rotating system under the action of a uniform magnetic field applied normal to the direction of flow. The temperature of the surface varies with time about a non-zero constant mean and the temperature at the free stream is constant. The porous medium is, in fact, a non-homogenous medium which may be replaced by a homogenous fluid having dynamical properties equal to those of a non-homogenous continuum. Also, we assume that the fluid properties are not affected by the temperature and concentration differences except by the density ρ in the body force term; the influence of the density variations in the momentum and energy equations is negligible. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Initially the plate and the fluid are of same temperature T_∞^* and concentration C_∞^*. The plate temperature is raised to T_w^* and the species concentration level near the plate is made raise to C_w^*.

We consider that the vertical infinite porous plate rotates in unison with a viscous fluid occupying the porous region with the constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. The Cartesian co-ordinate system is chosen such that *x, y* axes, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z = 0$ while z-axis is normal to it as shown in fig. 1 with the above frame of reference and assumptions, the physical variables, except the pressure *p*, are functions of *z* and time *t* only. Consequently, the equations expressing the conservation of mass, momentum, and energy and the equation of mass transfer, neglecting the heat due to viscous dissipation which is valid for small velocities, are given by:

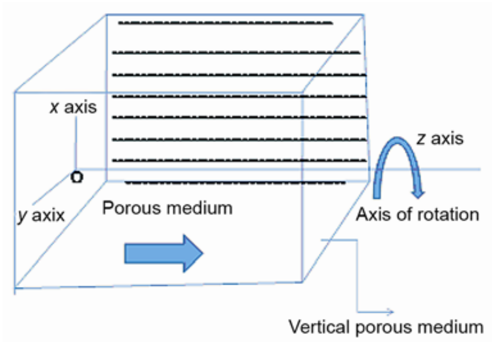

Figure 1. Sketch of the physical problem

$$\frac{\partial w^*}{\partial z^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega v^* = g\beta (T^* - T_\infty^*) + g\beta^* (C^* - C_\infty^*) + \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu}{K^*} u^* - \frac{\sigma B_0^2 u^*}{\rho} \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega u^* = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu}{K^*} v^* - \frac{\sigma B_0^2 v^*}{\rho} \tag{3}$$

$$0=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}-\frac{\nu}{K^*}w^* \tag{4}$$

$$\frac{\partial T^*}{\partial t^*}+w^*\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial z^{*2}} \tag{5}$$

$$\frac{\partial C^*}{\partial t^*}+w^*\frac{\partial C^*}{\partial z^*}=D\frac{\partial^2 C^*}{\partial z^{*2}}-K_\ell C^* \tag{6}$$

with the boundary conditions

$$\begin{gathered}u^*=0,\quad v^*=0,\quad T^*=T_w^*+\varepsilon(T_w^*-T_\infty^*)e^{iwt},\quad C^*=C_w^*\quad \text{at } z^*=0\\ u^*,v^*\to 0,\quad T^*\to T_\infty^*,\quad C^*\to C_\infty^*\quad \text{as}\quad z^*\to\infty\end{gathered} \tag{7}$$

In a physically realistic situation, we cannot ensure perfect insulation in any experimental se-tup. There will always be some fluctuations in the temperature. The plate temperature is assumed to vary harmonically with time. It varies from $T_w^*\pm\varepsilon(T_w^*-T_\infty^*)$ as t varies from 0 to $2\pi/w$. Since ε is small, the plate temperature varies only slightly from the mean value T_w^*.

For constant suction, we have from eq. (1) in view of (7):

$$w^*=-w_0 \tag{8}$$

Considering $u^*+iv^*=U^*$ and taking into account eq. (8), then eqs. (2) and (3) can be written as:

$$\frac{\partial U^*}{\partial t^*}-w_0\frac{\partial U^*}{\partial z^*}+2\Omega iU^*=\mathrm{g}\beta(T^*-T_\infty^*)+\mathrm{g}\beta^*(C^*-C_\infty^*)+\nu\frac{\partial^2 U^*}{\partial z^{*2}}-\frac{\nu}{K^*}U^* \tag{9}$$

We introduce the following non-dimensional quantities:

$$z=\frac{w_0 Z^*}{\nu},\ U=\frac{U^*}{w_0},\ t=\frac{t^* w_0^{2*}}{\nu},\ w'=\frac{\nu w^*}{w_0^2},\ T'=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*},\ C'=\frac{C^*-C_\infty^*}{C_w^*-C_\infty^*},\ \mathrm{Sc}=\frac{\nu^*}{D},$$

$$P=\frac{\rho\nu C_p}{K},\ k_0=\frac{w_0^2 K^*}{\nu^2},\ \mathrm{Gr}=\frac{\nu \mathrm{g}\beta(T_w^*-T_\infty^*)}{w_0^3},\ \mathrm{Gm}=\frac{\nu \mathrm{g}\beta^*(C_w^*-C_\infty^*)}{w_0^3},\ R=\frac{\Omega\nu}{w_0^2},\ K=\frac{K_\ell \nu}{w_0^*}$$

By introducing non-dimensional quantities, eqs. (9), (5), and (6) will be reduced to:

$$\frac{\partial U}{\partial t}-\frac{\partial U}{\partial z}+i2RU=\mathrm{Gr}T+\mathrm{Gm}C+\frac{\partial^2 U}{\partial z^2}-\left(\frac{1}{k_0}+M^2\right)U \tag{10}$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial z}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial z^2} \tag{11}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial z}=\frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial z^2}-KC \tag{12}$$

and the boundary conditions (7) become:

$$U=0,\quad T=1+\varepsilon e^{i\omega t},\quad C=1\quad \text{at}\quad z=0;\quad U\to 0,\quad T\to 0,\quad C\to 0\quad \text{as}\quad z\to\infty \tag{13}$$

Method of solution

In order to reduce the system of partial differential, eqs. (10)-(12) under their boundary conditions (13), to a system of ordinary differential equations in the non-dimensional form, we assume the following for velocity, temperature and concentration of the flow field as the amplitude $\varepsilon\,(\ll 1)$ of the permeability variations is very small:

$$U(z,t)=U_0(z)+\varepsilon\,\mathrm{e}^{i\omega t}U_1(z);\quad T(z,t)=T_0(z)+\varepsilon\,\mathrm{e}^{i\omega t}T_1(z);\quad C(z,t)=C_0(z)+\varepsilon\,\mathrm{e}^{i\omega t}C_1(z) \tag{14}$$

Substituting system (14) into the system (10)-(12) and equating harmonic and non-harmonic terms we get:

$$U_0''+U_0'-2iRU_0-\left(\frac{1}{k_0}+M^2\right)U_0=-(\mathrm{Gr}T_0+\mathrm{Gm}C_0) \tag{15}$$

$$U_1''+U_1'-U_1\left[\frac{1}{k_0}+M^2+i(\omega+2R)\right]=-(\mathrm{Gr}T_1+\mathrm{Gm}C_1) \tag{16}$$

$$T_0''+\mathrm{Pr}T_0'=0 \tag{17}$$

$$T_1''+\mathrm{Pr}T_1'-i\omega\mathrm{Pr}T_1=0 \tag{18}$$

$$C_0''+\mathrm{Sc}\,C_0'-K\,\mathrm{Sc}\,C_0=0 \tag{19}$$

$$C_1''+\mathrm{Sc}\,C_1'-\mathrm{Sc}\,C_1(i\omega+K)=0 \tag{20}$$

The appropriate boundary conditions reduce to:

$$\begin{gathered}U_0(0)=0,\quad T_0(0)=1,\quad C_0(0)=1;\quad U_1(0)=0,\quad T_1(0)=1,\quad C_1(0)=0;\\ U_0(\infty)\to 0,\quad T_0(\infty)\to 0,\quad C_0(\infty)\to 0;\quad U_1(\infty)\to 0,\quad T_1(\infty)\to 0,\quad C_1(\infty)\to 0\end{gathered} \tag{21}$$

Thus, the solution of the problem is:

$$U(z,t)=L_1\mathrm{e}^{-Prz}+L_2\mathrm{e}^{-Scz}+L_3\mathrm{e}^{M_3z}+\varepsilon\,\mathrm{e}^{i\omega t}\mathrm{Gr}\frac{(\mathrm{e}^{M_5z}-\mathrm{e}^{M_1z})}{(M_1-M_4)(M_1-M_5)} \tag{22}$$

$$T(z,t)=\mathrm{e}^{-Prz}+\varepsilon\,\mathrm{e}^{i\omega t}\mathrm{e}^{M_1z} \tag{23}$$

$$C(z,t)=\mathrm{e}^{-m_2z} \tag{24}$$

It is convenient to write the primary and secondary velocity fields, in terms of the fluctuating parts, separating the real and imaginary part from eqs. (22) and (23) and taking only the real parts as they have physical significance, the velocity and temperature distribution of the flow field can be expressed in fluctuating parts as:

$$\frac{u}{w_0} = u_0 + \varepsilon(N_r \cos\omega t - N_i \sin\omega t) \tag{25}$$

$$\frac{v}{w_0} = v_0 + \varepsilon(N_r \sin\omega t + N_i \cos\omega t) \tag{26}$$

where $u_0 + iv_0 = U_0$ and $N_r + iN_i = U_1$.

Hence, the expressions for the transient velocity profiles for $\omega t = \pi/2$ are given by:

$$\frac{u}{w_0}\left(z, \frac{\pi}{2\omega}\right) = u_0(z) - \varepsilon N_i(z) \quad \text{and} \quad \frac{v}{w_0}\left(z, \frac{\pi}{2\omega}\right) = v_0(z) - \varepsilon N_r(z)$$

Skin friction

The skin friction at the plate $z = 0$ in terms of amplitude and phase is given by:

$$\left.\frac{\mathrm{d}U}{\mathrm{d}Z}\right|_{(z=0)} = \left.\frac{\mathrm{d}U_0}{\mathrm{d}Z}\right|_{z=0} + \varepsilon \mathrm{e}^{iwt}\frac{\mathrm{d}U_1}{\mathrm{d}Z} = -\mathrm{Pr}L_1\mathrm{e}^{-\mathrm{Pr}z} - L_2\mathrm{Sce}^{-\mathrm{Sc}z} + L_3M_3\mathrm{e}^{M_3z} + \varepsilon\mathrm{e}^{i\omega t}\frac{\mathrm{Gr}(M_5\mathrm{e}^{M_5z} - M_1\mathrm{e}^{M_1z})}{(M_1 - M_4)(M_1 - M_5)}$$

$$\left(\frac{\mathrm{d}u}{\mathrm{d}Z}\right)_{\text{at } z=0} = (-\mathrm{Pr}L_1 - L_2\mathrm{Sc} + L_3M_3) + \varepsilon\mathrm{e}^{i\omega t}\frac{\mathrm{Gr}(M_5 - M_1)}{(M_1 - M_4)(M_1 - M_5)} =$$

$$= (-\mathrm{Pr}L_1 - L_2\mathrm{Sc} + L_3M_3) - \varepsilon\mathrm{e}^{i\omega t}\frac{\mathrm{Gr}}{(M_1 - M_4)} \tag{27}$$

The skin friction coefficient for various values of Gr, Gm, k_0, R, and M are given in tab. 3 after separating the real and imaginary parts of the eq. (28).

Rate of heat transfer

The heat transfer coefficient in terms of the Nusselt number at the plate $z = 0$ in terms of amplitude and phase is given by:

$$\left.\frac{\mathrm{d}T}{\mathrm{d}Z}\right|_{(z=0)} = \left.\frac{\mathrm{d}T_0}{\mathrm{d}Z}\right|_{z=0} + \varepsilon\mathrm{e}^{iwt}\frac{\mathrm{d}T_1}{\mathrm{d}Z}, \quad \left.\frac{\mathrm{d}T}{\mathrm{d}Z}\right|_{(z=0)} = -\mathrm{Pr} + +\varepsilon\mathrm{e}^{iwt}M_1 \tag{28}$$

The constants L_1, L_2, L_3, M_1, M_2, M_3, M_4, M_5, N_i, N_r are given in the Appendix.

Rate of mass transfer

The rate of mass transfer coefficients in terms of Schmidt number at the plate $z = 0$ is given by:

$$\left.\frac{\mathrm{d}C}{\mathrm{d}Z}\right|_{(z=0)} = \left.\frac{\mathrm{d}C_0}{\mathrm{d}Z}\right|_{z=0} + \varepsilon\mathrm{e}^{iwt}\frac{\mathrm{d}C_1}{\mathrm{d}Z} = -m_2$$

Results and discussions

The problem of unsteady MHD free convective flow with heat and mass transfer effects in a rotating porous medium has been considered. The solutions for primary and secondary velocity field, temperature field, and concentration profiles are obtained using the perturbation technique. The effects of flow parameters such as the magnetic parameter M, Grashof numbers for heat and mass transfer, and porosity parameter k_0, Prandtl number, and the rotation parameter R on the velocity field have been studied analytically and presented with the help of figs. 2 and 3. The effects of flow parameter on concentration profiles have been presented with the help of fig. 4. The effects of flow parameters on the transient velocity profiles u/w_0 and v/w_0 have been presented in tab. 2. Further, the effects of flow parameters on the skin friction coefficient and rate of heat transfer have been discussed with the help of tabs. 3 and 4.

Primary velocity profile (u/w_0)

From eqs. (22), (23), and (24), it is observed that the steady part of the velocity field has a three layer character. These layers may be identified as the thermal layer arising due to interaction of the thermal field and the velocity field and is controlled by the Prandtl number; the concentration layer arising due to the interaction of the concentration field and the velocity field, and the suction layer as modified by the rotation and the porosity of the medium. On the other hand, the oscillatory part of the velocity field exhibits a two layer character. These layers may be identified as the modified suction layers, arising as a result of the triangular interaction of the Coriolis force and the unsteady convective forces with the porosity of medium.

The dimensionless primary and secondary velocity components for different values of Gr, Gm, k_0, and R are shown in figs. 2 and 3 considering Pr = 0.71 (air), $\omega = 5$, $\omega t = \pi/2$ $\varepsilon = 0.002$, and Sc = 0.6. The value of Sc = 0.6 is chosen in such a way to represent water vapor at approximately 25 °C and 1 atm. It is clear from fig. 2 that the primary velocity profiles increases whenever there is either an increase in the Grashof number or the modified Grashof number for mass transfer whereas the profiles show the reverse trend whenever there is an increase in either of the rotation parameter, the permeability of the porous medium or the Hartmann number. This shows that the rotation, permeability of the porous medium and the magnetic field exert retarding influence on the primary flow.

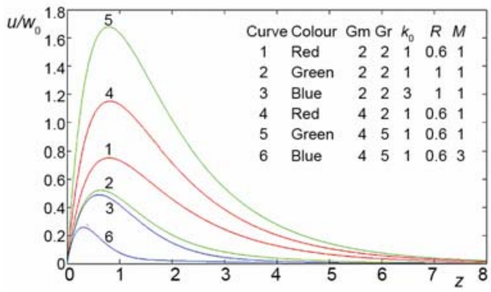

Figure 2. Effects of Gm, Gr, k_0, R, M on primary velocity profiles with Pr = 0.71, $\omega = 5$, $\omega t = \pi/2$, $\varepsilon = 0.002$, and Sc = 0.6
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From fig. 2 it is noted that all the velocity profiles increase steadily near the lower plate and thereafter they show a constant decrease and reach the value zero at the other plate, but the profiles show a reverse trend in fig. 3. The magnetic parameter is found to decelerate the velocity of the flow field to a significant amount due to the magnetic pull of the Lorentz force acting on the flow field. In the case of Singh [2], the magnetic parameter shows the reverse effect.

Secondary velocity profile (v/w_0)

The secondary velocity profiles are shown in fig. 3 for various values of the modified Grashof number, permeability of the porous medium, rotation parameter and the Hartmann-

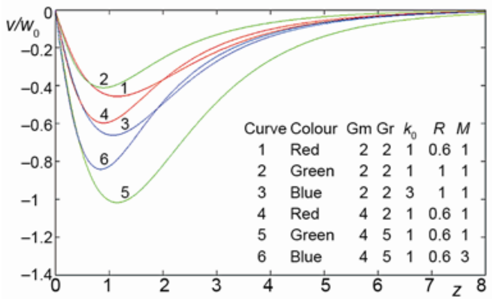

Figure 3. Effects of Gm, Gr, k_0, *R*, and *M* on secondary velocity profiles with Pr = 0.71, $\omega = 5$, $\omega t = \pi/2$, $\varepsilon = 0.002$, and Sc = 0.6
(for color image see journal web-site)

number or magnetic parameter. It is observed that the magnitude of the secondary velocity profiles increases whenever there is an increase in either of the Grashof number or the modified Grashof number for mass transfer or the permeability of the porous medium. On the other hand, the velocity profiles show the opposite trend whenever there is an increase in the rotation parameter or the Hartmann number.

Transient velocity profiles

The numerical values of the transient velocity component u/ω_0 and v/ω_0 for different values of the frequency ω are given in tab. 1. It is seen that for fixed values of k_0 and R, the components of the primary velocity and the magnitude of the secondary velocity decrease as the frequency parameter ω increases. This is in keeping with the view that the frequency of the oscillation of the plate temperature has an accelerating effect on the flow field. It is also remarked that since the permeability parameter k_0 involves the suction velocity ω_0, the results discussed and displayed in figs. 2 and 3 for the variations of the parameter k_0 correspond also to the variations in the suction velocity at the porous surface in the manner $\omega_0 \propto (k_0)^{-1/2}$.

Table 1. Variations of velocities u/ω_0 and v/ω_0 when Gm = 2.0, Gr = 2.0, Pr = 0.71, Sc = 0.6, $\varepsilon = 0.2$, $k_0 = 1.0$, $R = 1.0$

Z	$\omega = 5$ u/ω_0	v/ω_0	$\omega = 10$ u/ω_0	v/ω_0
0	0	0	0	0
1	0.5244	–0.4982	0.5172	–0.4964
2	0.2752	–0.3720	0.2657	–0.3696
3	0.1328	–0.2072	0.1301	–0.2068
4	0.0672	–0.1091	0.0652	–0.1088
5	0.0350	–0.0572	0.0345	–0.0568
6	0.0185	–0.0301	0.0180	–0.0295
7	0.0098	–0.0159	0.0090	–0.0155
8	0.0052	–0.0084	0.0050	–0.0080

Concentration profiles

The concentration profiles for various values of the Schmidt number are plotted in fig. 4. It is noted from fig. 4 that the concentration profiles decreases with an increase in the Schmidt number. The values of the Schmidt number are chosen in such a way that they represent the diffusing chemical species of most common interest in air. For example, the values of Sc for H_2, H_20, NH_3, propyl benzene and helium in air are 0.22, 0.60, 0.78, 2.62, and 0.30, respectively, as reported by Perry [21]. It is noted that for heavier diffusing foreign species, *i. e.*, increasing the Schmidt number reduces the velocity in both magnitude and extent and thinning of thermal boundary layer occurs. Substantial increase in the velocity profiles is observed near the plate with decreasing values of the Schmidt number (lighter diffusing particle). This shows that the heavier diffusing species have greater retarding effects on the concentration profiles of the flow field. The concentration profiles agrees well with the results of Das *et al.* [3].

It is clear from fig. 5. that the concentration profiles decrease with an increase in the chemical reaction parameter.

Skin friction

It is noted from tab. 2 that the primary skin-friction component increases due to an increase in either of the Grashof number or the modified Grashof number for mass transfer. On

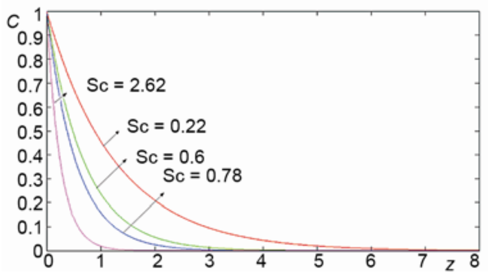

Figure 4. Effects of Sc on concentration distribution with ε = 0.002, ω = 5 and ωt = π/2
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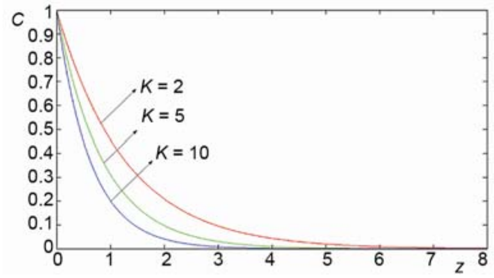

Figure 5. Effects of *K* on concentration distribution with ε = 0.002, ω = 5 and ωt = π/2
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Table 2. Variations of skin friction (primary and magnitude of secondary) when ε = 0.002, *Pr* = 0.71, *Sc* = 0.6, and various values of *Gm, Gr, R,* k_0*, M*

Gm	Gr	k_0	*R*	*M*	Primary skin friction d/dz (u/ω_0)	Secondary skin friction d/dz (v/ω_0)	Magnitude of secondary skin friction
2	2	1	0.6	1	2.7087	–1.0428	1.0428
2	2	1	1	1	2.2342	–1.1789	1.1789
2	2	3	1	1	2.2006	–1.5409	1.5409
4	2	1	0.6	1	4.1182	–1.6098	1.6098
4	5	1	0.6	1	6.0669	–2.3233	2.3233
4	5	1	0.6	3	2.1285	–1.8165	1.8165

the other hand, it decreases due to an increase in either of the rotation parameter, permeability of the porous medium or the Hartmann number. It is also noted from the above table that the magnitude of the secondary skin friction increases due to an increase in either of the rotation parameter *R*, modified Grashof number, permeability of the porous medium k_0 or the Grashof number. However, it decreases with an increase in the Hartmann number. The effects of all parameters except closely agrees with the results of Das *et al*. [3].

Table 3. Variations of heat transfer when ε = 0.2, in the case air and water

Pr = 0.71 (air)					Pr = 7.0 (water)			
ω	*T*	*M*	*Nu*		ω	*T*	*M*	*Nu*
2	2	2	0.6716		2	2	2	6.4251
2	2	3	0.6300		2	2	3	6.4032
3	2	3	1.1343		3	2	3	8.6623
3	3	3	0.2930		3	3	3	5.3585
3	3	4	0.2472		3	3	4	5.3383
4	3	4	1.2119		4	3	4	8.7379
4	4	4	0.3830		4	4	4	5.6434

Rate of heat transfer

The magnitude of the heat transfer coefficient for various values of ω, *t,* and *M* are given in tab. 3. It is clear from tab. 3 that the magnitude of the heat transfer coefficient in the case of both air and water decrease whenever there is an increase in either the Hartmann number or time *t*. But, they increase due to an increase in the frequency parameter ω. It is also observed from tab. 3 that the heat transfer coefficient in the case of water for any particular values of ω, *t,* and *M* is significantly higher when compared with that of air. Our results are in good agreement with the results of Mahato and Maiti [22].

Table 4. Rate of mass transfer for several of Sc and *K*

Sc	Rate of mass transfer	*K*	Rate of mass transfer
0.22	−0.1646	2	−3.5377
0.6	−2.0578	5	−4.6764
0.78	−2.4030	10	−6.0164
2.26	−4.6764	15	−7.0610

Rate of mass transfer

The rate of mass transfer for various values of Sc and *K* are given in tab. 4.

The rate of mass transfer decreases when either there is an increase in the Schmidt number or there is an increase in the chemical reaction parameter.

Conclusions

Following results are of physical interest for analyses the velocity (primary and secondary), temperature and concentration profiles.

- The modified Grashof number for mass transfer and Grashof number have the effect of accelerating the primary velocity profiles, the magnitude of the secondary velocity profiles and the skin friction whereas the Hartmann number has the effect of decreasing the flow field at all the points due to the magnetic pull of the Lorentz force acting on the flow field.
- The rotation parameter and the frequency parameter have the effect of decreasing the primary velocity profiles as well as the magnitude of the secondary velocity profiles whereas they have the effect of increasing the skin friction and the rate of heat transfer.
- The permeability parameter has the influence of decreasing the primary velocity and the primary skin friction whereas it has the influence of increasing the magnitude of the secondary velocity profiles and the secondary skin friction.
- The presence of foreign species reduces the velocity as well as the thermal boundary layer and further reduction occurs with increasing values of the Schmidt number.
- The velocity of the fluid layer decreases and the thickness of the thermal boundary layer increases with increasing values of the Schmidt number.
- An increase in the magnetic parameter causes decreases in both the primary velocity profiles and the magnitude of the secondary velocity profiles.
- When the magnetic parameter is neglected. *i. e.*, ($M = 0$) and the frequency of oscillation is kept constant, the results obtained in this paper coincide with the result obtained by Mahato and Maiti [22]. In the absence of magnetic field, the primary velocity and the magnitude of the secondary velocity obtained by the above researchers increase as the frequency parameter increases. However, when the frequency parameter increases, the components of the primary velocity profiles and the magnitude of the secondary velocity profiles decrease as could be seen from tab. 1 in this paper. This is due to the presence of the magnetic field.
- The concentration profile decreases with an increase in the chemical reaction parameter but as far as the velocity profiles are concerned (both primary and secondary velocity) there is no change in them when it increases.

Appendix

$$\mathrm{L}_1 = -\frac{\mathrm{Gr}}{(\mathrm{Pr}+\mathrm{M}_2)(\mathrm{Pr}+\mathrm{M}_3)},\ L_2 = -\frac{\mathrm{Gm}}{(\mathrm{Sc}+M_2)(\mathrm{Sc}+M_3)},\ L_3 = -(L_1+L_2)$$

$$M_1 = \frac{1}{2}(-\mathrm{Pr} - \sqrt{\mathrm{Pr}^2 + 4i\omega\mathrm{Pr} + M^2}),\quad M_2 = \frac{1}{2}\left[-1+\sqrt{1+4\left(2iR+M^2+\frac{1}{k}\right)}\right],$$

$$M_3 = \frac{1}{2}\left[-1 - \sqrt{1 + 4\left(2iR + M^2 + \frac{1}{k}\right)}\right], \quad M_4 = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(2iR + M^2 + \frac{1}{k} + i\omega\right)}\right],$$

$$M_5 = \frac{1}{2}\left[-1 - \sqrt{1 + 4\left(2iR + M^2 + \frac{1}{k} + i\omega\right)}\right],$$

$$N_r = \text{Real part}\left[\mathrm{e}^{i\omega t}\frac{\mathrm{Gr}}{M_5 - M_1}(\mathrm{e}^{M_5 z} - \mathrm{e}^{M_1 z})\right], N_i = \text{Imaginary part}\left[\mathrm{e}^{i\omega t}\frac{\mathrm{Gr}}{M_5 - M_1}(\mathrm{e}^{M_5 z} - \mathrm{e}^{M_1 z})\right]$$

$$m_2 = \frac{1}{2}\mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + 4K\,\mathrm{Sc}}$$

Nomenclature

- C – species concentration, [kg]
- C' – dimensionless species concentration, [–]
- C_p – specific heat at constant pressure, [$\mathrm{Jkg^{-1}K^{-1}}$]
- D – chemical molecular diffusivity, [$\mathrm{m^2s^{-1}}$]
- g – acceleration due to gravity, [$\mathrm{ms^{-2}}$]
- Gm – dimensionless modified Grashof number, [–]
- Gr – dimensionless Grashof number, [–]
- M – dimensionless Hartmann number (magnetic parameter), [–]
- K – chemical reaction parameter in non-dimensional form, [–]
- K_ℓ – chemical reaction parameter in dimensional form, [–]
- k_0 – permeability of the porous medium, [$\mathrm{m^2}$]
- k – thermal conductivity, [$\mathrm{Jm^{-1}s^{-1}K^{-1}}$]
- k^* – dimensionless permeability parameter, [–]
- Pr – dimensionless Prandlt number, [–]
- p – pressure, [mmHg]
- R – dimensionless rotation parameter, [–]
- Sc – dimensionless Schmidt number, [–]
- T – temperature, [K]
- T' – dimensionless temperature, [–]
- t – time, [s]
- t' – dimensionless time, [–]
- U' – dimensionless velocity, [–]
- u, v, w – velocity components in x-, y- and z-directions, respectively, [$\mathrm{ms^{-1}}$]
- w_0 (> 0) – constant suction velocity of the liquid through the porous plane surface, [$\mathrm{ms^{-1}}$]
- z – dimensionless normal direction of vertical porous plane surface, [–]
- z' – dimensionless normal distance, [–]

Greek symbols

- β – volumetric coefficient of thermal expansion, [$\mathrm{K^{-1}}$]
- β^* – volumetric coefficient of expansion with concentration, [$\mathrm{kg^{-1}}$]
- $\varepsilon\,(0 < \varepsilon < 1)$ – constant, [–]
- ν – kinematic viscosity, [$\mathrm{m^2s^{-1}}$]
- ρ – density, [$\mathrm{kgm^{-3}}$]
- Ω – angular velocity of the rotating frame of reference, [$\mathrm{s^{-1}}$]
- w – frequency of oscillation of the plate temperature, [$\mathrm{s^{-1}}$]
- w' – dimensionless frequency, [–]

Superscript

- U' – derivative of U with respect to z, [–]

Subscripts

- w – conditions on the porous plane surface, [–]
- ∞ – conditions away from the porous plane surface, [–]

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