Non-similar solutions for mixed convection along a wedge embedded in a porous medium saturated by a non-Newtonian nanofluid: Natural convection dominated regime

Ali J. Chamkha M. Rashad Rama Subba Reddy Gorla

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Non-similar solutions for mixed convection along a wedge embedded in a porous medium saturated by a non-Newtonian nanofluid

Natural convection dominated regime

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Abstract

Purpose – The purpose of this paper is to present a boundary layer analysis for the mixed convection past a vertical wedge in a porous medium saturated with a power law type non-Newtonian nanofluid. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the buoyancy ratio parameter Nr, Brownian motion parameter Nb, thermophoresis parameter Nt, Lewis number Le and the power law exponent n. The dependency of the friction factor, surface heat transfer rate (Nusselt number) and mass transfer rate on these parameters has been discussed.

Design/methodology/approach – This general non-linear problem cannot be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. An implicit, tri-diagonal finite-difference method has proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present study. Variable step sizes were used. The convergence criterion employed in this study is based on the difference between the current and the previous iterations. When this difference reached $10^{-5}$ for all the points in the $\eta$ directions, the solution was assumed to be converged, and the iteration process was terminated.

Findings – The results indicate that as the buoyancy ratio parameter (Nr) and thermophoresis parameter (Nt) increase, the friction factor increases whereas the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decrease. As the Brownian motion parameter (Nb) increases, the friction factor and surface mass transfer rates increase whereas the surface heat transfer rate decreases. As Le increases, mass transfer rates increase. As the power law exponent n increases, the heat and mass transfer rates increase.

Research limitations/implications – The analysis is valid for natural convection dominated regime. The combined forced and natural convection dominated regimes will be reported in a future work.

Practical implications – The approach used is useful in optimizing the porous media heat transfer problems in geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage and flow through filtering media.

Originality/value – The results of the study may be of some interest to the researchers of the field of porous media heat transfer. Porous foam and microchannel heat sinks used for electronic cooling are...
optimized utilizing the porous medium. The utilization of nanofluids for cooling of microchannel heat sinks requires understanding of fundamentals of nanofluid convection in porous media.

**Keywords** Nanofluid, Mixed convection, Porous medium, Non-Newtonian fluid

**Paper type** Research paper

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_B$</td>
<td>Brownian diffusion coefficient</td>
</tr>
<tr>
<td>$D_T$</td>
<td>Thermophoretic diffusion coefficient</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Rescaled nano-particle volume fraction</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration vector</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Effective thermal conductivity of the porous medium</td>
</tr>
<tr>
<td>$K$</td>
<td>Permeability of porous medium</td>
</tr>
<tr>
<td>$Le$</td>
<td>Lewis number</td>
</tr>
<tr>
<td>$n$</td>
<td>Power-law index</td>
</tr>
<tr>
<td>$Nr$</td>
<td>Buoyancy Ratio</td>
</tr>
<tr>
<td>$Nb$</td>
<td>Brownian motion parameter</td>
</tr>
<tr>
<td>$Nt$</td>
<td>Thermophoresis parameter</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$q''$</td>
<td>Wall heat flux</td>
</tr>
<tr>
<td>$Ra_s$</td>
<td>Local Rayleigh number</td>
</tr>
<tr>
<td>$S$</td>
<td>Dimensionless stream function</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_W$</td>
<td>Wall temperature at vertical wedge</td>
</tr>
<tr>
<td>$T_{\infty}$</td>
<td>Ambient temperature attained as $y$ tends to infinity</td>
</tr>
<tr>
<td>$U$</td>
<td>Reference velocity</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Velocity components</td>
</tr>
<tr>
<td>$(x, y)$</td>
<td>Cartesian coordinates</td>
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</tbody>
</table>

### Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_m$</td>
<td>Thermal diffusivity of porous medium</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Dimensionless distance</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dimensionless temperature</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity of fluid</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Fluid density</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Nano-particle mass density</td>
</tr>
<tr>
<td>$(\rho c)_f$</td>
<td>Heat capacity of the fluid</td>
</tr>
<tr>
<td>$(\rho c)_m$</td>
<td>Effective heat capacity of porous medium</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Porosity</td>
</tr>
</tbody>
</table>

### Introduction

Several engineering applications require high heat transfer performance. Over the last several decades, scientists and engineers have tried to develop fluids, which provide better performances for a variety of thermal applications. Applying nanotechnology to heat transfer, the new concept of “nanofluid,” introduced by Choi (1995), has been proposed to meet the new heat transfer challenges. This new kind of fluid is manufactured by dispersing an amount of solid nanoparticles in traditional heat transfer fluids. Maxwell (1881) was the first to show the possibility of increasing thermal properties, particularly conductivity, of a liquid by including a volume fraction of solid particles. Several investigations revealed that the dispersion of a small amount of different kinds of nanoparticles (i.e. $\text{Al}_2\text{O}_3$, CuO, $\text{TiO}_2$) in water or ethylene glycol exhibit enhanced thermal conductivity, as reviewed in Wang and Mujumdar (2007), Murshed et al. (2008), Kakac et al. (2010). Different concepts have been proposed to explain this enhancement in thermal performance, which results to be higher with respect to that of classical mixtures. Li and Xuan (2000) and Xuan and Roetzel (2000) attributed the enhancement of heat transfer to the increased thermal dispersion...
resulting from the chaotic movements of nanoparticles, which accelerates the exchange of energy. Keblinski et al. (2002) proposed different mechanisms that contribute to the increase of nanofluids heat transfer, among which are Brownian motion of the particles and molecular level layering at the liquid/particle interface. Also Wang et al. (1999) explained the heat transfer enhancement with the interface layer between liquid and particles. Buongiorno (2006) developed a very in-depth analysis of all the possible mechanisms of fluid particles slip during convection of nanofluids, concluding that the abnormal increase of heat transfer coefficient in turbulent regime is due to the variation of thermophysical properties within the boundary layer, because of the effect of the temperature gradient and thermophoresis.

Porous media heat transfer problems have several engineering applications such as geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage and flow through filtering media. Cheng and Minkowycz (1977) presented similarity solutions for free convective heat transfer from a vertical plate in a fluid-saturated porous medium. Gorla and co-workers (Gorla and Tornabene, 1988; Gorla and Zinoledini, 1987) solved the non-similar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. The problem of combined convection from vertical plates in porous media was studied by Minkowycz et al. (1985) and Ranganathan and Viskanta (1984). All these studies were concerned with Newtonian fluid flows. The boundary layer flows in nano fluids have been analyzed recently by Nield and Kuznetsov (2009a,b). A clear picture about the nanofluid boundary layer flows is still to emerge.

Nanofluids in porous media constitute an emerging topic. Porous foam and microchannel heat sinks used for electronic cooling are optimized utilizing the porous medium. The utilization of nanofluids for cooling of microchannel heat sinks requires understanding of fundamentals of nanofluid convection in porous media.

The present work has been undertaken in order to analyze the mixed convection past a non-isothermal wedge in a porous medium saturated by a power law type non-Newtonian nanofluid. The effects of Brownian motion and thermophoresis are included for the nanofluid. Numerical solutions of the boundary layer equations are obtained and discussion is provided for several values of the nanofluid parameters governing the problem.

**Analysis**

We consider the steady mixed convection boundary layer flow past a vertical wedge placed in a nanofluid saturated porous medium. The co-ordinate system is selected such that x-axis is aligned vertically upwards. We consider the two-dimensional problem. We consider a vertical plate at \( y = 0 \). At this boundary the temperature \( T \) and the nano-particle fraction \( \phi \) take constant values \( T_w \) and \( \phi_w \), respectively. The ambient values, attend as \( y \) tends to infinity, of \( T \) and \( \phi \) are denoted by \( T \) and \( \phi \), respectively.

The Oberbeck-Boussinesq approximation is employed and the homogeneity and local thermal equilibrium in the porous medium are assumed. We consider the porous medium whose porosity is denoted by \( \varepsilon \) and permeability by \( K \).

Here \( \rho_f, \mu \) and \( \beta \) are the density, viscosity and volumetric volume expansion coefficient of the fluid; \( \rho_p \) the density of the particles; \( g \) the gravitational acceleration; \( (\rho c)_m \) the effective heat capacity and \( k_m \) effective thermal conductivity of the porous medium and \( D_B \) the Brownian diffusion coefficient and \( D_T \) the thermophoretic diffusion coefficient.
We now make the standard boundary layer approximation based on a scale analysis and write the governing equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]

\[
\frac{\partial u^*}{\partial y} = \frac{(1 - \phi_\infty)\rho_\infty \beta g_s K \frac{\partial T}{\partial y} - (\rho_p - \rho_\infty)g_s K \frac{\partial \phi}{\partial y}}{\mu}, \quad (2)
\]

\[
u \frac{\partial^2 T}{\partial x^2} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right], \quad (3)
\]

\[
\frac{1}{\varepsilon} \left( \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = D_B \frac{\partial^2 \phi}{\partial y^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2}, \quad (4)
\]

where:

\[
\alpha_m = \frac{k_m}{(\rho c)_m}, \quad \tau = \frac{\varepsilon (\rho c)_p}{(\rho c)_m}, \quad (5)
\]

where, \(\rho, \mu, \) and \(\beta\) are the density, viscosity, and volumetric volume expansion coefficient of the fluid, while \(\rho_p\) is the density of the particles. The gravitational acceleration is denoted by \(g\). We have introduced the effective heat capacity \((\rho c)_m\) and effective thermal conductivity, \(k_m\), of the porous medium. The coefficients that appear in Equations (3) and (4) are, respectively, the Brownian diffusion coefficient, \(D_B\), and the thermophoretic diffusion coefficient, \(D_T\).

The boundary conditions are taken to be:

\[
v = 0, \quad T = T_w, \quad \phi = \phi_w, \quad \text{at} \quad y = 0, \quad (6)
\]

\[
u \to u_\infty, \quad T \to T_\infty, \quad \phi \to \phi_\infty, \quad \text{at} \quad y \to \infty \quad (7)
\]

We introduce a stream line function \(\psi\) defined by:

\[
\frac{\partial u}{\partial y} = \frac{\partial \psi}{\partial x}, \quad \frac{\partial v}{\partial x} = - \frac{\partial \psi}{\partial y}, \quad (8)
\]

so that Equation (1) is satisfied identically. We are then left with the following three equations:

\[
n \left( \frac{\partial \psi}{\partial y} \right)^{n-1} \frac{\partial^2 \psi}{\partial y^2} = \frac{(1 - \phi_\infty)\rho_\infty \beta g_s K}{\mu} \frac{\partial T}{\partial y} - \frac{(\rho_p - \rho_\infty)g_s K}{\mu} \frac{\partial \phi}{\partial y}, \quad (9)
\]
\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[ \frac{D_B}{\partial y} \frac{\partial T}{\partial y} + \left( \frac{D_T}{T_{\infty}} \right) \left( \frac{D_T}{\partial y} \right)^2 \right],
\]

where \( T \) represents the temperature. The term \( T_{\infty} \) in the denominator is the reference temperature.

\[
\frac{1}{\varepsilon} \left( \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} \right) = \frac{D_B}{\partial y} \frac{\partial^2 \phi}{\partial y^2} + \left( \frac{D_T}{T_{\infty}} \right) \frac{\partial^2 T}{\partial y^2}.
\]

Proceeding with the analysis, we introduce the following dimensionless variables:

\[
\eta = \frac{y}{x} R a_{x}^{1/2}, \quad \zeta = \left( \frac{P e_x}{R a_x} \right)^n, \quad P e_x = \frac{u_\infty x}{x_m}, \quad R a_x = \frac{(1 - \phi_\infty) \rho_{\infty} \beta g_x K x(T_w - T_{\infty})}{\mu x_m},
\]

\[
S = \frac{\psi}{x_m R a_x^{1/2}}, \quad \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad f = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}.
\]

where \( u_\infty = c x^n \) and \( g_x = g \cos \phi \) represents the \( x \)-component of the acceleration due to gravity.

Substituting the expressions in Equation (12) into the governing Equations (9)-(11), we obtain the following transformed equations:

\[
n(S')^{n-1} S'' - \theta' + N_r f' = 0,
\]

\[
\theta'' + \frac{1}{2} S \theta' + N_b f' \theta' + N_t (\theta')^2 = m n \xi \left[ S' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial S}{\partial \xi} \right],
\]

\[
f'' + \frac{1}{2} L e S f' + \frac{N_t}{N_b} \theta' = L e m n \xi \left[ S' \frac{\partial f}{\partial \xi} - f' \frac{\partial S}{\partial \xi} \right],
\]

where the parameters are defined as:

\[
N_r = \frac{(\rho_p - \rho_{\infty})(\phi_w - \phi_\infty)}{\rho_{\infty} \beta (T_w - T_{\infty})(1 - \phi_\infty)}, \quad N_b = \frac{\varepsilon (\rho c)_p D_B (\phi_w - \phi_\infty)}{(\rho c)_t x_m},
\]

\[
N_t = \frac{\varepsilon (\rho c)_p D_T (T_w - T_{\infty})}{(\rho c)_t x_m T_{\infty}}, \quad L e = \frac{x_m}{\varepsilon D_B}.
\]

The transformed boundary conditions are:

\[
\eta = 0: \quad S = 0, \quad \theta = 0, \quad f = 1
\]

\[
\eta \to \infty: \quad S' = \xi^{1/n}, \quad \theta = 0, \quad f = 0
\]

It is noted that the \( \xi \) parameter here represents the forced flow effect on free convection. The case of \( \xi = 0 \) corresponds to pure free convection, and the limiting case of \( \xi = 1 \)
corresponds to pure forced convection. The above system of Equations (13)-(15) was solved over the region covered by $\xi = 0$ to provide the other half of the solution for the entire mixed convection regime. Moreover, it may be remarked that the system of Equations (13)-(15) with the boundary conditions (17) reduces to the equations of combined convection along an isothermal wedge in a porous medium; when \((N_r = N_b = N_t = 0)\), this case has been studied by Gorla and Kumari (1999).

The local friction factor is given by:

$$C_{f_x} = \frac{2\mu \left( \frac{\partial u(x, 0)}{\partial y} \right)^n}{\rho f u_{\infty}^2} = 2\Pr Ra_x^{1/2} Pe_x^{-2} \left( S''(\xi, 0) \right)^n$$  \hspace{1cm} (18)

The heat transfer rate is given by:

$$q_w = \left( -k_i \frac{\partial T}{\partial y} \right)_{y=0}$$  \hspace{1cm} (19)

The heat transfer coefficient is given by:

$$h = \frac{q_w}{(T_w - T_\infty)}$$  \hspace{1cm} (20)

Local Nusselt number is given by:

$$Nu_x = \frac{h_x}{k_i} = -Ra_x^{1/2} \frac{\partial}{\partial x} f'(\xi, 0)$$  \hspace{1cm} (21)

The mass transfer rate is given by:

$$N_w = \left( -D \frac{\partial \phi}{\partial y} \right)_{y=0} = h_m (\phi_w - \phi_\infty),$$  \hspace{1cm} (22)

where \(h_m\) = mass transfer coefficient:

$$M_w = -D(\phi_w - \phi_\infty) Ra_x^{1/2} \frac{\partial}{\partial x} f'(\xi, 0)$$  \hspace{1cm} (23)

and Sherwood number is given by:

$$Sh = \frac{h_m x}{D} = -Ra_x^{1/2} f'(\xi, 0).$$  \hspace{1cm} (24)

**Numerical method and validation**

Equations (13)-(15) represent an initial-value problem with $\xi$ playing the role of time. This general non-linear problem cannot be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. The implicit, tri-diagonal finite-difference method similar to that discussed by Blottner (1970) has
proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present study. All the first-order derivatives with respect to $\xi$ are replaced by two-point backward-difference formulae when marching in the positive $\xi$ direction. Then, all the second-order differential equations in $\eta$ are discretized using three-point central difference quotients. This discretization process produces a tri-diagonal set of algebraic equations at each line of constant which is readily solved by the well-known Thomas algorithm Blottner (1970). During the solution, iteration is employed to deal with the nonlinearity aspect of the governing differential equations. The problem is solved line by line starting with line $\xi = 0$ where similarity equations are solved to obtain the initial profiles of velocity, temperature and concentration, and marching forward (or backward) in $\xi$ until the desired line of constant $\xi$ is reached. Variable step sizes in the $\eta$ direction with $\Delta \eta_0 = 0.001$ and a growth factor $G = 1.035$ such that $\Delta \eta_n = G \Delta \eta_{n-1}$ and constant step sizes in the $\xi$ direction with $\Delta \xi = 0.01$ are employed. These step sizes are arrived at after many numerical experimentations performed to assess grid independence. The convergence criterion employed in this study is based on the difference between the current and the previous iterations. When this difference reached $10^{-5}$ for all the points in the $\eta$ directions, the solution was assumed to be converged, and the iteration process was terminated.

Results and discussion

Equations (9)-(11) were solved numerically to satisfy the boundary conditions (13) for parametric values of Le, Nr (buoyancy ratio number), Nb (Brownian motion parameter), Nt (thermophoresis parameter) and power law exponent $n$ using finite difference method.

In order to assess the accuracy of our results, we compared our results for friction factor, as well as heat and mass transfer rates with those of Gorla and Chamkha (2011) for a flat plate geometry ($m = 0$). This is shown in Table I. The agreement between the two is excellent and therefore, our results are highly accurate (Figure 1).

Figure 2 indicates that as the power law index $n$ increases, the velocity increases whereas the temperature and concentration decrease. Figure 3 shows that as $n$ increases, the friction factor decreases whereas the heat and mass transfer rates increase. Power law fluids are also known as the Ostwald-de Waele fluids describe the behavior of a real non-Newtonian fluid. For example, if $n$ were less than one, the power law predicts that the effective viscosity would decrease with increasing shear rate indefinitely, requiring a fluid with infinite viscosity at rest and zero viscosity as the shear rate approaches infinity, but a real fluid has both a minimum and a maximum effective viscosity that depend on the physical chemistry

<table>
<thead>
<tr>
<th>$n$</th>
<th>$S^n(0^n)$</th>
<th>$-\theta(0)$</th>
<th>$-f(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.423683E-01 (5.423684E-01)</td>
<td>2.226416E-01 (2.226418E-01)</td>
<td>1.032334 (1.032335)</td>
</tr>
<tr>
<td>0.7</td>
<td>4.875134E-01 (4.875135E-01)</td>
<td>2.621866E-01 (2.621869E-01)</td>
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<td>1.0</td>
<td>3.742727E-01 (3.742729E-01)</td>
<td>2.980915E-01 (2.980915E-01)</td>
<td>1.344727 (1.344729)</td>
</tr>
<tr>
<td>1.2</td>
<td>2.970928E-01 (2.970928E-01)</td>
<td>3.146485E-01 (3.146487E-01)</td>
<td>1.407003 (1.407001)</td>
</tr>
<tr>
<td>1.5</td>
<td>1.978428E-01 (1.978430E-01)</td>
<td>3.318324E-01 (3.318324E-01)</td>
<td>1.472220 (1.472218)</td>
</tr>
</tbody>
</table>

Table I.

Effects of $n$ on $S^n(0^n)$, $-\theta(0)$ and $-f(0)$ for $N_b = 0.3$, $N_w = 0.5$, $N_t = 0.1$ and $Le = 10$
at the molecular level. Therefore, the power law is only a good description of fluid behavior across the range of shear rates to which the coefficients were fitted. Power-law fluids can be subdivided into three different types of fluids based on the value of their flow behavior index \( n \): less than 1, pseudoplastic; \( n = 1 \), Newtonian; and \( n > 1 \), dilatant. Pseudoplastic, or shear-thinning fluids have a lower apparent viscosity at higher shear rates, and are usually solutions of large, polymeric molecules in a solvent with smaller molecules. It is generally supposed that the large molecular chains tumble at random and affect large volumes of fluid under low shear, but that they gradually align themselves in the direction of increasing shear and produce less resistance. Dilatant, or shear-thickening fluids increase in apparent viscosity at higher shear rates. One common example is an uncooked paste of cornstarch and water. Under high shear the water is squeezed out from between the starch molecules, which are able to interact more strongly.

Figure 4 shows the variation of velocity, temperature and concentration distributions within the boundary layer as \( \text{Nt} \) varies. Thermophoresis parameter, \( \text{Nt} \)
Natural convection dominated regime

Figure 3.
Friction factor, heat transfer rate and mass transfer rate
(n as parameter)

Figure 4.
Velocity, temperature and concentration distributions
(Nt as parameter)
appears in the thermal and concentration boundary layer equations. As we note, it is coupled with temperature function and plays a strong role in determining the diffusion of heat and nanoparticle concentration in the boundary layer. From Figure 4, we note that the velocity, temperature and nanoparticle concentration are elevated as \( N_t \) increases. Figure 5 shows that the friction factor, heat transfer rate and mass transfer rates decrease with \( N_t \).

\( Nb \) is the Brownian motion parameter. Brownian motion decelerates the flow in the nanofluid boundary layer. Brownian diffusion promotes heat conduction. The nanoparticles increase the surface area for heat transfer. Nanofluid is a two phase fluid where the nanoparticles move randomly and increase the energy exchange rates. Brownian motion reduces nanoparticle diffusion. From Figure 6, we note that velocity and concentration increase with increasing values of \( Nb \). The temperature profiles are decreasing functions of Brownian motion number \( Nb \). Figure 7 shows that the friction factor and mass transfer rates increase with \( Nb \) whereas the heat transfer rates decrease with \( Nb \).

Figure 8 shows that as \( Nr \) increases, the velocity decreases and the temperature and concentration increase. The parameter \( Nr \) appears only in the momentum boundary layer equation. Buoyancy is principally a macroscale effect. The buoyancy influences the velocity and temperature fields, however, has a minor effect on nanoparticle diffusion. This explains the minor influence of buoyancy on concentration profiles. Figure 9 shows that the friction factor increases with \( Nr \) whereas the heat and mass transfer rates decrease.

**Figure 5.** Friction factor, heat transfer rate and mass transfer rate \((N_t\) as parameter)
Figure 10 shows that as the wedge flow parameter \( m \) increases, the velocity, temperature and concentration within the boundary layer decrease. Figure 11 shows that the friction factor and heat and mass transfer rates increase with \( m \).

Figure 12 illustrates the variation of velocity, temperature and concentration within the boundary layer as \( \text{Le} \) increases. The velocity increases as \( \text{Le} \) increases.
We also observe that as Le increases, the temperature and concentration within the boundary layer decrease and the thermal and concentration boundary layer thicknesses decrease. Figure 13 indicates that as Le increases, the heat transfer rate decreases whereas the mass transfer rate increases.
Concluding remarks
In this paper, we presented a boundary layer analysis for the mixed convection past a vertical wedge in a porous medium saturated with a non-Newtonian nano fluid. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the buoyancy ratio parameter $N_r$, Brownian motion parameter $N_b$, thermophoresis parameter $N_t$ and Lewis number $Le$.

**Figure 10.** Velocity, temperature and concentration distributions ($m$ as parameter)

**Figure 11.** Friction factor, heat transfer rate and mass transfer rate ($m$ as parameter)
The results indicate that as Nr and Nt increase, the friction factor increases whereas the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decrease. As Nb increases, the friction factor and surface mass transfer rates increase whereas the surface heat transfer rate decreases. As Le increases, mass transfer rates increase. As the power law exponent n increases, the heat and mass transfer rates increase.
_AIAA J_, Vol. 8 No. 2, pp. 193-205.

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