Numerical Modeling of Natural Convection of a Nanofluid Between Two Enclosures

M. A. Mansour¹, M. A. Y. Bakeir¹, and A. J. Chamkha²*

¹Department of Mathematics, Faculty of Sciences, Assiut University, Assiut, 71515, Egypt
²Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuwaikh, 70654, Kuwait

Natural convection fluid flow and heat transfer between two enclosures filled with a water-based nanofluid (mainly Cu-water nanofluid) has been investigated numerically using the finite difference method. A parametric study is conducted and the effects of pertinent parameters such as the Rayleigh number, the aspect ratio of the two intertwined enclosures, and the volume fraction of nanoparticles on the flow and temperature fields and the rate of heat transfer inside the enclosure are investigated. It is found from the obtained results that the mean Nusselt number increases with the increase in the Rayleigh number and the volume fraction of nanoparticles regardless of the aspect ratio of the enclosure. Moreover, the obtained results show that the rate of heat transfer increases with decreasing values of the aspect ratio of the cavity. Also, it is found that the rate of heat transfer increases with the increase in the nanoparticles volume fraction. In addition, at low Rayleigh numbers, the effect of nanoparticles on the enhancement of heat transfer for narrow enclosures is more than that for wide enclosures.

KEYWORDS: Natural Convection, Nanofluid, Two Intertwined Enclosures, Nusselt Number, Rayleigh Number.

1. INTRODUCTION

The significantly enhanced transport properties of the nanofluids, which is a mixture of nano-sized particles (nanoparticles) suspended in a base fluid, have enormous implications in industrial processes, such as the cooling of very small electronic components, which will comprise the next generation of computer chips, absorption of gases by liquid carriers, chemical reactions, combustion for electricity generation, cooling of IC engines, directed-energy weapons, boiling under microgravity conditions, nuclear reactor cooling, and biomedicine. Many researchers have reported that a nanofluid enhances the rate of heat transfer through its higher thermal conductivity compared to the base fluid. Mahmoodi and Hashemi¹ investigated numerically free convection of Cu-water nanofluid in L-shaped and C-shaped cavities. They found that the effect of the presence of nanoparticles on heat transfer enhancement is more apparent for narrow L-shaped and C-shaped cavities. Mansour et al.² showed that the rate of heat transfer increased with decreasing aspect ratio of the cavity and nanoparticles volume fraction inside T-shaped enclosures filled with Cu-Water nanofluid. Ignacio et al.³ showed that the enhancement of thermal conductivity of water reached 50% after dispersing nanosilica powder at different volume concentrations measured at temperatures ranging 30-70 °C. The enhancement of thermo-physical properties of nanosilica in fluid dispersion suggests the potential use of the nanofluids as a heat transfer fluid.

However, the continuing miniaturization of electronic devices requires further heat transfer improvements from an energy saving viewpoint. In recent years, thermo-physical properties of nanofluids have been investigated by many researchers.⁴-⁸ Investigation of different applications of nanofluids can be found in the literature such as forced convection of nanofluids (Namburu et al.⁹, Santra et al.¹⁰ and Strandberg and Daa¹¹), boiling heat transfer of nanofluids (Das et al.¹²), mixed convection of nanofluids (Akbarinia and Behzadmehr.¹³ Akbari et al.¹⁴ Ghaffari et al.¹⁵ Mahmoodi,¹⁶ Arefianesh and Mahmoodi¹⁷ and Abu-Nada and Chamkha¹⁸), natural convection of nanofluids (Abu-Nada and Chamkha,¹⁹ Parvin et al.²⁰ Basak and Chamkha²¹ and Ben Cheikh et al.²²) and conjugate heat transfer (Chamkha and Ismael²³).

There are a number of recent studies on free convection inside cavities containing nanofluid. Khanfera et al.²⁴ investigated numerically the problem of free convection of nanofluid in rectangular cavities with cold right wall, hot left wall and insulated horizontal walls. They found that the rate of heat transfer increased with the increase...
in nanoparticles volume fraction. Since the free convection heat transfer of nanofluids in the thermal engineering applications will be extending the existing knowledge, Ghalambaz et al.\textsuperscript{25} studied the problem of natural convective flow over a vertical plate embedded in a porous medium saturated with a nanofluid and reported numerical results for the friction factor, surface heat transfer rate and mass transfer rate were presented for various parametric conditions. On other hand, Tongkrako et al.\textsuperscript{26} deduced that volume fraction, effective viscosity, and effective thermal conductivity can enhance the heat transfer performance in nanofluid flows not only with the single-phase model considered but also with the mixing models examined. Pat et al.\textsuperscript{27} predicted that the skin-friction coefficient increases for both stretching/shrinking sheet with increase in volume fraction of the nanoparticles. Malavgri and Ganji\textsuperscript{28} reported that nanofluids can transfer heat more efficiently in a slip condition than in a no-slip condition.

Motivated by the work referenced above, the present study considers a numerical solution for natural convection flow and heat transfer of a water-based nanofluid (mainly Cu-water nanofluid) between two interwoven enclosures. The most convenient application of O-shaped enclosures can be cooling of some electronic parts in the manufactures. The governing equations in terms of the primitive variables in Cartesian coordinate system are discretized using the finite difference method. The effects of the aspect ratio of enclosure, nanoparticles volume fraction and the Rayleigh number on the flow and temperature fields and the heat transfer characteristics are investigated and discussed.

2. MATHEMATICAL MODELING

Consider two-dimensional natural convection fluid flow and heat transfer of various water-based nanofluids (mainly Cu-water nanofluid) between two interwoven square enclosures. Figure 1 shows the schematic illustration of the problem under consideration. The inner enclosure is kept isothermally at a higher temperature \(T_H\). The outer enclosure boundaries are kept adiabatic except a part of the bottom wall which is heated at a constant temperature \(T_H\) and a part of the top wall where it is cooled at constant temperature \(T_c\). The length of the heated part (called heat source) and cooled part (called cold rib) is \(B\). The width of the inner enclosure is \(W\) while the width of the outer enclosure is \(H\). A water-based nanofluid occupies the region separating the boundaries of both enclosures. The nanofluid is assumed incompressible. The governing equations based on these assumptions together with adopting the Boussinesq approximation and known nanofluid model\textsuperscript{1} can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

(1)

(2)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\alpha_{nf}}{\rho_{nf}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(3)

(4)

where all parameters are defined in the Nomenclature section.

Numerous formulations for the thermo-physical properties of nanofluids are proposed in the literature. In the present study, we are adopting the relations which depend on the nanoparticles volume fraction only and which were proven and used in many previous studies as follows:

The nanofluid density, heat capacity and the product of density with thermal expansion coefficient (see Mahmoodi and Hashemi\textsuperscript{1}) are given by

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s
\]

(5)

\[
(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s
\]

(6)

\[
(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s
\]

(7)

The thermal diffusivity and thermal expansion coefficient of the nanofluid (see Mahmoodi and Hashemi\textsuperscript{1}) are defined as

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}
\]

(8)

\[
\beta_{nf} = \frac{(1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s}{(1 - \phi) \rho_f + \phi \rho_s}
\]

(9)

The nanofluid viscosity based on the Brinkman model (see Mahmoodi and Hashemi\textsuperscript{1}) is given by

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^3}
\]

(10)
The nanofluid thermal conductivity, based on Maxwell-Garnetts model (see Mahmodi and Hashemi)\(^1\) is given by

\[
\frac{k_{nf}}{k_f} = \frac{(k_1 + 2k_f) - 2\phi(k_f - k_1)}{(k_1 + 2k_f) + \phi(k_f - k_1)}
\]  

where \(k_{nf}\) is the nanofluid thermal conductivity, \(k_f\) is the fluid thermal conductivity, \(k_1\) is the base fluid thermal conductivity, \(\phi\) is the volume fraction, and \(\alpha_f\) is the thermal diffusivity of the fluid.

Introducing the following dimensionless set:

\[
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha_f}, \quad V = \frac{vH}{\alpha_f}
\]

\[
P = \frac{\rho H^2}{\alpha_f}, \quad \theta = \frac{T - T_c}{T_H - T_c}
\]

into Eqs. (1)–(4) yields the following dimensionless equations:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho \alpha_f} \frac{\partial \rho}{\partial X} + \frac{\mu_{nf}}{\rho \alpha_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

\[
\frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{\rho \alpha_f} \frac{\partial \rho}{\partial Y} + \frac{\mu_{nf}}{\rho \alpha_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{\rho \beta_{nf} \alpha_f}{\rho \alpha_f^2} \right) \rho \alpha_f
\]

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_f}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

where

\[
Ra = \frac{g \beta_f (T_a - T_c) H^3}{\alpha_f \alpha_f}, \quad Pr = \frac{\nu f}{\alpha_f}
\]

are the Rayleigh number and the Prandtl number, respectively.

The proper dimensionless boundary conditions for the problem are given by:

\[
\begin{cases}
\text{on walls:} & \begin{cases} 
U = V = 0, \quad \theta = 1; \\
D - 0.5B \leq X \leq D + 0.5B & D - 0.5B \leq X \leq D + 0.5B \\
U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0; \quad \text{Otherwise}
\end{cases} \\
\text{on walls bc and da:} & U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \\
\text{on walls ef, fg, gh and he:} & U = V = 0, \quad \theta = 1
\end{cases}
\]

The local Nusselt number along the heated side can be written as:

\[
Nu_{local} = \frac{hH}{k_f}
\]

where \(h\) is the heat transfer coefficient is given by

\[
h = \frac{q_w}{T_H - T_C}
\]

The thermal conductivity at the different enclosure walls can be written as:

\[
k_{nf} = \begin{cases} 
\frac{-q_w}{\alpha_f \partial T/\partial Y} & \text{on wall ef and gh} \\
\frac{-q_w}{\alpha_f \partial T/\partial Y} & \text{on wall ab and} \\
\frac{-q_w}{\alpha_f \partial T/\partial X} & \text{on wall bc and da}
\end{cases}
\]

The local Nusselt number at the different enclosure walls are given by

\[
Nu_i = \begin{cases} 
\left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial Y} & \text{on wall ef and gh} \\
\left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial Y} & \text{on wall ab s.t.} \\
\left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial X} & \text{on wall bc and da}
\end{cases}
\]

The average Nusselt numbers of interest can be calculated from:

\[
\begin{align*}
Nu_{m1} &= \frac{1}{4} \left( \int_{X_0}^{X_f} Nu_i \, dX + \int_{Y_0}^{Y_f} Nu_i \, dY \right) \\
&\quad + \int_{X_0}^{X_f} Nu_i \, dY + \int_{Y_0}^{Y_f} Nu_i \, dX \\
Nu_{m2} &= \int_{D - 0.5B}^{D + 0.5B} Nu_i \, dX \bigg|_{X = 0}\bigg|_{X = 0} \\
Nu_m &= \frac{1}{2} (Nu_{m1} + Nu_{m2})
\end{align*}
\]

### 3. NUMERICAL METHOD

In this investigation, the finite difference method (Mansour et al.\(^{29}\)) was employed to solve the governing equations with the boundary conditions. Central difference quotients were used to approximate the second derivatives in both the \(X\) and \(Y\) directions. Then, the obtained discretized equations are solved using a Gauss-Seidel iteration technique (Grosan et al.\(^{30}\)). The solution procedure is iterated until the following convergence criterion is satisfied:

\[
\sum_{i,j} |X_{i,j}^{m+1} - X_{i,j}^{m}| \leq 10^{-7}
\]

where \(\chi\) is the general dependent variable. The numerical method was implemented in FORTRAN software.

The finite difference method uses fine sets of grids: \(26 \times 26, 51 \times 51, 76 \times 76, 101 \times 101, 121 \times 121\). There is a good
agreement was found between \(76 \times 76\) and \((121 \times 121)\) grids, so the numerical computations were carried out for \(76 \times 76\) and \((121 \times 121)\) grid nodal points.

In order to verify the accuracy of the present method, the obtained results under special cases are compared with the results obtained by Grosan et al.\(^{39}\) and Haajizadeh et al.\(^{31}\) Table II shows that a good agreement exists between the present results and the results reported by previous investigators. These favorable comparisons lend confidence in the numerical results to be reported subsequently.

### 4. RESULTS AND DISCUSSION

The numerical results for the problem under consideration are presented in terms of the streamlines, isotherms, local Nusselt number and the average Nusselt number in Figures 2–21 and Tables III and IV.

Figure 2 illustrates effect of increase in the aspect ratio AR on the flow pattern and the temperature distribution inside the enclosures filled with a pure fluid (\(\phi = 0\)) at \(Ra = 10^5\). As can be seen from the streamlines in the figure, the fluid is heated by the heated source and expands as it moves upward. Then the fluid is cooled by the cold ribs and compressed as it moves near the cooled ribs. Hence, there is a symmetrical behavior for both of the streamlines and the isotherms. It is clear that the left part and the right part are congruent. Also, the top and the bottom parts, so there are 6 sets of concentric shaped circulating eddies or cells. It is seen that the heat transfer increases as AR decreases. From the observed result, the maximum and minimum limits are found for developing the Rayleigh-Bénard cells in the gap between the heated source and the cold rib.

Figure 3 shows the streamlines and isotherms between the two enclosures filled with a pure fluid at \(AR = 0.4, D = 0.5, Ra = 10^5\), and for various lengths of the heat source B. It is shown that the heat transfer increases as the heat source length B increases.

Figure 4 shows the streamlines and the isotherms between the two enclosures filled with a pure fluid (\(\phi = 0.0\)) at \(Ra = 10^5, AR = 0.4, B = 1/3\) and for different heat source locations D. The noticeable change is on

**Table I.** Thermo-physical properties of water and nanoparticles.

<table>
<thead>
<tr>
<th></th>
<th>Pure water</th>
<th>Copper (Cu)</th>
<th>Silver (Ag)</th>
<th>Alumina Al₂O₃</th>
<th>Titanium oxide (TiO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho) (kg/m³)</td>
<td>(997.1)</td>
<td>(893.3)</td>
<td>(10500)</td>
<td>(3970)</td>
<td>(4250)</td>
</tr>
<tr>
<td>(C_p) (J/kg K⁻¹)</td>
<td>(4179)</td>
<td>(385)</td>
<td>(235)</td>
<td>(765)</td>
<td>(688.2)</td>
</tr>
<tr>
<td>(k) (W/m K⁻¹)</td>
<td>(0.613)</td>
<td>(401)</td>
<td>(429)</td>
<td>(40)</td>
<td>(8.9538)</td>
</tr>
<tr>
<td>(\beta) (K⁻¹)</td>
<td>(21 \times 10^{-3})</td>
<td>(1.67 \times 10^{-3})</td>
<td>(1.89 \times 10^{-3})</td>
<td>(0.85 \times 10^{-3})</td>
<td>(0.9 \times 10^{-3})</td>
</tr>
</tbody>
</table>

**Table II.** Comparison of \(\psi_{\text{max}}\)

<table>
<thead>
<tr>
<th>(Ra)</th>
<th>Haajizadeh et al.(^{39})</th>
<th>Grosan et al.(^{39})</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.078</td>
<td>0.079</td>
<td>0.0799</td>
</tr>
<tr>
<td>(10^5)</td>
<td>4.880</td>
<td>4.833</td>
<td>4.8266</td>
</tr>
</tbody>
</table>

**Fig. 2.** Streamlines (top) and isotherms (down) between two enclosure filled with pure fluid (\(\phi = 0.0\)) at \(Ra = 10^5\), \(B = 1/3\), \(D = 0.5\). (a) \(AR = 0.2\), (b) \(AR = 0.4\), (c) \(AR = 0.6\), (d) \(AR = 0.8\).
both the top and bottom parts. It is clear that there is no symmetric behavior as the heat source moves to the middle and that the heat transfer increases near the heat source or the heat sink.

Figure 5 shows the streamlines and the isotherms between the two enclosures filled with a pure fluid ($\phi = 0.0$) at $D = 0.5$, $AR = 0.4$, $B = 1/3$ and for different values of the Rayleigh number $Ra$. As for the streamlines, they show the fluid is heated by the heat source and expands as it moves upward. For the isotherms, the contour lines tend to be more compressed as $Ra$ increases, so the heat transfer for higher values of $Ra$ becomes irregular and may cause a loss of the symmetrical behavior.

Figure 6 shows the streamlines and the isotherms between the two intertwined enclosures filled with a Cu-water nanofluid ($\phi = 0.1$) at $Ra = 10^5$, $B = 1/3$, $D = 0.5$ for different aspect ratios of inner enclosure $AR$.

It can be noticed that a great difference exists in the

![Fig. 3. Streamlines (top) and isotherms (down) between two enclosures filled with pure fluid ($\phi = 0.0$) at $Ra = 10^5$, $AR = 0.4$, $D = 0.5$, (a) $B = 0.2$, (b) $B = 0.4$, (c) $B = 0.6$, (d) $B = 0.8$.](image)

![Fig. 4. Streamlines (top) and isotherms (down) between two enclosures filled with pure fluid ($\phi = 0.0$) at $AR = 0.4$, $Ra = 10^5$, $B = 1/3$, (a) $D = 0.2$, (b) $D = 0.3$, (c) $D = 0.4$, (d) $D = 0.5$.](image)
contour lines for small values of the aspect ratio AR (such as $AR = 0.2$) because of the nanoparticles existence, where the streamlines and the isotherms are scattered in a fine way. In spite of that, there is no great difference between the pure and “nano” cases.

Figure 7 shows the streamlines and the isotherms between the two intertwined enclosures filled with a Cu-water nanofluid ($\phi = 0.1$) at $Ra = 10^3$, $AR = 0.4$ and for different heat source lengths $B$. It is clearly seen that the heat transfer increases as $B$ increases similar to the pure fluid case.

Figure 8 shows the streamlines and the isotherms between the two intertwined enclosures filled with a Cu-water nanofluid at $Ra = 10^3$, $AR = 0.4$, $D = 0.5$, $B = 1/3$ and for various values of solid volume fraction $\phi$. The effect of the solid volume fraction $\phi$ on the streamlines and the isotherms is seen to be relatively weak as is clear from the contours.

Fig. 5. Streamlines (top) and isotherms (down) between two enclosures filled with pure fluid ($\phi = 0.0$) at $AR = 0.4$, $D = 0.5$, $B = 1/3$. (a) $Ra = 10^3$, (b) $Ra = 10^4$, (c) $Ra = 10^5$, (d) $Ra = 10^6$.

Fig. 6. Streamlines (top) and isotherms (down) between two enclosures filled with nanofluid ($\phi = 0.1$) at $Ra = 10^3$, $B = 1/3$, $D = 0.5$. (a) $AR = 0.2$, (b) $AR = 0.4$, (c) $AR = 0.6$, (d) $AR = 0.8$. 

Mansour et al.
Numerical Modeling of Natural Convection of a Nanofluid Between Two Enclosures
Figure 7 shows the streamlines and the isotherms between the two intertwined enclosures filled with a water-based nanofluid at $Ra = 10^5$, $AR = 0.4$, $D = 0.5$, $B = 1/3$ and for various types of nanoparticles (Cu, Ag, Al₂O₃ and TiO₂). As seen from contours, the various nanofluids do not have great difference from each other.

Figure 10(i) presents the profiles of the local Nusselt number $Nu_{X}$ along the gh wall for different values of the aspect ratio $AR$ and a Cu-water nanofluid. It is clear that each curve is symmetric around the middle, where the heat transfer reaches its minimum value. It is also seen that the local Nusselt number decreases as the aspect ratio $AR$ increases. Figure 10(ii) presents the profiles of the local Nusselt number $Nu_{X}$ along the gh wall for different values of the aspect ratio $AR$ and a Cu-water nanofluid. Also, it is clear that each curve is symmetric around the middle. The heat transfer reaches its minimum value in the middle for $AR$ less than or equal 0.4, but for the other $AR$ values,
**Fig. 9.** Streamlines (top) and isotherms (down) between two enclosures filled with nanofluid at \( Ra = 10^5 \), \( AR = 0.4 \), \( D = 0.5 \), \( B = 1/3 \). \( \phi = 0.1 \).
(a) Cu-water, (b) Ag-water, (c) \( Al_{2}O_{3} \)-water, (d) \( Ti_{2}O_{3} \)-water.

**Fig. 10.** Profile of local Nusselt number along (i) along the ef wall, (ii) along the gh wall, (iii) the heat source, (iv) the eh (or fg) wall at various aspect ratios (Cu-water, \( D = 0.5 \), \( Ra = 10^5 \), \( B = 1/3 \) and \( \phi = 0.1 \)).
the local Nusselt number has a little increment in the middle because the heat wall, gh, is close to the heat sink. In addition, the local Nusselt number is observed to decrease as the aspect ratio AR increases. Figure 10(iii) presents the profiles of the local Nusselt number \( N_u - X \) along the heat source for different values of the aspect ratio AR and a Cu-water nanofluid. Again, it is clear that each curve is symmetric around the middle where the heat transfer reaches its minimum value. The local Nusselt number decreases as the aspect ratios increase. Also, the difference between the maximum point and the minimum point decreases as the aspect ratio increases so that at \( AR = 0.8 \), the change in heat transfer tends to zero. Figure 10(iv) presents the profiles of the local Nusselt number \( N_u - X \) along the eh (or fg) wall for different values of the aspect ratio AR and a Cu-water nanofluid. Similar to the other walls, it is clear that each curve is symmetric around the middle where the heat transfer reaches its minimum value. From this and the previous figures, it can be concluded that, in general, the local Nusselt number decreases as the aspect ratio AR increases.

Figure 11 (left) shows the variation of the local Nusselt number \( N_u - X \) along the bottom heat source for different values of the nanoparticles or solid volume fraction \( \phi \) and a Cu-water nanofluid. It is clear that each curve is symmetric around the middle where the heat transfer reaches its minimum value. It also observed that the local Nusselt number decreases as the solid volume fraction \( \phi \) increases. Figure 11 (right) presents the profiles of the local Nusselt number \( N_u - X \) along the heat source for different values of the heat source length B and a Cu-water nanofluid. It is clear that the effect of the heat source length on the local Nusselt number is similar to that of the aspect ratio. That is, the local Nusselt number decreases as heat source length B increases.

Figure 12(i) presents the effect of the heat source length B on the relation between the average Nusselt number \( N_u \) and the solid volume fraction \( \phi \) for a Cu-water nanofluid. It is shown clearly that the average Nusselt number increases as B increases but it decreases gradually as the solid volume fraction \( \phi \) increases. Figure 12(ii) presents the effect of the aspect ratio AR on the relation between the average Nusselt number \( N_u \) and the solid volume fraction \( \phi \) for a Cu-water nanofluid. It is shown that the average Nusselt number increases as AR decreases. Also, the average Nusselt number decreases gradually for \( AR < 0.8 \) as the solid volume fraction increases while for \( AR = 0.8 \), the average Nusselt number tends to be constant. Figure 12(iii) shows the profile of average Nusselt number versus solid volume fraction \( N_u - \phi \) for various values of the Rayleigh number and a Cu-water nanofluid. It is predicted that the heat transfer increases as the Rayleigh number increases for all solid volume fraction levels. It is also noticed that for \( Ra < 10^4 \), there is no great change in the average Nusselt number with the solid volume fraction.

Figure 13(i) shows the variations of the average Nusselt number \( N_u \) versus the heat source location D for various aspect ratios when \( B = 1/3 \), \( Ra = 10^4 \) and \( \phi = 0.1 \) and a Cu-water nanofluid. The average Nusselt number is shown to decrease as AR increases. Also, there exists a small change in \( N_u \) as the heat source moves to the center. Figure 13(ii) presents the effects of the length of heat source B on the average Nusselt number \( N_u \) for various values of the aspect ratio AR. As stated before, the average Nusselt number decreases as AR increases. This occurs for all values of B. Also, it is predicted that the average Nusselt number increases as the length of the source B increases. Figure 13(iii) shows the profile of the average Nusselt number \( N_u \) with the solid volume fraction \( \phi \) for different types of nanofluids. The profiles are obtained for all water-based nanofluids (Cu, Ag, Al\(_2\)O\(_3\) and TiO\(_2\)) with the lowest Nusselt number for the middle of the heat source. Table I shows that TiO\(_2\) has the lowest value of thermal conductivity compared to other nanoparticles, hence, it has the highest absolute value of the average

![Fig. 11](image-url). Profile of local Nusselt number along the heat source for various solid volume fraction (left) and for various heat source lengths (right) (Cu-Water, \( D = 0.5 \), \( Ra = 10^4 \), \( B = 1/3 \) and \( AR = 0.4 \)).
Nusselt number, as verified from Eq. (22), Cu and Ag, on the other hand, have the lowest values. In addition, the thermal conductivity of Al₂O₃ is approximately one tenth of those for Cu and Ag (Table I), thus, the average Nusselt number for Al₂O₃ is greater than those for Cu and Ag. As shown in Figure 12, the heat transfer decreases linearly as the solid volume fraction increases for Cu and Ag but it increases in a nonlinear way for the others.

Comparisons of the average Nusselt number for different values of the aspect ratio changes with different nanoparticles. For all types of nanoparticles considered, the average Nusselt number Nuₚ decreases as AR increases and the values for Cu and Ag are less than the others. The global maximum value for the average Nusselt number is for TiO₂ and the global minimum is for Ag. This can be observed from Table III. For the maximum values of the stream function, for all considered nanoparticles, they also decrease as AR increases. It is clear that the stream function has the same behavior as that of Nuₚ. In Table IV, as the nanoparticle changes, Nuₚ has slight differences, then as Ra increases, the average increases and
Table III. Values for average Nusselt number, maximum heat source and maximum streamline for various aspect ratios.

<table>
<thead>
<tr>
<th>AR</th>
<th>$Nu_m$</th>
<th>$\psi_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>Cu</td>
<td>6.33968</td>
</tr>
<tr>
<td>Cu</td>
<td>Ag</td>
<td>6.12332</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>8.06216</td>
<td>20.47484</td>
</tr>
<tr>
<td>TiO$_2$</td>
<td>8.31180</td>
<td>21.01814</td>
</tr>
</tbody>
</table>

| 0.4 | Cu  | 4.55838 | 7.89033 |
| Cu  | Ag   | 4.32589 | 7.64721 |
| Al$_2$O$_3$ | 6.15272 | 14.58552 |
| TiO$_2$ | 6.36326 | 15.12800 |

| 0.6 | Cu  | 2.39257 | 4.12087 |
| Cu  | Ag   | 2.28851 | 3.76009 |
| Al$_2$O$_3$ | 3.71948 | 9.95825 |
| TiO$_2$ | 3.94565 | 10.52895 |

| 0.8 | Cu  | 1.49222 | 0.40601 |
| Cu  | Ag   | 1.48976 | 0.38015 |
| Al$_2$O$_3$ | 1.53783 | 1.03093 |
| TiO$_2$ | 1.55067 | 1.11337 |

the difference increases between the different nanoparticles. Its arrangement is such as: Ag is less than or equal to Cu and less than Al$_2$O$_3$ and less than TiO$_2$. Then, as AR increases, $Nu_m$ decreases for every type of nanoparticle. When the heat source length $B$ is equal to 0.8, the value of $Nu_m$ falls to its third value. The values of $\psi_{max}$ at $B = 0.2$, have an ascending order as Ag < Cu < Al$_2$O$_3$ < TiO$_2$. Then as Ra increases, $\psi_{max}$ jumps to high values, then as AR increases the maximum value decreases.

5. CONCLUSION

Natural convection in a partially heated two intertwined enclosures from the bottom and the top and filled with different types of water-based nanofluids was numerically investigated by the finite difference method. The effects of Rayleigh number, solid volume fraction, heat source length and location and the type of nanofluid on the enclosure cooling performance were studied. The increase of Rayleigh numbers strengthened the natural convection flows which resulted in the reduction of the heat source temperature. Also, the increase of the solid volume fraction of nanoparticles caused the heat source maximum temperature to decrease particularly at low Rayleigh numbers where conduction is the main heat transfer mechanism. The increase of the heat source length increased the heat transfer to the nanofluid and therefore, increased the surface temperature of the heat source and the strength of natural convection circulating cells within the enclosure. As the heat source moved from the left wall towards the middle of the bottom wall of the enclosure, at low Rayleigh numbers, the heat source maximum temperature continuously increased. At high Rayleigh numbers, the minimum rate of heat transfer occurred from the top wall and its minimum occurred from the bottom wall while at low Rayleigh numbers similar rates of heat transfer from top and bottom walls were observed. As the two intertwined square enclosures became narrower, the rate of heat transfer decreased. However, as the Rayleigh number increased, the rate of heat transfer increased for a constant value of the aspect ratio.

**NOMENCLATURE**

- $H$ Length of external square [m]
- $W$ Length of internal square [m]
- $AR$ Aspect ratio $AR = W/H$
- $C_p$ Specific heat at constant pressure [Jkg$^{-1}$ K$^{-1}$]
- $g$ Acceleration due to gravity [ms$^{-2}$]
- $k$ Thermal conductivity [Wm$^{-1}$ K$^{-1}$]
- $Nu$ Local Nusselt number
- $p$ Pressure of the fluid [Nm$^{-2}$]
- $Pr$ Prandtl number
- $Ra$ Rayleigh number
- $q_w$ Heat flux at the surface [Wm$^{-2}$]
- $T$ Temperature of the fluid [K]
$T_H$ Maximum temperature on wall [K]
$T_0$ Minimum temperature on wall [K]
$u, v$ Dimensional velocity components along the (x, y) axes [m s$^{-1}$]
$x, y$ Axis in the direction along and normal to the tangent of the surface
$B$ Heat source length
$D$ Heat source location.

Greek symbols
$\alpha$ Thermal diffusivity
$\beta$ Volumetric coefficient of thermal expansion [K$^{-1}$]
$\theta$ Dimensionless temperature function
$\phi$ Solid volume fraction
$\mu$ Viscosity of the fluid [kg m$^{-1}$ s$^{-1}$]
$\nu$ Kinematic viscosity [m$^2$ s$^{-1}$]
$\rho$ Density of the fluid [kg m$^{-3}$]
$\chi$ General dependent variable.

Subscripts
$i, j$ Positions in the grid
$l$ Local value
eff Effective value
$C$ The minimum temperature
$H$ The maximum temperature
$f$ Fluid particle
$s$ Solid particle
$nf$ Nanofluid property
$m$ Mean value.

References and Notes