A finite element analysis on combined convection and conduction in a channel with a thick walled cavity

M.M. Rahman
Department of Mathematics, BUET, Dhaka,
Bangladesh and Tower Faculty of Engineering University of Malaya,
Kuala Lumpur, Malaysia

Hakan Oztop
Department of Mechanical Engineering, Firat University, Elazig, Turkey

S. Mekhilef
Department of Electrical Engineering, University Malaya,
Kuala Lumpur, Malaysia

R. Saidur
Manufacturing Engineering Department,
The Public Authority for Applied Education and Training, Shuweikh, Kuwait

A. Chamkha
Manufacturing Engineering Department,
The Public Authority for Applied Education and Training, Safat, Kuwait

A. Ahsan
Department of Civil Engineering, University Putra Malaysia,
Selangor, Malaysia, and
Khaled S. Al-Salem
Department of Mechanical Engineering, King Saud University,
Riyadh, Saudi Arabia

Abstract

Purpose – The purpose of this paper is to examine the effects of thick wall parameters of a cavity on combined convection in a channel. In other words, conjugate heat transfer is solved.

Design/methodology/approach – Galerkin weighted residual finite element method is used to solve the governing equations of mixed convection.

Findings – The streamlines, isotherms, local and average Nusselt numbers are obtained and presented for different parameters. It is found heat transfer is an increasing function of dimensionless thermal conductivity ratio.

Originality/value – The literature does not have mixed convection and conjugate heat transfer problem in a channel with thick walled cavity.

Keywords Combined convection, Conjugate heat transfer, Nonlinear, Open channel

Paper type Research paper

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>dimensional thickness of solid wall (m)</td>
</tr>
<tr>
<td>( \frac{D}{H} )</td>
<td>dimensionless thickness of the solid wall</td>
</tr>
<tr>
<td>( g )</td>
<td>gravitational acceleration (ms(^{-2}))</td>
</tr>
<tr>
<td>( H )</td>
<td>height of the enclosure (m)</td>
</tr>
</tbody>
</table>
1. Introduction
Analysis of both conduction and convection is an important problem in engineering applications such as district heating installation, solar photovoltaic/thermal, refrigerator, radiator, electronic cooling, channels and heat exchangers (Teo et al., 2012, Hasanuzzaman et al., 2009). In most of these applications, forced and natural convection can be occurred in the same engineering systems which are called as mixed or combined convection. However, conduction heat transfer is taken into account in thick walls and the problem is solved for both convection and conduction and is called as conjugate heat transfer problem.

Conjugate mixed convection heat transfer in a lid-driven enclosure with a thick bottom wall is analyzed by Oztop et al. (2008) at different Richardson numbers and thermal conductivity ratios. In another work (Oztop et al., 2009), lid-driven cavity was divided into two chests by using a solid conductive divider. This problem is highly interesting that both mixed convection and natural convection are formed in one system. Visualization of heat flow using Bejan’s heatlines due to natural convection of water near 4°C in thick walled porous cavity is studied by Varol et al. (2010). They indicated that the dimensionless thickness of the wall is an effective parameter on heatlines.

Kanna and Das (2006) studied the conjugate heat transfer characteristics for backward-facing step flow problem by using alternating direction implicit method. They compared the obtained results with the non-conjugate heat transfer case. Chiu et al. (2001) made both experimental and numerical study on conjugate heat transfer in a horizontal channel heated from below. In their work, they observed that the conjugate heat transfer revealed that the addition of conjugate heat transfer significantly affects the temperature and heat transfer rates at the surface of the heated region. Yang and Tsai (2007) presented a numerical study of transient conjugate heat transfer in a high turbulence air jet impinging over a flat circular disk. Conjugate heat transfer in a rectangular channel with lower and upper wall-mounted obstacles is investigated using the Lattice Boltzmann Method by Pirouz et al. (2011). Their results showed that reducing the distance between obstacles made the flow deviate and accelerate in the vicinity of faces, and caused an increase in the rate of convective heat transfer from obstacles. Other related works with conjugate heat transfer for natural or mixed convection can be found in literature as Mobedi and Oztop (2008) and Varol et al. (2008).

Greek symbols
\[ \alpha \] thermal diffusivity (m²s⁻¹)
\[ \beta \] coefficient of thermal expansion (K⁻¹)
\[ \mu \] dynamic viscosity (Pa.s)
\[ \nu \] kinematic viscosity (m²s⁻¹)
\[ \theta \] non-dimensional temperature
\[ \psi \] streamfunction

Subscripts
\[ i \] inlet state
\[ s \] Solid
\[ av \] average

Finite element analysis

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_f )</td>
<td>fluid conductivity (Wm⁻¹K⁻¹)</td>
</tr>
<tr>
<td>( k_s )</td>
<td>solid conductivity (Wm⁻¹K⁻¹)</td>
</tr>
<tr>
<td>( K )</td>
<td>thermal conductivity ratio (( k_s/k_f ))</td>
</tr>
<tr>
<td>( L )</td>
<td>length of cavity (m)</td>
</tr>
<tr>
<td>( Nu )</td>
<td>average Nusselt number (-)</td>
</tr>
<tr>
<td>( n )</td>
<td>any direction</td>
</tr>
<tr>
<td>( P )</td>
<td>dimensional pressure (Nm⁻²)</td>
</tr>
<tr>
<td>( P )</td>
<td>non-dimensional pressure (-)</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number (-)</td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds number (-)</td>
</tr>
<tr>
<td>( Ri )</td>
<td>Richardson number (-)</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>( T_h )</td>
<td>hot wall temperature (K)</td>
</tr>
<tr>
<td>( u, v )</td>
<td>velocity components (ms⁻¹)</td>
</tr>
<tr>
<td>( U, V )</td>
<td>dimensionless velocity component</td>
</tr>
<tr>
<td>( w )</td>
<td>height of the channel (m)</td>
</tr>
<tr>
<td>( x, y )</td>
<td>dimensional coordinates (m)</td>
</tr>
<tr>
<td>( X, Y )</td>
<td>dimensionless coordinates</td>
</tr>
</tbody>
</table>
The detailed hydrodynamic and thermal problem for channel flow with cavity is studied by Rahman et al. (2011a, b). Ozalp et al. (2010) made an experimental study to compare the cavity shapes inside the open channel by using PIV technique. They selected the circular, rectangular and triangular cavities and found that rectangular and triangular cavities had the largest amplitudes while semi-circular cavity had the smallest. Natural convection in trapezoidal and triangular cavities has been studied for heat transfer investigation (Hasanuzzaman et al. (2012a, b)). Then, the flow past a triangular cavity in a channel by including magnetic force is numerically investigated by Rahman et al. (2011a, b). An experimental work was performed by Migeon et al. (2000) to study the flow establishment phase inside square, rectangular and semi-circular closed cavities submitted to impulsive translation, from rest, of one of their walls at a Reynolds number of 1000. Flow induced vibrations and oscillations were investigated by Lin and Rockwell (2001). Some other works related with channel with cavity flow can be found in Rahman et al. (2012a, b).

The main aim of this work is to examine the conduction-mixed convection heat transfer of laminar flow in a channel with thick walled cavity. Based on the authors’ knowledge of the above literature survey and cited references, most of the work on channel flow with cavity is done for thin walled boundaries. Thus, solution of conjugate problem of conduction, forced and natural convection in a single system is obtained in this work. This problem is important from the application of cooling of electronic equipments and space heating systems in engineering.

2. Model description
The physical configuration for the problem with boundary conditions is depicted in Figure 1(a). The flow enters to the channel with a constant temperature and velocity. It is assumed that the height of the channel \( w = 0.25L \). The channel is heated from the bottom wall of cavity with a heater at a constant temperature. The gravity also acts in the y-direction. All of the boundaries are taken as adiabatic. Cyclic boundary condition is applied to the exit of the channel. The thickness of the solid wall is depicted by \( d \) and the dimensionless thickness of the solid wall is taken as \( D = d/(d + H) \).

3. Governing equations
The fundamental laws used to solve the fluid flow and heat transfer problems are the conservation of mass (continuity equations), conservation of momentums (momentum equations), and conservation of energy (energy equations), which constitute a set of coupled, nonlinear, partial differential equations. For laminar incompressible thermal flow, the buoyancy force is included here as a body force in the \( \nu \)-momentum equation. The governing equations for the two-dimensional steady flow after invoking the Boussinesq approximation and neglecting radiation and viscous dissipation can be expressed as:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \nabla^2 U \tag{2}
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \nabla^2 V + Ri \theta \tag{3}
\]
Figure 1.
(a) Physical configuration of the considered problem; (b) grid distribution; (c) optimum grid
For the solid wall the energy equation is:

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0$$

(5)

where $Re = u_i L / \nu$, $Pr = \nu / \alpha$ and $Ri = g \beta \Delta T L / u_i^2$ are Reynolds number, Prandtl number and Richardson number, respectively.

The above equations were non-dimensionalized by using the following dimensionless quantities:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_i}, \quad V = \frac{v}{u_i}, \quad P = \frac{(\rho + \rho g y)L^2}{\rho u_i^2}, \quad D = \frac{d}{L}, \quad \theta = \frac{(T - T_i)}{(T_h - T_i)}$$

where $X$ and $Y$ are the coordinates varying along horizontal and vertical directions, respectively, $U$ and $V$ are the velocity components in the $X$ and $Y$ directions, respectively, $\theta$ is the dimensionless temperature and $P$ is the dimensionless pressure.

The boundary conditions for the present problem are specified as follows:

- At the inlet: $U = 1, \quad V = 0, \quad \theta = 0$
- At the outlet: $\partial U / \partial X = 0, \quad V = 0, \quad \partial \theta / \partial X = 0$
- At the bottom wall of the cavity: \[ \begin{align*}
    U &= 0, \quad V = 0, \quad \frac{\partial \theta}{\partial n} = K \frac{\partial \theta}{\partial n}, \quad \text{at } y = d / H, \quad 0 < x < 1 \\
    U &= 0, \quad V = 0, \quad \theta = 1, \quad \text{at } y = 0, \quad 0 < x < 1 
\end{align*} \]
- At the channel as well as cavity walls (left and right): $U = V = \partial \theta / \partial n = 0$

where $N$ is the non-dimensional distances either $X$ or $Y$ direction acting normal to the surface of the solid block and $K$ is the dimensionless ratio of the thermal conductivity ($K = k_s / k_f$). Small values of $K$ ($K < 1$) are only valid mathematically.

The local heat transfer rates on the surface of heat source is defined as

$$Nu_x = -\frac{\partial \theta}{\partial Y}|_{y=0}.$$

The average heat transfer rate on the surface of heat sources can be evaluated by the average Nusselt, which is defined as $Nu_{av} = - \int_0^1 \partial \theta / \partial Y \, dX$.

4. Numerical solution

4.1 Solution procedure

In this investigation, the Galerkin weighted residual method of finite-element formulation is used as a numerical procedure. The finite-element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. In the present study, erratic grid size system is considered especially near the walls to capture the rapid changes in the dependent variables. The coupled governing Equations (2)-(5) are transformed into sets of algebraic equations using the finite-element method to reduce the continuum domain into discrete triangular domains. The system of algebraic
equations is solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied: \(|\Gamma^{m+1} - \Gamma^m| \leq 10^{-6}\) where \(m\) is number of iteration and \(\Gamma\) is the general dependent variable.

4.2 Grid refinement check
Grid distribution for the physical model is depicted in Figure 1(b). As seen from the figure, grid connections are given very regularly on the boundary of solid and fluid part. Grid refinement check is also made to show the optimum grid in this work. It is given in Figure 1(c) to get optimum grid for \(D = 0.1, K = 0.2\) and \(Ri = 1\) and 16,962 elements are used for executions.

4.3 Code validation
Results of studied code are validated with an experimental work by done Ozalp et al. (2010) which is performed for flow past different shaped cavities. This study is performed in a water tunnel and measurements by done PIV devices and laser-camera. In their work, just flow motion was performed and heat transfer is not included to work. A rectangular cavity without thick wall is chosen to compare with available data. The results are compared and showed in Figure 2. The experimental data are taken from Ozalp et al. (2010). The flow is visualized by using PIV technique. The comparison of the two figures indicates that the obtained results from the written code are acceptable.

5. Results and discussion
A computational analysis has been performed in this work to investigate the effects of dimensionless value of thermal conductivity, Richardson number and dimensionless thickness value of the solid wall in a channel with a solid walled cavity. In this work, Prandtl number is taken as 7.0 for whole work, respectively.

Figure 3(a) show streamlines (on the left) and isotherms (on the right) for \(D = 0.1\) and \(K = 0.2\) at different values of Richardson numbers. It is noticed that non-dimensional parameter of \(K\) is defined as the ratio of thermal conductivity of solid (\(k_s\)) to fluid (\(k_f\)). As well known from the literature, Richardson number is a measurement for the ratio of forced convection to natural convection. When forced convection becomes dominant to natural convection a huge circulation cell is formed in clockwise direction with \(\psi_{\text{min}} = -0.199\). However, the flow inside the cavity behaves as lid-driven cavity due to low Richardson number. Other two cells are formed in right bottom corner and near the left corner. Physically, the flow tries to go into cavity from left and go out from right vertical walls. In this case, the main flow through top side of the channel does not affected from the presence of cavity. For \(K = 0.2\), the thick wall has high conductivity and isotherms are formed as parallel to horizontal walls. They are mostly cumulated inside the cavity. The flow starts to heating from left vertical wall of the cavity. This result is supported with many studies in literature as Hasanuzzaman et al. (2012a). Thus, the heat is mostly transferred into fluid due to high conductivity of bottom thick bottom wall. The distribution of isotherm also behaves as lid-driven cavity heated from the bottom wall. The main cell becomes smaller with increasing of Richardson number to \(Ri = 1\) and the cell moves to right bottom corner. The values of minimum stream function becomes as \(\psi_{\text{min}} = -0.021\). It means that the flow strength decreases with increasing of Richardson number. This is clear from Figure 3(b). The flow goes into channel from inlet and it circulates due to presence of buoyancy.
In this case, temperature contours are more parallel to horizontal walls as seen from isotherms. Temperature value becomes higher inside the thick wall due to domination of mixed convection. The circulating cell is disappeared with the highest value of Richardson number due to increasing of domination of natural convection. Conduction mode of heat transfer is increased inside the cavity. The shape of the flow fits with the geometry of the channel. As a general observation, variation of Richardson number becomes more effective on flow field than that of temperature field.

Figure 4 illustrates the streamlines (on the left) and isotherms (on the right) for $D = 0.1$ and $K = 1$. In this case, fluid and solid have the same value of thermal

**Figure 2.**
Comparison of results with earlier work
(a) results from present study; (b) results from experimental work

**Source:** Ozalp *et al.* (2010)
conductivity. When this figure compares with Figure 3 for $K = 0.1$, flow field shows almost same behavior due to lower effects of buoyancy forces. Flow strength is increased with increasing of thermal conductivity ratio for $Ri = 0.1$. However, presence of solid thick wall of the bottom wall of the cavity becomes insignificant from the temperature point of view. Thus, flow is more heated according to case of $K = 0.1$. This distribution proves the temperature distribution inside the cavity heated from bottom and cooled from top because of flowing fluid from open side of the cavity. Non-linear temperature distribution is observed. Figure 5 presents the streamlines and isotherms for $K = 10$. In this case, the cavity is heated from the top side of the thick wall due to higher thermal conductivity value of solid material.

As seen from the Figure 6 which is plotted for the case of $D = 0.25$ and $K = 0.2$, both flow field and temperature distribution are completely affected. There is no circulation cell inside the cavity due to domination of forced convection flow at $Ri = 0.1$. In this case, solid wall is chosen as insulated material. Thus, the fluid inside the cavity is not well heated and the top point of the cavity the fluid is equal to inlet fluid temperature. When buoyancy induced flow becomes effective two weak circulation cells are formed. The biggest one rotates in clockwise and the other one rotates counterclockwise rotating direction which sits at the bottom wall. Figures 7 and 8 illustrate the streamlines and isotherms for $D = 0.25$ and $K = 1$ and $10$, respectively. The thick wall becomes insignificant from the temperature distribution.
A huge circulation cell is formed inside the cavity with $c_{\text{min}} = 0.144$. The flow is pushed to corner and impinges to bottom wall and jumped. Thus, a mini cell is formed on the bottom wall with $c_{\text{max}} = 0.006$ as seen from Figure 7(c). The mini cell is dissapeared for $Ri = 1$ and circle shaped cell is formed at the right corner (Figure 7(b)). Variation of Richardson number becomes less effective on isotherms. The solid wall is well conducted for $K = 10$ and the fluid is heated from the top side of solid wall in Figure 8. The Richardson number affects the temperature distribution inside the solid region. It is mostly effective at the right side of the solid wall due to recirculating flow inside the cavity. When thermal conductivity ratio increases, temperature values are also increased. Thermal boundary layer is a function of thermal conductivity ratio. Figure 9 shows streamlines (on the left) and isotherms (on the right) for $D = 0.5$ and $Ri = 10$ at different values of thermal conductivity ratio. In this case, half of the cavity is occupied by solid wall. Thus, most of heat is captured inside the solid wall and top side temperature of the solid wall becomes colder according to other case of D values. However, natural convection heat transfer becomes effective as seen from Figure 9(a). The fluid exhibits wavy variation and two circulation cells are formed, one on the middle and the other at the right corner of the cavity. These circulations affect the thermal boundary layer at that points. These cells are dissapeared with increasing of
dimensionless value of thermal conductivity. Figure 9(c) is plotted to show streamlines (left) and isotherms (right) for \( D = 0.5 \) and \( K = 10 \). In this case, solid wall occupies half of the cavity. The flow inlets to channel from input side of the duct and wavy variation is obtained above the solid wall. Mini circulation cell in streamline is observed on the wall but the flow strength becomes very weak. Boundary layer becomes very thin around the inlet and exit part of main duct. Isotherms are also showed wavy variation inside the cavity. Temperatures become constant inside the solid wall due to higher thermal conductivity value. Higher values of Richardson number become insignificant due to higher thickness of the wall.

Figure 10 illustrates the local Nusselt number along the heated side for different Richardson number and thermal conductivity ratio at \( D = 0.1 \). As seen from the figure, local Nusselt values are linearly increased along the heated side for all values of Richardson numbers. As an expected results, values of local Nusselt number increases with increasing of thermal conductivity ratios. Also, higher values are obtained when natural convection becomes dominant to forced convection, namely higher values of Richardson number. Values are closer to each other with increasing of thermal conductivity ratio due to increasing of heat transfer from bottom to top. Cold fluid passes the cavity at lowest value of the Richardson number. At the left side of the cavity, values are almost same at \( \text{Ri} = 0.1 \) and 1 due to stagnant flow at that side. Similarly, Figure 11 gives an opportunity to compare results on local Nusselt numbers.
between dimensionless solid wall thicknesses for different dimensionless value of thermal conductivity. Results are decreased with increasing of wall thickness due to low heat transfer from outside to inside of the cavity. Nevertheless, values are increased along the bottom wall. In case of dominant regime of mixed convection and forced convection values are very close to each other for all values of thermal conductivity due to high flow velocity which passes from the cavity. On the contrary a huge differences are obtained for Ri = 10 especially near the outlet port of the cavity due to increasing of flow velocity at that part. Figure 12 is plotted to show variation of local Nusselt number for D = 0.5. In this case, all values are close to each other which are opposite trend of other thickness values as given in Figures 11 and 12.

Table I lists the variation of average Nusselt number for different parameters for studied parameters such as dimensionless thickness of the solid wall, dimensionless thermal conductivity and Richardson number. For higher value of dimensionless thickness, Richardson number is not an effective parameter for the lower value of the thermal conductivity such as K = 0.01 and 0.2 due to low heat transfer. It must be noted that studied value of K < 1 is only valid for mathematically. Also, average Nusselt number decreases with increasing of dimensionless thermal conductivity value for all values of Richardson numbers. However, average Nusselt number increases with increasing of Richardson number except D = 0.5. Average Nusselt number decreases with increasing of dimensionless wall thickness.
Figure 7. Streamlines (on the left) and isotherms (on the right) for $D = 0.25$ and $K = 1$, (a) $Ri = 0.1$, (b) $Ri = 1$, (c) $Ri = 10$

Figure 8. Streamlines (on the left) and isotherms (on the right) for $D = 0.25$ and $K = 10$, (a) $Ri = 0.1$, (b) $Ri = 1$, (c) $Ri = 10$
6. Conclusions

A numerical analysis has been performed to investigate the conjugate mixed convection problem in an open channel with thick walled cavity. Important findings can be listed as follows:

- The presence of thick wall inside the cavity affects both flow distribution and temperature distribution. The thickness of the cavity is directly an effective parameter with flowing fluid inside cavity. It is important for lower Richardson number, namely, domination of forced convection.

- The number of circulation cells inside the cavity is directly related with the Richardson number for all values of the thermal conductivity and the solid wall thickness. The cell number is decreased with decreasing of the Richardson number and cells are disappeared in the case of forced convection dominated regime.

- The local Nusselt numbers are very close to each other at the left side of the cavity almost for all studied parameters. The values increase to right side of the cavity.

- Heat transfer is an increasing function of the thermal conductivity ratio for all cases. It is calculated from the top side of solid wall. Thus, it is increased with increasing of Richardson number.
Figure 10. Local Nusselt number for different Richardson number at $D = 0.1$, (a) $K = 0.2$; (b) $K = 1$ and (c) $K = 10$
Figure 11. Local Nusselt number for different Richardson number at $D = 0.25$, (a) $K = 0.2$; (b) $K = 1$ and (c) $K = 10$.
Finite element analysis

Figure 12. Local Nusselt number for different Richardson number at $D = 0.5$, (a) $K = 0.2$; (b) $K = 1$ and (c) $K = 10$
The results of this theoretical work can be used for applications of cooling of electronic equipments in the range of Richardson numbers. The study can be extended to turbulent flow and different boundary conditions as a future work.

### References


**Corresponding author**
Dr Hakan Oztop can be contacted at: hfoztop1@yahoo.com

To purchase reprints of this article please e-mail: reprints@emeraldinsight.com
Or visit our web site for further details: www.emeraldinsight.com/reprints