UNSTEADY HEAT AND MASS TRANSFER BY MHD MIXED CONVECTION FLOW FROM A ROTATING VERTICAL CONE WITH CHEMICAL REACTION AND SORET AND DUFOUR EFFECTS

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This work focused on the study of Soret and Dufour effects on unsteady coupled heat and mass transfer by mixed convection flow over a vertical cone rotating in an ambient fluid with a time-dependent angular velocity in the presence of a magnetic field and chemical reaction. The cone surface is maintained at variable temperature and concentration. The resulting governing equations are non-dimensionalised and transformed into a non-similar form and then solved numerically by an implicit, iterative, finite-difference method. Comparisons with previously published work are performed and excellent agreement is obtained. A parametric study showing the effects of the buoyancy parameter, magnetic field, chemical reaction parameter, Soret and Dufour numbers on the local tangential and azimuthal skin friction coefficients, and the local Nusselt and Sherwood numbers are conducted. These results are illustrated graphically to depict special features of the solutions.

Keywords: rotating cone, mixed convection, unsteady flow, MHD, Soret and Dufour effects, chemical reaction

INTRODUCTION

Combined heat and mass transfer flows are of considerable importance in many applications. These include atmospheric and oceanic circulation, transport phenomena in power transformer electronics, heating and cooling processes in semiconductor electronics, absorption reactors, binary diffusion systems, solar energy systems and polymer processing in the plastics industry. Double-diffusive mixed convective over rotating bodies are very important in manufacturing industries for the design of reliable equipment, nuclear plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. Such a study is important in the design of turbines and turbo-machines, in estimating the flight path of rotating wheels and spin-stabilised missiles and in the modelling of many geophysical vortices. As explained by Takhar et al.,\cite{1} when an axi-symmetric body rotates in a forced flow field, the fluid near the surface of the body is forced outward in the radial direction due to the action of the centrifugal force. Ostrach and Braun\cite{2} have investigated the possibility of cooling the nose-cone of re-entry vehicles by spinning the nose. Also, rotating heat exchangers are extensively used by the chemical and automobile industries. Early investigations of flow and heat transfer in rotating systems are given by Dorfman\cite{3} and Kreith.\cite{4} The effect of the axial magnetic field on the flow and heat transfer over a rotating disk was considered by Sparrow and Cess.\cite{5} Tarek et al.\cite{6} have obtained an asymptotic solution of the flow problem over a rotating disk with a weak axial magnetic field. The similarity solution of the mixed convection from a rotating vertical cone in an ambient fluid was obtained Himasekhar et al.\cite{7} Wang\cite{8} has investigated the flow and heat transfer on rotating cones, disks and axi-symmetric bodies with concentrated heat sources. The laminar natural convection from a non-isothermal cone was analysed by Hering and Groshi\cite{9} and Roy.\cite{10} An approximate method of solution for the overall heat transfer from vertical cones in laminar natural convection was reported by Alamgir.\cite{11} Roy and Anilkumar\cite{12} considered the problem of unsteady mixed convection from a rotating cone in a rotating fluid due to the combined effects of thermal and mass diffusion for the conditions of prescribed wall temperature and heat flux. Chamkha and Al-Mudhaf\cite{13} studied the unsteady heat and mass transfer by mixed convection flow over a vertical permeable cone rotating in an ambient fluid with a time-dependent angular velocity.

On other hand, the growing need for chemical reactions in chemical and hydro-metallurgical industries requires the study of heat and mass transfer with chemical reaction. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of the chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors, as well as metallurgical and chemical engineering. Their other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, and the combustion of atomised liquid fuels. The presence of a foreign mass in water or air causes some kind of chemical reaction. This mass may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, for example, polymer production, manufacturing of ceramics or glassware, and food processing. Diffusion and chemical reactions in an isothermal

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laminar flow along a soluble flat plate were studied by Fairbanks and Wike.\cite{14} The effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied by Das et al.\cite{15} Andersson et al.\cite{16} studied the flow and mass diffusion of a chemical species with first order and higher order reactions over a linearly stretching surface. Chamkha\cite{17} studied coupled heat and mass transfer on MHD flow of a uniformly stretched vertical permeable surface with the effects of heat generation/absorption and chemical reaction. Anjali and Kandasamy\cite{18} investigated the double-diffusive convective flow over a wedge in the presence of chemical species concentration. Kandasamy et al.\cite{19} reported on the heat and mass transfer along a wedge with a heat source and a concentration under a chemical reaction effect. Eldahab and Salem\cite{20} investigated the MHD flow and heat transfer of a non-Newtonian fluid on a moving cylinder with diffusion and chemical reaction. Raptis and Perdikis\cite{21} examined the effects of viscous flow over a nonlinearly stretching sheet in the presence of chemical reaction and magnetic field. Patil et al.\cite{22} investigated the double diffusive mixed convection flow over a moving vertical plate in the presence of a homogeneous nth order chemical reaction. Chamkha et al.\cite{23} studied the effect chemical reaction on the coupled heat and mass transfer by MHD natural convection over a truncated cone with variable surface temperature and concentration.

All of the above studies neglected the Soret and/or Dufour effects. Such effects are significant when density differences exist in the flow regime. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering. When species are introduced at a surface in a fluid domain, with a different (lower) density than the surrounding fluid, both Soret (thermo-diffusion) and Dufour (diffuso-thermal) effects can become influential. It is also found that the diffusion thermo effect is much stronger with the injection of hydrogen and helium (molecular weight less than air) than with the injection of argon, carbon dioxide, and xenon (molecular weight greater than air). It is observed that, for a light gas such as helium or hydrogen injected into the boundary layer, the temperature-induced buoyancy forces are augmented and the heat transfer rates are increased. Therefore, the heat transfer rate (Nusselt number) increases in the presence of thermal diffusion and diffusion thermo effects with blowing rate. Weaver and Viskanta\cite{24} investigated numerically the influence of species inter diffusion, Soret and Dufour effects on the free convection heat and mass transfer in a cavity showing that species inter diffusion reduces the overall heat transfer, but boosts the mass transfer through the cavity for specific geometrical aspect ratios. They also showed that Soret diffusion can supplement total mass flux through the cavity by more than 10% and that the heat transfer because of the Dufour effects is comparable to that by the heat conduction. Harstad and Belan\cite{25} developed a model for simulating the isolated fluid drop entry of a single compound immersed into another compound in finite, quiescent surroundings at supercritical conditions including both Soret and Dufour effects with applications in liquid oxygen–hydrogen conditions in rocket chambers. Li et al.\cite{26} studied the thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects in a strongly endothermic chemically reacting flow in a porous medium. Osalusi et al.\cite{27} discussed the Soret and Dufour effects on simultaneous heat and mass transfer in steady hydromagnetic convective and slip flow from a rotating disk with viscous dissipation and Joule heating. Abd El-Aziz\cite{28} investigated the effects of thermal diffusion and diffusion-thermo on combined heat and mass transfer by hydromagnetic three-dimensional free convection over a permeable stretching surface with radiation. EL-Kabeir et al.\cite{29} discussed the thermal-diffusion and diffusion-thermo effects on coupled heat and mass transfer by MHD mixed convection stagnation-point flow of a non-Newtonian fluid towards a stretching surface in the presence of chemical reaction. Makinde and Olanejo\cite{30} studied the unsteady mixed convection with the Dufour and Soret effects past a semi-infinite vertical plate moving through a binary mixture of chemically reacting fluid. The diffusion-thermo and thermal-diffusion effects on heat and mass transfer by MHD free convection along a flat plate with streamwise temperature and the species concentration were considered numerically by Chamkha et al.\cite{31}

In this paper, the problem of unsteady heat and mass transfer by mixed convective flow over a rotating vertical cone in the presence of the magnetic field, thermal-diffusion and diffusion-thermo and chemical reaction effects is considered. The order of chemical reaction in this work is taken as a first-order reaction. The unsteadiness in the flow field is due to the angular velocity of the cone which varies arbitrarily with time. The coupled nonlinear parabolic partial differential equations governing the flow and heat and mass transfer problem have been solved numerically using an implicit iterative finite-difference scheme.

**PROBLEM FORMULATION**

Consider unsteady, laminar, incompressible boundary-layer heat and mass transfer by mixed convective axi-symmetric flow of an electrically conducting fluid over a heated vertical cone rotating in an ambient fluid with a time-dependent angular velocity, \( \Omega(t) = \Omega_0 \phi(t') \), \( t' = (\Omega_0 \sin \alpha t) \), around the axis of the cone in the presence of magnetic field, chemical reaction, thermal-diffusion and the diffusion-thermo effects. The physical model and the coordinate system are shown in Figure 1. A uniform magnetic field of strength \( B_0 \) is applied in the \( z \)-direction (normal direction) and the gravitational acceleration \( g \) acts downward parallel to the axis of the cone. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. In addition, the Hall effect and the electric field are assumed negligible. The small magnetic Reynolds number assumption uncouples the Navier–Stokes equations from the Maxwell’s equations. The rectangular curvilinear fixed coordinate system \((x, y, z)\), where \( x \) is measured along a meridional section,

![Figure 1. Physical model and coordinate system.](image-url)
the $y$-axis is along a circular section and the $z$-axis is normal to the cone surface have been employed. Let $u$, $v$ and $w$ be the velocity components along the $x$ (tangential), $y$ (circumferential or azimuthal) and $z$ (normal) directions, respectively. The wall temperature $T_w$ and wall concentration $c_w$ are assumed to vary linearly with the distance $x$ and the ambient temperature ($T_\infty$) and concentration ($c_\infty$) are constant. In addition, the concentration of diffusing species thermal-diffusion and diffusion-thermal energy effects are taken into account, and a first-order homogeneous chemical reaction is assumed to take place in the fluid. Under the above assumptions and using the Boussinesq approximation, the boundary-layer equations governing this heat and mass transfer convective flow on the rotating cone are given by:

$$u_x + x^{-1}u + w_z = 0 \quad (1)$$

$$u_t + uu_x + uw_z - \frac{v^2}{x} = vv_{zz} + g\beta_T \cos \alpha (T - T_\infty) + g\beta \cos \alpha (c - c_\infty) - \sigma B_0 \frac{u}{\rho} \quad (2)$$

$$v_t + uv_x + vw_z - \frac{u^2}{x} = vv_{zz} - \sigma B_0 \frac{v}{\rho} \quad (3)$$

$$T_t + uT_x + wT_z = \frac{v}{\rho_T} T_{zz} + \frac{DkT}{C_i c_p} c_{zz} \quad (4)$$

$$c_t + uc_x + wc_z = Dc_{zz} + \frac{DkT}{T_m} T_{zz} - K_1 (c - c_\infty) \quad (5)$$

The initial conditions are given by the steady-state equations:

$$u(x, z, 0) = u_0(x, z), \quad v(x, z, 0) = v_0(x, z), \quad w(x, z, 0) = w_0(x, z) \quad (6)$$

The boundary conditions for this problem are given by:

$$u(x, 0, t) = 0, \quad w(x, 0, t) = 0, \quad v(x, 0, t) = \Omega_0 x \sin \alpha f(t), \quad T(x, 0, t) = T_w(x), \quad c(x, 0, t) = c_w(x), \quad w(x, \infty, t) = v(x, \infty, t) = 0, \quad T(x, \infty, t) = T_\infty, \quad c(x, \infty, t) = c_\infty, \quad u(\infty, z, t) = v(\infty, z, t) = 0, \quad T(\infty, z, t) = T_\infty, \quad c(\infty, z, t) = c_\infty, \quad z > 0 \quad (7)$$

Here $T$ is the temperature; $c$ is the concentration; $\beta_T$ is the coefficient of the thermal expansion; $\beta_i$ is the coefficient of the concentration expansion; $\alpha$ is the semi-vertical angle of the cone; $v$ is the kinematic viscosity; $C_P$ is the specific heat at constant pressure; $\rho$ is the density of the fluid; $t$ and $t^*$ ($t^* = \Omega_0 t$) are the dimensional and dimensionless times, respectively; $\Omega_0$ is the angular velocity of the cone at $t = 0$; $Pr$ is the Prandtl number; $D$ is the mass diffusivity, $K_1$ is the chemical reaction rate; $T_m$, $K_T$ and $C_i$ are the mean fluid temperature, thermal diffusion ratio and concentration susceptibility; $\phi(t^*)$ is a continuous function having continuous first-order derivative; the subscripts $x$, $y$ and $z$ denote partial derivatives with respect to $x$, $y$ and $z$, respectively; the subscript $t$ denotes the initial conditions; and the subscripts $w$ and $\infty$ denote the wall and ambient conditions, respectively. The last term on the right-hand side of each of the energy Equation (3) and the diffusion Equation (4) signifies the Dufoff or diffusion-thermo effect and the Sorot or thermal-diffusion effect, respectively.

It is convenient to transform Equations (1)–(5) into the $(\eta, \tau)$ system by applying the following transformations:

$$\eta = \left( \Omega_0 \sin \frac{\alpha}{\nu} \right)^{1/2} z, \quad \tau = \left( \Omega_0 \sin \alpha \right) t$$

$$u(x, z, t) = z^{-1} (\Omega_0 x \sin \alpha) H'(\eta, \tau) \phi(\tau)$$

$$v(x, z, t) = (\Omega_0 x \sin \alpha) G(\eta, \tau) \phi(\tau)$$

$$w(x, z, t) = (\nu \Omega_0 \sin \alpha / 2H(\eta, \tau) \phi(\tau))$$

$$T(x, z, t) - T_\infty = (T_w - T_\infty) \frac{x}{L}$$

$$c(x, z, t) - c_\infty = (c_w - c_\infty) C(\eta, \tau)$$

$$Gr_L = \frac{\beta_i \sin \alpha}{\nu} (T_w - T_\infty) L^3 / \rho, \quad Re_L = \Omega_0 L^2 \left( \frac{\sin \alpha}{\nu} \right)$$

$$N = \frac{\beta_i (c_w - c_\infty)}{\beta_i (T_w - T_\infty)}, \quad M = \frac{H \Omega^2}{Re_L}, \quad Sc = \frac{\nu}{D}, \quad Df = \frac{DK (c_w - c_\infty)}{c_w c_p (T_w - T_\infty)}$$

$$Ha^2 = \frac{\sigma B_0^2 L^2}{\nu}, \quad \tau = \frac{Dk (c_w - c_\infty)}{\nu} \frac{T_w - T_\infty}{v F_m (c_w - c_\infty)}$$

$$\gamma = \frac{K_1}{(\Omega_0 \sin \alpha)} \left( \frac{(\sigma B_0 \sin \alpha)/\nu}{(\Omega_0 \sin \alpha)} \right)$$

where $T_w$ and $c_w$ are the wall temperature and concentration at time $t^* = 0$, respectively. With these transformations, Equation (1) is identically satisfied and Equations (2)–(5) reduce to the following system of equations:

$$H'' = \phi HH'' + 2^{-1} \phi H'2 - 2\phi G^2 - 2\phi^{-1} \lambda (\theta + NC) - MH'$$

$$- \phi^{-1} \left( \frac{d\phi}{dt^*} \right) H' \frac{dH'}{dt^*} = 0 \quad (9)$$

$$G'' - \phi HG' - H'G - MG - \phi^{-1} \left( \frac{dG}{dt^*} \right) G \frac{dG}{dt^*} = 0 \quad (10)$$

$$\theta' - Pr (H\theta' - 2^{-1} H'\theta) \phi + PrDf C'' - Pr \frac{d\theta}{dt^*} = 0 \quad (11)$$

$$C'' - Sc(HC' - 2^{-1} H' C) \phi + Sc\theta'' - ScgC - Sc \frac{dC}{dt^*} = 0 \quad (12)$$

The boundary conditions (7) can be rewritten as:

$$H(0, t^*) = H'(0, t^*) = 0, \quad G(0, t^*) = \theta(0, t^*) = C(0, t^*) = 0$$

$$H'(\infty, t^*) = G(\infty, t^*) = \theta(\infty, t^*) = C(\infty, t^*) = 0 \quad (13)$$

Equations (9)–(12) reduce to those of Takhar et al. when Equation (12) is ignored and $N = 0$. Also, they reduce to those reported by Roy and Anilkumar for the case of prescribed wall temperature when $M = Df = Sg = \gamma = 0$.

The initial conditions (i.e. conditions at $t^* = 0$) are given by the steady state equations obtained from (9) to (12) by putting $\phi = 1, \frac{d\phi}{dt^*} = 0, \frac{dG}{dt^*} = 0, \frac{dH'}{dt^*} = 0, \frac{dG}{dt^*} = 0, \frac{dH'}{dt^*} = 0$ when $t^* = 0$. The steady-state equations are:

$$H'' = H' H'' + 2^{-1} (H')^2 - 2G^2 - 2\lambda (\theta + NC) - MH' = 0 \quad (14)$$

$$G'' - (HG' - H'G) - MG = 0 \quad (15)$$

$$\theta' - Pr (H\theta' - 2^{-1} H'\theta) + PrDf C'' = 0 \quad (16)$$

$$C'' - Sc(HC' - 2^{-1} H' C) + Sc\theta'' - ScgC = 0 \quad (17)$$
with boundary conditions:

\[ H(0) = 0, \quad H'(0) = 0, \quad G(0) = \theta(0) = C(0) = 1 \]
\[ H'(\infty) = G'(\infty) = \theta'(\infty) = C'(\infty) = 0 \]

Here \( \eta \) and \( t' \) are the transformed coordinates; \( H', G \) and \( H \) are the dimensionless velocity components along the tangential, azimuthal and normal directions, respectively; \( \theta \) is the dimensionless temperature; \( C \) is the dimensionless concentration; \( Gr_\lambda \) is the Grashof number; \( Re_\lambda \) is the Reynolds number; \( \lambda \) is the dimensionless buoyancy parameter; \( N \) is the buoyancy ratio such that \( N < 0 \) corresponds to opposing flow while \( N > 0 \) corresponds to aiding flow; \( M \) is the dimensionless magnetic parameter; \( Ha_\lambda \) is the Hartmann number; \( \gamma \) is the dimensionless chemical reaction parameter; \( Pf \) is the Dufour number, \( Sr \) is Soret number; \( L \) is a characteristic length; \( Sc \) is the Schmidt number; \( \mu \) is the coefficient of viscosity; and a prime denotes a derivative with respect to \( \eta \).

It may be remarked that the steady-state Equations (14)–(16) in the absence of all of the magnetic field \( (M = 0) \), buoyancy ratio \( (N = 0) \) and Dufour effects \( (Pf = 0) \) are identical to those of Himasekhar et al.[7] if we replace \( H' \) by \( -2F \) and \( \lambda \) by \( Gr/Re^2 \). Furthermore, Equations (14)–(16) in the absence of the buoyancy and Dufour effects \( (\lambda = 0, Pf = 0) \) and for constant wall temperature case are the same as those of Sparrow and Cess[5] if the term \( PrH'/2 \), which is the contribution due to the linear variation of the wall temperature with the distance \( x \) is omitted.

The quantities of physical interest are local skin friction coefficients in the tangential and azimuthal directions, the local Nusselt number and the local Sherwood number, and these are respectively given by:

\[ C_{rx} = 2\mu \frac{(\partial u/\partial z)_{2=0}}{\rho(\Omega_0 x \sin \alpha) \gamma} = -Re_\lambda^{1/2} \phi'(0,t') \]
\[ C_{ry} = -2\mu \frac{(\partial v/\partial z)_{2=0}}{\rho(\Omega_0 x \sin \alpha) \gamma^2} = -2Re_\lambda^{1/2} \phi'(0,t') \]
\[ Nu_x = -x \frac{(\partial T/\partial z)_{2=0}}{T_w - T_\infty} = -Re_\lambda^{1/2} \theta'(0,t') \]
\[ Sh_x = -x \frac{(\partial c/\partial z)_{2=0}}{c_w - c_\infty} = -Re_\lambda^{1/2} C'(0,t') \]

where \( Re_\lambda = \Omega_0 x^2 \sin \alpha/\gamma \) is the local Reynolds number.

**NUMERICAL METHOD**

The coupled nonlinear parabolic partial differential Equations (9)–(12) under boundary conditions (13) and initial conditions (14)–(18) have been solved numerically using an implicit, iterative tri-diagonal finite-difference scheme similar to that discussed by Blottner.[32]

All first-order derivatives with respect to \( t' \) are replaced by two-point backward difference formulae of the form:

\[ \frac{\partial R}{\partial t'} = \frac{R_{i,j} - R_{i-1,j}}{\Delta t'} \]

where \( R \) represents the dependent variables \( H \) or \( \theta \) or \( C \) and \( i \) and \( j \) are node locations in the \( \eta \) and \( \theta \) directions, respectively. First, the third-order partial differential Equation (9) is converted into a second-order partial differential equation by substituting

**Table 2. Comparison of surface shear stresses \([-H'(0), -G'(0)]\) and ambient velocity \([-H'(\infty)]\) for \( t' = \lambda = \gamma = Df = Sr = 0 \)**

<table>
<thead>
<tr>
<th>( M )</th>
<th>(-H'(0))</th>
<th>(-H'(\infty))</th>
<th>(-G'(0))</th>
<th>(-G'(\infty))</th>
<th>(-H'(\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.021</td>
<td>1.0207</td>
<td>0.6159</td>
<td>0.8848</td>
<td>0.616</td>
</tr>
<tr>
<td>0.5</td>
<td>0.770</td>
<td>0.773</td>
<td>0.8488</td>
<td>0.4587</td>
<td>0.849</td>
</tr>
<tr>
<td>1</td>
<td>0.619</td>
<td>0.6194</td>
<td>1.0692</td>
<td>0.2531</td>
<td>1.069</td>
</tr>
<tr>
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<td>0.4613</td>
<td>1.4418</td>
<td>0.0109</td>
<td>1.441</td>
</tr>
<tr>
<td>3</td>
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<td>0.3813</td>
<td>1.7477</td>
<td>0.0617</td>
<td>1.748</td>
</tr>
<tr>
<td>4</td>
<td>0.331</td>
<td>0.3308</td>
<td>2.0097</td>
<td>0.0406</td>
<td>2.010</td>
</tr>
</tbody>
</table>
$H = N$. Then the second-order partial differential equations for $N$, $G$, $\theta$ and $C$ are discretised using three-point central difference formulae, while first-order derivatives with respect to $\eta$ are discretised by employing the trapezoidal rule. At each line of constant $r^*$, a system of algebraic equations is obtained. The nonlinear terms of these equations are evaluated at the previous iteration and the system of algebraic equations is then solved with iteration by using the well known Thomas algorithm (see Ref. [32]). The same procedure is repeated for the next $r^*$ value and the problem is solved line by line until the desired convergence criterion based on the relative difference between the current and previous iterations is employed. When this difference reaches $10^{-5}$, the solution is assumed to have converged and the iteration process is terminated.

In order to assess the accuracy of the numerical method, the present results were compared with the steady-state results for the surface shear stresses in the tangential and azimuthal directions $[-H'(0), -G'(0)]$, and the surface heat transfer $[-\theta'(0)]$ in the absence of the magnetic field ($M = 0$), mass transfer ($N = 0$), Dufour effect ($DF = 0$), Soret effect ($Sr = 0$) and chemical reaction ($\gamma = 0$) with those of Himasekhar et al. [7] and Chamkha and Al-Mudhaf [13] and the results were found to be in excellent agreement. This comparison is given in Table 1. In addition, the surface shear stresses and heat transfer $[-H'(0), -G'(0), -\theta'(0)]$ and ambient velocity $-H(\infty)$ for the steady-state case in the absence of buoyancy effects ($\lambda = 0$) were compared with those given by Sparrow and Cess [5] and Chamkha and Al-Mudhaf [13] and the results were found to be in excellent agreement. These comparisons are shown in Tables 2 and 3.

### RESULTS AND DISCUSSION

In this section, numerical results for the local skin friction coefficients in the tangential and azimuthal directions, the local Nusselt number and the local Sherwood number based on the finite-difference methodology discussed earlier are presented and/ or discussed for various values of the physical parameters. The results have been obtained for both increasing and decreasing angular velocities $[\phi(t^*) = 1 + at^2, \epsilon = \pm 0.2, 0 \leq t^* \leq 2]$ for several values of parameters namely; the magnetic parameter $M$, the buoyancy ratio $N$, the dimensionless chemical reaction parameter $\gamma$, Dufour number $DF$ and the Soret number $Sr$. These conditions are intended for an electrically conducting fluid such as metal ammonia suspensions ($Pr = 0.78$) polluted by water vapour ($Sc = 0.6$). The value of the Schmidt number $(Sc)$ is chosen to represent the most common diffusing chemical species, which are of interest and the values of the Dufour and Soret numbers are chosen in such a way that their product is constant provided that the mean temperature $T_m$ is kept constant as well. The results of this parametric study are shown in Figures 2–17.

Figures 2–5 display the effect of the magnetic parameter $M$ on the local skin friction coefficients in the tangential and azimuthal directions ($Re^{1/2}_x C_x$, $2^{1/6} Re^{1/2}_x C_x$), the local Nusselt number ($Re^{1/2}_c C_x$) and the local Sherwood number ($Re^{1/2}_c S_x$), respectively for increasing and decreasing angular velocities $[\phi(t^*) = 1 + at^2, \epsilon = \pm 0.2]$. It is noted that the effect of the time variation is more pronounced for large $t^*$ ($t^* > 2$). Moreover, for a fixed $M$, the skin friction coefficients and the Nusselt number increase with an increasing angular velocity, but the reverse

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**Table 3. Comparison of surface heat transfer $[-\theta'(0)]$ for $t^* = \lambda = y = DF = Sr = 0$**

| $M$ | 0 | 0.1 | 1 | 10 | 0.1 | 1 | 10 | 0.1 | 1 | 10 |
|-----|---|-----|---|----|-----|---|----|-----|---|----|----|
| $Pr$ |   |     |   |    |     |   |    |     |   |    |    |
| $M$ | 0  | 0.0766 | 0.396 | 1.134 | 0.0763 | 0.3958 | 1.1337 | 0.0763 | 0.3958 | 1.1337 |
| 0.5 | 0.0428 | 0.282 | 0.956 | 0.0426 | 0.2819 | 0.9558 | 0.0426 | 0.2819 | 0.9558 |
| 1   | 0.0244 | 0.194 | 0.801 | 0.0282 | 0.1939 | 0.8011 | 0.0282 | 0.1939 | 0.8011 |
| 2   | 0.0108 | 0.0982 | 0.571 | 0.0107 | 0.0981 | 0.5712 | 0.0107 | 0.0981 | 0.5712 |
| 3   | 0.00614 | 0.0588 | 0.422 | 0.00612 | 0.0587 | 0.4218 | 0.00612 | 0.0587 | 0.4218 |
| 4   | 0.00407 | 0.0395 | 0.318 | 0.00406 | 0.0344 | 0.3181 | 0.00406 | 0.0344 | 0.3181 |

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**Figure 2. Effect of $M$ on the tangential skin-friction coefficient.**

**Figure 3. Effect of $M$ on the azimuthal skin-friction coefficient.**
trend is observed for a decreasing angular velocity. However, these are not a mirror reflection of each other. When the angular velocity increases with time, the skin-friction coefficients \((Re^{1/2}C_{f\theta}, 2^{-1}Re^{1/2}C_{f\nu})\), Nusselt number \((Re^{1/2}Nu_{x})\) and the local Sherwood number \((Re^{1/2}Sh_{x})\) for \(M = 2\) increase as \(t'\) increases from zero to 2. In addition, the effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive type force called the Lorentz force in the azimuthal direction. This force has the tendency to speed up the motion of the fluid. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the mixed convection flow. The application of a magnetic field moving with the free stream has the tendency to induce a motive force which decreases the motion of the fluid and increases its boundary layer. Therefore, the local skin friction coefficient in the tangential direction \((Re^{1/2}C_{f\theta})\), the local Nusselt number \((Re^{1/2}Nu_{x})\) and the local Sherwood number \((Re^{1/2}Sh_{x})\) decrease with an increasing magnetic parameter \(M\), whereas a pronounced increase in the coefficient in the azimuthal direction \((2^{-1}Re^{1/2}C_{f\nu})\) occurs.

Figures 6–9 illustrate the effect of the buoyancy ratio \(N\) on the local skin friction coefficients in the tangential and azimuthal directions \((Re^{1/2}C_{f\theta}, 2^{-1}Re^{1/2}C_{f\nu})\), the local Nusselt number \((Re^{1/2}Nu_{x})\) and the local Sherwood number \((Re^{1/2}Sh_{x})\) for increasing and decreasing angular velocities \(\phi(t') = 1 + \epsilon t^2, \epsilon = \pm 0.2\), respectively. The effect of the time variation is more pronounced for large \(t' (t' > 2)\). For a fixed value of \(N\), the local skin friction coefficients, the local Nusselt number and the local Sherwood number increase with an increasing angular velocity, but the reverse trend is predicted for a decreasing angular velocity, and consequently, the effect of decreasing the angular velocity of the cone is seen to cause reductions in all of \(C_{f\theta}, C_{f\nu}, Nu_{x}\) and \(Sh_{x}\) due to the decreases in the wall gradient of \(H\) and negative wall gradients of \(G, \theta\) and \(C\). These and all previous trends are obvious from Figures 3–6. However, these are not a mirror reflection of each other. On the other hand, it should be noted that the curve for \(N < 0\) corresponds to opposing flow while those for \(N > 0\) correspond to aiding flow. As expected, increases in the buoyancy ratio (or solutal buoyancy effect) cause an increase in the buoyancy-induced flow along and normal to the cone and decreases in circumferential flow and the thermal and solutal level within the boundary layer for all times. Physically speaking, as the cone rotates, the fluid near the surface of the cone is forced outward along the tangential direction due to the action of the centrifugal force. Therefore, it can be seen that the skin-friction coefficients, the Nusselt and Sherwood numbers are found to be strongly dependent on \(N\) for all \(t'\). Moreover, the skin-friction coefficients in the tangential and azimuthal directions \((Re^{1/2}C_{f\theta}, 2^{-1}Re^{1/2}C_{f\nu})\) and the Nusselt and Sherwood numbers \((Re^{1/2}Nu_{x}, Re^{1/2}Sh_{x})\) increase with increasing values of \(N\).
The effects of the Dufour number $Df$ (reciprocal of diffusion-thermo effect) and the Soret number $Sr$ (reciprocal of thermal-diffusion effect) on the local skin-friction coefficients in the tangential and azimuthal directions ($Re^{1/2}C_{fx}$, $2Re^{1/2}C_{fy}$), the Nusselt and Sherwood numbers ($Re^{1/2}Nu_x$, $Re^{1/2}Sh_x$) for $\phi(t') = 1 + e t^2$, $e = \pm 0.2$ in the time range $0 \leq t' \leq 2$ are displayed in Figures 10–13, respectively. It can be observed that the local Sherwood number is predicted to increase while the local skin-friction coefficients in the tangential and azimuthal directions and the local Nusselt number are predicted to decrease with increases in the values of the Dufour number $Df$. The reason for these trends in the local Nusselt and Sherwood numbers are that
either the increase in the concentration difference or the decrease in the temperature difference leads to an increase in the value of $D_\text{f}$ resulting in trends similar to the above observation. Similarly, either a decrease in the concentration difference or an increase in the temperature difference leads to an increase in the value of the Soret number $S_r$. Therefore, increasing the parameter $S_r$ causes increases in the skin-friction coefficients and the local Nusselt number while it produces decreases in the local Sherwood number. On other hand, it can be seen that the effect of reducing the angular velocity of the cone results in significant reductions in $C_{fx}$ and $N_{ux}$ and slight reductions in $C_{fy}$ and $S_{hx}$ for times $t^* > 1$.

Finally, Figures 14–17 present the effect of the chemical reaction parameter $\gamma$ on the local skin-friction coefficients in the tangential and azimuthal directions ($Re_t^{1/2} C_{fx}, 2^{1/4} Re_t^{1/2} C_{fy}$), and the Nusselt and Sherwood numbers ($Re_s^{1/2} N_{ux}, Re_s^{1/2} S_{hx}$) for $\phi(t^*) = 1 + \epsilon t^2, \epsilon = \pm 0.2$ in the time range $0 \leq t^* \leq 2$, respectively. It can be seen that as $\gamma$ increases, the local Sherwood number increases, while the opposite effect is found for the local skin-friction coefficients in the tangential and azimuthal directions and the local Nusselt number. This is because as $\gamma$ increases, the concentration difference between the cone surface and the fluid decreases and so the rate of mass transfer at the cone surface must increase, while a higher values of $\gamma$ leads to acceleration in the fluid which results reductions in the skin-friction coefficients and the rate of heat transfer. Furthermore, for a fixed $\gamma$, it can be observed that the effect of the time variation is found to be more pronounced on the skin-friction coefficients in the tangential and azimuthal directions and the Nusselt number than on the Sherwood number.

CONCLUSIONS

An analysis was performed to study the Soret and Dufour effects on the unsteady heat and mass transfer by mixed convection flow over a vertical cone rotating in an ambient fluid with a time-dependent angular velocity in the presence of a magnetic field and chemical reaction effects. The cone surface was maintained at variable temperature and concentration. The coupled nonlinear partial differential equations governing the thermosolutal mixed convective flow were solved numerically using an implicit, iterative finite-difference scheme. Comparisons with previously published work were performed and the results were found to be in excellent agreement. It was found that the local tangential and azimuthal skin-friction coefficients and local Nusselt and Sherwood numbers increased with time when the angular velocity of the cone increased, but the reverse trend was observed for a decreasing angular velocity. However, these were not a mirror reflection of each other. Increasing the magnetic field leads to reduce the skin friction coefficient in the tangential direction and local Nusselt and Sherwood numbers but increases the skin-friction coefficient in the azimuthal direction. Also, increasing the buoyancy ratio was
predicted to increase the skin-friction coefficients and the Nusselt and Sherwood numbers. Furthermore, increases in the values of the Dufour number (reciprocal of diffusion-thermo effect) produced decreases in the skin-friction coefficients and the local Nusselt number and decreases in the local Sherwood number. However, the opposite behaviour was predicted as the Dufour number (reciprocal of diffusion-thermo effect) increased. On the other hand, the local tangential and azimuthal skin-friction coefficients and the Nusselt and Sherwood number decreased, while the local Sherwood number increased as the chemical reaction parameter was increased. It is hoped that the present work will serve as a motivation for future experimental work which seems to be lacking at the present time.

**NOMENCLATURE**

\[ B_0 \quad \text{magnetic induction} \]

\[ c \quad \text{concentration (kg/m}^3) \]

\[ c_{w0} \quad \text{wall concentration at time } t = 0 \]

\[ C \quad \text{dimensionless concentration} \]

\[ C_{ex} \quad \text{tangential skin friction coefficient} \]

\[ C_{iy} \quad \text{azimuthal skin friction coefficient} \]

\[ C_p \quad \text{specific heat at constant pressure (kJ/kg K)} \]

\[ C_s \quad \text{concentration susceptibility} \]

\[ D \quad \text{mass diffusivity (m}^2/s) \]

\[ Df \quad \text{Dufour number} \]

\[ G, H \quad \text{dimensionless similarity velocity functions} \]

\[ Gr_L \quad \text{Grashof number} \]

\[ Ha^2 \quad \text{square of Hartmann number, } \alpha B_0^2 L^2/\mu \]

\[ k \quad \text{fluid thermal conductivity (W/m K)} \]

\[ k_t \quad \text{thermal diffusion ratio (kg/m}^3) \]

\[ K_1 \quad \text{dimensional chemical reaction (s}^{-1}) \]

\[ L \quad \text{characteristic length (m)} \]

\[ M \quad \text{magnetic parameter} \]

\[ N \quad \text{buoyancy ratio} \]

\[ Nu_x \quad \text{local Nusselt number} \]

\[ Pr \quad \text{Prandtl number} \]

\[ Re_L \quad \text{Reynolds number} \]

\[ Re_x \quad \text{local Reynolds number} \]

\[ Sc \quad \text{Schmidt number} \]

\[ Sh_x \quad \text{local Sherwood number} \]

\[ Sr \quad \text{Soret number} \]

\[ t \quad \text{dimensional time (s)} \]

\[ t' \quad \text{dimensionless time} \]

\[ T \quad \text{temperature (K)} \]

\[ T_m \quad \text{mean fluid temperature (K)} \]

\[ T_{w0} \quad \text{wall temperature at time } t = 0 \text{ (K)} \]

\[ u, v, w \quad \text{velocity components (m/s)} \]

\[ x, y, z \quad \text{curvilinear coordinates} \]

\[ \mu \quad \text{dynamic viscosity (kg/m}^1\text{s}) \]

\[ \nu \quad \text{kinematic viscosity (m}^2\text{s}) \]

\[ \rho \quad \text{fluid density (kg/m}^3) \]

\[ \sigma \quad \text{electrical conductivity} \]

\[ \theta \quad \text{dimensionless temperature} \]

\[ \Omega \quad \text{angular velocity of the cone (s}^{-1}) \]

\[ \Omega_0 \quad \text{angular velocity of the cone at } (t = 0, \text{ s}^{-1}) \]

**Subscripts**

\[ i \quad \text{initial condition} \]

\[ t, x, z \quad \text{partial derivatives with respect to } t, x \text{ and } z \text{ respectively} \]

\[ \text{w condition at the wall} \]

\[ \infty \quad \text{condition at free stream} \]

**REFERENCES**


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