MHD Flow and Heat Transfer of a Nanofluid Embedded with Dust Particles Over a Stretching Sheet

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A study has been carried out on boundary layer flow and heat transfer of an incompressible nanofluid with homogeneously suspended dust particles. The flow is generated due to linear stretching surface in the presence of uniform magnetic field. The governing partial differential equations are transformed into a set of non-linear ordinary differential equations using suitable similarity variables. Similarity solutions are obtained for these equations using efficient numerical technique Runge-Kutta-Fehlberg fourth-fifth order method. The similarity solutions which depend on nanoparticle volume fraction, magnetic parameter, fluid particle interaction parameter, Prandtl number, Eckert number are presented through graphs and tables and are discussed in detail.


1. INTRODUCTION

Heat transfer due to a continuously stretching surface through an ambient fluid is one of the thrust areas of current research. As they find their application over a broad spectrum of science and engineering processes, such as cooling of metallic plates in a cooling bath, the aerodynamic extrusion of plastic sheets, polymer sheet extruded continuously from a die and heat-treated materials that travel between feed and wind-up rolls or on a conveyer belt possesses or on the characteristics of a mobbing continuous surface. In doing so, it is important to investigate cooling and heat transfer for improvement of the final products. The conventional fluids such as water and air are amongst the most widely used fluids as a cooling medium.

With these existing demands on boundary layer flow and heat transfer Sakiadis1 in his pioneering work introduced the boundary layer flow of incompressible fluid over a stretched moving surface by employing similarity transformation to obtain numerical solution for the flow problem. Crane4 considered the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. Anderson et al.11 extended the work1 and investigated the magnetohydrodynamic flow of an electrically conducting power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field using an exact similarity transformation. Ariel et al.25 made an analytical approach using homotopy perturbation method for the problem of axisymmetric flow over a stretching sheet. Chakrabarti and Gupta5 have discussed the hydromagnetic flow and heat transfer over a stretching sheet. Grubka and Bobba6 analyzed the problem related with power law temperature distribution by obtaining linear differential equations and expressed their solutions in terms of Kummer’s function. Chiam15 in his work considered the boundary layer heat transfer in a two dimensional Newtonian fluid flow caused by linear and porous stretching sheet in the presence of blowing/suction. Banks4 presented a class of similarity solutions of the boundary-layer equations corresponding to the flow engendered solely by a stretching surface. An analytical solution to the equation of motion is given for steady laminar flow of an uniformly conducting incompressible non-Newtonian fluid was given by Sarpakaya.7

An analysis has been carried out by Abel et al.20 to study an unsteady, two-dimensional MHD (magnetohydrodynamic) boundary layer flow of a viscous incompressible and electrically conducting visco-elastic fluid under the influence of transverse magnetic field over a linearly stretching sheet.

In determining the particle accumulation and impingement on the surface, the study on boundary layer flow of fluid-particle suspension flow finds its importance. In view of this, Saffman3 formulated governing equations for the
flow of dusty fluid and has discussed the stability of the laminar flow of a dusty gas in which dust particles are uniformly distributed. Based on this model, Chakrabarti analyzed the boundary layer flow for a dusty gas. Datta and Mishra have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Numerical investigations were carried out by Vajravelu and Nayfeh analyzed the hydromagnetic flow of a dusty fluid over a stretching sheet which was carried out with a view to throw adequate light on the effect of fluid-particle interaction, particle loading, and suction on flow characteristics. Girasesha et al. in their work MHD flow and heat transfer of a dusty fluid over a stretching sheet.

For the last decade most of the Mathematicians gave their attention to the studies on nanofluid, since nanofluids are expected to have superior properties compared to conventional heat transfer fluids. These nanofluids are the mixture of nanoparticles/layers in a base fluid. Based on these, Choi et al. conducted the initial investigations on thermal conductivity of nanoparticle fluid mixture and found the enhancement in the thermal conductivity for fluid with nanoparticles, which lead to the research development in the field of nanofluids. The thermal conductivity of nanofluids has dominated the literature in the past decade, though this pattern has changed slightly over last few years. Temperature dependence of thermal conductivity enhancement for nanofluids was investigated by Das et al. and Khan and Pop analyzed the development of steady boundary layer flow, heat transfer and nanoparticle fraction over a stretching surface in a Nanofluid. Hassan et al. investigated the analytical solutions for the boundary layer flow of a nanofluid past a stretching surface.

To the best of author’s knowledge, the problem of magneto-hydro-dynamic flow of nanofluid with uniform distribution of dust particles has remained unexplored. So the main objective of this paper is to present numerical solution to the problem on steady, laminar, two dimensional boundary layer flow of an incompressible conducting dusty nanofluid with heat transfer over a linear stretching sheet in the presence of magnetic field.

2. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional, laminar, magnetohydrodynamic, boundary layer flow of a dusty nanofluid over a stretching sheet coinciding with the plane \( y = 0 \) and the flow being confined to \( y > 0 \). The schematic diagram of the problem is shown in Figure 1. The flow is generated due to the linear stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the \( x \)-axis. Keeping the origin fixed, the sheet is then stretched with a velocity \( U(x) = cx \), where \( c \) is constant. Further the flow field is exposed to the influence of an external transverse magnetic field of strength \( B_0 \) (along \( y \)-axis). The dust particles are assumed to be uniform in size and number density of the dust particle is taken as a constant throughout the flow.

![Figure 1. Schematic representation of boundary layer flow.](image)

Under the above assumption, the basic two-dimensional boundary layer equations are as

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_f}{\rho_f} \frac{\partial^2 u}{\partial y^2} + \frac{K}{\rho_f} (u_p - u) - \frac{\sigma B_0^2}{\rho_f} u
\]

\[
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u_p}{\partial x} + \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p)
\]

where \((u, v)\) and \((u_p, v_p)\) are the velocity components of nanofluid and dust phases along \( x \) and \( y \) directions, respectively. \( K = 6\pi \mu a \) is called Stokes drag constant, \( N \) is the number density of the dust particles, \( m \) is the mass of the dust particle, \( \sigma \) is the electrical conductivity, \( B_0 \) is uniform magnetic field strength, \( \rho_f \) is the effective density of the nanofluid, \( \mu_f \) is the effective dynamic viscosity of the nanofluid, \( k_{nf} \) is the thermal conductivity and \((\rho c_p)_{nf}\) is the heat capacitance of the nanofluid, which are defined as

The effective density of the nanofluid \((\rho_{nf})\) and the heat capacitance of the nanofluid \((\rho c_p)_{nf}\) are given by

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p
\]

and

\[
(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_p
\]

Also, the effective dynamic viscosity of the nanofluid by Brinkman and the thermal conductivity of the nanofluid \( k_{nf} \) for spherical nanoparticles as by Maxwell are defined as

\[
\mu_{nf} = \frac{\mu_f (1 - \phi)^{\frac{2}{3}}}{\frac{k_f}{k_f} + 2\phi (k_f - k_p) + 2\phi (k_f - k_p)}
\]

\[
k_{nf} = \frac{k_f + 2k_f - 2\phi (k_f - k_p)}{k_f + 2k_f + 2\phi (k_f - k_p)}
\]
Table I. Thermophysical properties of water and nanoparticle.

<table>
<thead>
<tr>
<th></th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C_p ) (j/kgK)</th>
<th>( k ) (W/mK)</th>
<th>( \beta ) (k(^{-1}))</th>
<th>( \sigma ) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>21 \times 10(^{-5})</td>
<td>0.05</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>8933</td>
<td>385</td>
<td>401</td>
<td>1.67 \times 10(^{-5})</td>
<td>5.96 \times 10(^{-7})</td>
</tr>
</tbody>
</table>

Here, \( \phi \) is the solid volume fraction, \( \mu_j \) is the dynamic viscosity of the base fluid, \( \rho_j \) and \( \rho \) are the densities of the base fluid and nanoparticle, respectively, \( k_j \) and \( k \) are the thermal conductivities of the base fluid and nanoparticle, respectively. The thermophysical properties of water and the Copper nanoparticles are given in Table I.

The boundary conditions are

\[
\begin{align*}
  u &= U_{w}(x), \quad v = 0, \quad T = T_w \quad \text{at } y = 0 \\
  u &= v = 0, \quad u_p \rightarrow 0, \quad v_p \rightarrow v, \quad \text{as } y \rightarrow \infty
\end{align*}
\]  

(6)

To convert governing equations into a set of similarity equations, we introduce following similarity transformations,

\[
\begin{align*}
  u &= cxf'(\eta), \quad v = \sqrt{v_f}cf(\eta), \quad \eta = \frac{c}{v_f}y \\
  u_p &= cxf'(\eta), \quad v_p = \sqrt{v_f}cF(\eta)
\end{align*}
\]  

(7)

Equation (7) identically satisfies (1) and (3) and on substituting these in (2) and (4), we obtain the following non-linear ordinary differential equations,

\[
\begin{align*}
  f'''' + (1 - \phi)^{2.5} \left[ \left( 1 - \phi \right) + \frac{\phi \beta}{\rho_j} \right] (f'' - f'^2) \\
  + (1 - \phi)^{2.5} \left[ \beta F'' - f' \right] - Mf' &= 0 \\
  F'' - F'' - \beta F' - F' &= 0
\end{align*}
\]  

(8)

(9)

where \( \eta \), \( l = (mN)/\rho_j \) is the mass concentration, \( \tau_p = (m/K) \) is the relaxation time of the particle phase, \( \beta = (1/(cT_{p})) \) is the fluid particle interaction parameter for velocity \( M = (\sigma B_0^2)/\rho_{np} \) is the magnetic parameter and \( v_f \) is kinematic viscosity.

The boundary condition defined as in (6) will take the following form,

\[
\begin{align*}
  f'(\eta) &= 1, \quad f(\eta) = 0 \quad \text{at } \eta = 0 \\
  f''(\eta) &= 0, \quad F'(\eta) = 0, \quad F(\eta) = f(\eta) \quad \text{as } \eta = \infty
\end{align*}
\]  

(10)

3. HEAT TRANSFER ANALYSIS

The governing of nanofluid boundary layer heat transport equations for both nanofluid and dust phase are given by,

\[
\begin{align*}
  (\rho c_p)_{nf} \left[ \frac{\partial T}{\partial x} + \frac{v}{\tau_p} \frac{\partial T}{\partial y} \right] &= k_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{\tau_T} (T_p - T) \\
  \frac{u_p \partial T_p}{\partial x} + \frac{v_p \partial T_p}{\partial y} &= - \frac{c_{pf}}{\epsilon_{nf} \tau_T} (T_p - T)
\end{align*}
\]  

(11)

(12)

where \( T \) and \( T_p \) are the temperatures of the nanofluid and dust particles, \( c_{pf} \) and \( c_{nf} \) are the specific heat of nanofluid and dust particles, \( \tau_T \) is the thermal equilibrium time i.e., the time required by a dust cloud to adjust its temperature to the nanofluid, \( k_{nf} \) is the thermal conductivity, \( \tau_{p} \) is the relaxation time of the dust particle.

The appropriate boundary conditions are

\[
\begin{align*}
  T &= T_w = T_\infty + A \left( \frac{x}{l} \right)^2 \quad \text{at } y = 0 \\
  T &\rightarrow T_\infty, \quad T_p \rightarrow T_\infty \quad \text{as } y \rightarrow \infty
\end{align*}
\]  

(13)

where \( A \) is a positive constant, which depends on the properties of the fluid and \( l \) is the characteristic length and is given by \( l = \sqrt{(v_f/c)} \).

Defining the non-dimensional nano fluid phase temperature \( \theta(\eta) \) and dust phase temperature \( \theta_p(\eta) \) as,

\[
\begin{align*}
  \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}
\end{align*}
\]  

(14)

Using (13) and (14) into (11) and (12), we obtain

\[
\begin{align*}
  1 - k_{nf} \frac{\partial \theta}{\partial \eta} + 2 \left( 1 - \phi \right) + \frac{\phi \beta}{\rho_j} \left( \frac{f'''}{f''} - 2f'' \right) \\
  + \frac{\beta}{\rho_j} \left[ \beta F'' - f' \right] - Mf' &= 0 \\
  + \frac{\beta}{m} \left[ \beta F' - 2F' - \gamma \beta \right] + \frac{\beta}{m} \left[ \beta F'' - f' \right]^2 &= 0
\end{align*}
\]  

(15)

(16)

where \( Pr = \frac{(\mu c_p)_{nf}}{k_{nf}} \) is Prandtl number, \( Ec = \frac{c_{pf}^2}{\rho_{np} c_{nf}} \) is Eckert number, \( \beta_T = (1/(cT_{p})) \) is fluid particle interaction parameter for temperature and \( \gamma = \frac{c_{pf}}{c_{nf}} \) is ratio of specific heat.

The corresponding thermal boundary conditions becomes

\[
\begin{align*}
  \theta(\eta) &= 1 \quad \text{at } \eta = 0, \quad \theta(\eta) \rightarrow 0, \\
  \theta_p(\eta) &\rightarrow 0 \quad \text{as } \eta \rightarrow \infty
\end{align*}
\]  

(17)

Table II. Comparison of the results of \( f'(0) \) for Cu-water for various values of solid volume fraction of nanoparticles (\( \phi \)) when \( \beta = M = l = 0 \).

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>Vajravelu et al.\textsuperscript{13}</th>
<th>Kalidas et al.\textsuperscript{17}</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-1.001411</td>
<td>-1.001411</td>
<td>-1.001396</td>
</tr>
<tr>
<td>0.1</td>
<td>-1.175209</td>
<td>-1.175251</td>
<td>-1.175264</td>
</tr>
<tr>
<td>0.2</td>
<td>-1.218301</td>
<td>-1.218315</td>
<td>-1.218340</td>
</tr>
</tbody>
</table>
Table III. Comparison of the results for the dimensionless temperature gradient \( \Theta (0) \) for various values of \( Pr \) in the case of \( \beta = 0 \) and \( Ec = 0 \).

<table>
<thead>
<tr>
<th>Pr</th>
<th>Grubka et al.(^9)</th>
<th>Chen et al.(^14)</th>
<th>Abel et al.(^10)</th>
<th>Ishak et al.(^11)</th>
<th>Dural et al.(^13)</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>-1.0885</td>
<td>-1.0885</td>
<td>-1.0885</td>
<td>-</td>
<td>-</td>
<td>-1.0885</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.3333</td>
<td>-1.3333</td>
<td>-1.3333</td>
<td>-1.3333</td>
<td>-1.3333</td>
<td>-1.3333</td>
</tr>
<tr>
<td>10.0</td>
<td>-4.7968</td>
<td>-4.7968</td>
<td>-4.7968</td>
<td>-4.7968</td>
<td>-4.7968</td>
<td>-4.7968</td>
</tr>
</tbody>
</table>

Table IV. Values of wall temperature gradient \( \Theta (0) \) for different values of the parameters \( Pr, Ec, M, \phi, \) and \( \beta \).

<table>
<thead>
<tr>
<th>( Pr )</th>
<th>( Ec )</th>
<th>( \beta )</th>
<th>( l )</th>
<th>( M )</th>
<th>( \phi )</th>
<th>( \Theta (0) )</th>
</tr>
</thead>
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<tr>
<td>2.2</td>
<td>2.0</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.2</td>
<td>-1.481944</td>
</tr>
<tr>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2.195169</td>
</tr>
<tr>
<td>6.2</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
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<td>-3.865980</td>
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<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6.2</td>
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<td>1.0</td>
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<tr>
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<td></td>
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<td>6.2</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
<td>-2.713439</td>
</tr>
</tbody>
</table>

Fig. 3. Effect of \( \beta \) on temperature profiles.

4. NUMERICAL SOLUTION

Equations (8), (9), and (15), (16) together with the boundary conditions (10) and (17) forms highly non-linear ordinary differential equations. These equations are solved numerically using the well known Runge-Kutta-Fehlberg method of fourth-fifth order. In this method, we choose suitable finite values of \( \eta \rightarrow \infty \) as \( \eta = 5 \).

Comparative studies on \( f''(0) \) obtained results for these by Vajravelu et al.\(^{13}\) and Kalidas Das\(^{22}\) are shown in Table II, for various values of \( \phi \). Table III is framed to compare the result of \( \Theta (0) \) with Grubka et al.\(^9\) Abel et al.\(^{10}\) Ishak et al.\(^{11}\) and Dural et al.\(^{13}\) for various values of \( Pr \). With the present studies from these tables, one can notice that there is a close agreement with these approaches and thus verifies the accuracy of the method used. The results of thermal characteristics at the wall are examined for the values of the temperature gradient function \( \Theta (0) \), which is tabulated in Table IV. The effects of various physical parameters such as fluid particle interaction parameter (\( \beta \)), Prandtl number (\( Pr \)), Eckert number (\( Ec \)), magnetic parameter (\( M \)) and solid volume fraction parameter (\( \phi \)) are examined and are discussed in detail.
5. RESULTS AND DISCUSSION

In the present paper, investigations are carried out on the boundary layer flow and heat transfer of a nanofluid embedded with dusty particles over a stretching sheet. The boundary layer equations for momentum and heat transfer are solved numerically using Runge-Kutta-Pehlberg fourth-fifth order method. Fluid particle interaction parameter $\beta$, Prandtl number ($Pr$), Eckert number ($Ec$), Magnetic parameter ($M$), solid volume fraction of nanoparticle parameter $\phi$ are some of the parameters, whose effects on the flow and heat transfer analysis discussed below.

It is evident from the Figure 2 that the flow is parabolic in nature and we can see that the flow of nanofluid particles is parallel to that of dust. The velocity of both fluid and dust particles, which are nearer to the axis of flow, move with the greater velocity. Further observation shows the effect of nanofluid interaction parameter $\beta$ of nanoparticle
on velocity components \( f'(\eta) \) (fluid phase) and \( F'(\eta) \) (dust phase), i.e., if \( \beta \) increases, one can find the decrease in fluid phase velocity and increase in dust phase velocity. Also, it reveals that for large values of \( \beta \) the velocity of both nanofluid and dust particles will be same.

Figure 3 depicts that temperature profiles \( \theta(\eta) \) and \( \theta_d(\eta) \) for different values of fluid-particle interaction parameter \( \beta \). We infer from this figure, that the temperature increases with increase in fluid-particle interaction parameter \( \beta \) which indicates that the nanofluid-particle temperature is parallel to that of dust phase. Also it is observe that the nanofluid phase temperature is higher than that of dust phase.

Figures 4 and 5 show the velocity and temperature profiles for various values of the magnetic parameter in the case of both fluid and dust phase of nanofluid, respectively. It is noticed that in both the case, velocity along the surface decreases while the temperature increases with the increasing value of magnetic parameter. Thus the presence of magnetic field decreases the momentum boundary layer thickness and increases the thermal boundary layer thickness. It is also observed that, for a specific value of \( M \) and at each position, the corresponding value of velocity in presence of nanoparticles is smaller than the value of the velocity observed for pure nanofluid, while the value of for temperature profile is opposite to that of velocity profile. Physically, the temperature increases for Cu-nanoparticle, because copper has high thermal conductivity.

Figure 6 represents temperature profiles for different values of \( Pr \). The relative thickening of momentum and thermal boundary layers are controlled by Prandtl number \( (Pr) \). By analyzing the graph it reveals that the \( Pr \) is increased to decrease the temperature distribution in the flow region. Which implies that momentum boundary layer is thicker than the thermal boundary layer, in both nanofluid and dust phase and the temperature of these asymptotically approaches to zero in free stream region.

Figure 7 is plotted for the temperature distribution for different values of \( Ec \). We observed that the effect of increasing values of Eckert number is to increase the wall temperature of both nanofluid and dust phase temperatures.

Figures 8 and 9 are presented to show the effect of volume fraction of nanoparticles (\( \alpha \)) on velocity and temperature distributions for both fluid and dust phase. When the volume fraction of nanoparticles increases from 0 to 0.2, the velocity profile for both fluid and dust phase decreases inside the boundary layer, while they increase outside. Also, we can see that, by increasing volume fraction of nanoparticles, the thermal boundary layer is increased for both fluid and dust phase. This agrees with the physical behavior that when the volume fraction of copper increases, the thermal conductivity increases, and thermal boundary layer thickness increases.

Figure 10 shows the velocity profile for different type of fluid in the region of uniform magnetic field. It is observed that the velocity profile for normal fluid is much higher than that of nanofluid, dusty fluid and dusty nanofluid, respectively in their order. This result is well evident to say that dusty nanofluid has high thermal conductivity than dusty fluid, nanofluid and ordinary fluid.
Figure 11 is depicted to show the behavior of mass concentration of dust particles (J) on flow and heat transfer profiles. It can be seen that an increase in J decreases both the momentum and thermal boundary layer thickness in both nanofluid and dust phase. As a result of which fluid velocity and the temperature of nanofluid decreases with the increase in mass concentration.

6. CONCLUSIONS

The present work deals with the boundary layer flow and heat transfer of a nanofluid embedded with dust particles over a stretching sheet. The effect of magnetic parameter and solid volume fraction on the flow and heat transfer characteristics are determined for dusty fluid with Cu nanoparticles and the outcomes are listed below:

- Velocity of fluid phase decreases and dust phase increases as β increases.
- Increase in the value of M, decreases the fluid and dust phase velocity and increase the temperature field on these fluids.
- Increasing value of Pr, decreases the temperature of both nanofluid and dust phase.
- Temperature of nanofluid and dust phase increases with increasing values of Ec.
- Velocity of nanofluid and dust phase decreases and temperature of nanofluid and dust phase increases as solid volume fraction of nanoparticle φ increases.
- It is found that the dusty fluid with Copper (Cu) nanoparticle have the appreciable cooling performance.

References and Notes