Unsteady coupled heat and mass transfer by mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface in the presence of radiation and chemical reaction

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Abstract: The effects of radiation and chemical reaction on coupled heat and mass transfer by unsteady mixed convection boundary-layer flow of a micropolar fluid near the region of the stagnation point on a double-infinite vertical flat plate are studied. The free stream velocity and the surface temperature and concentration are assumed to vary linearly with the distance along the surface. The flow is impulsively set into motion and both of the temperature and concentration at the surface are also suddenly changed from that of the ambient fluid. The governing partial differential equations are transformed into a set of non-similar equations and solved numerically by an efficient implicit, iterative, finite-difference method. Various comparisons with previously published work are performed and the results are found to be in excellent agreement. A representative set of numerical results for the velocity, angular velocity, temperature and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number is presented graphically for various parametric conditions and discussed.

Keywords: heat and mass transfer; micropolar fluid; thermal radiation; chemical reaction; unsteady mixed convection.

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1 Introduction

In recent years, the dynamics of micropolar fluids, originated from the theory of Eringen (1966, 1972) has been a popular area of search. This theory takes into account the effect of local rotary inertia and couple stresses arising from practical microrotation. The theory is expected to provide a mathematical model for the non-Newtonian fluid behaviour observed in certain man-made liquids such as polymers, colloidal suspensions, fluids with additives, suspension solutions, and animal blood, etc. This theory is also capable of explaining the experimentally observed phenomena of drag reduction near a rigid body in fluids containing small amount of additives when compared with the skin friction in the same fluids without additives. Two-dimensional boundary layer flow caused by a moving surface or a stretching sheet is of interest in the manufacture of sheeting material through an extrusion process. An excellent early review about micropolar fluid mechanics was provided by Ariman et al. (1973, 1974).

EL-Hakiem et al. (2000) studied natural convection from combined thermal and mass diffusion buoyancy effects in micropolar fluids. Mansour et al. (2000) discussed heat and mass transfer on magnetohydrodynamic flow of micropolar fluids in a circular cylinder with uniform heat and mass flux. Bhargava and Takhar (2000) presented numerical study of heat transfer characteristics of the micropolar boundary layer near a stagnation point on a moving wall. Siddheshwar and Manjunath (2000) studied unsteady convective diffusion with heterogeneous chemical reaction in a plane-Poiseuille flow of a micropolar fluid. Ibrahim and Hassanien (2001) analysed a local non-similarity solutions for mixed convection boundary layer flow of a micropolar fluid on horizontal flat plates with variable surface temperatures. The problem of heat and mass transfer by free convection flow over a cone with uniform suction or injection in micropolar fluid was considered by EL-Kabeir et al. (2006). Cheng (2008) presented a boundary-layer analysis about the natural convection heat transfer near a vertical truncated cone with power-law variation in surface temperature in a micropolar fluid. EL-Kabeir et al. (2011) studied heat transfer in a micropolar fluid flow past a permeable continuous moving surface.

Moreover, combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet-cooling tower, and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of chemical reactions. A homogeneous chemical reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous chemical reaction is analogous to internal source of heat generation. In contrast, a heterogeneous chemical reaction takes place in a restricted region or within the boundary of a phase. It can, therefore, be treated as a boundary condition similar to the heat flux condition in heat transfer. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Muthucumarswamy and Ganesan (2001, 2002) solved the problem of unsteady flow past an impulsively started vertical plate with uniform heat and mass flux and variable temperature and mass flux, respectively. EL-Kabeir and Modather (2007) presented a study on chemical reaction, heat and mass transfer on MHD flow over a vertical isothermal cone surface in micropolar fluids with heat generation/absorption. Modather et al. (2009) studied the effect of chemical reaction on MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. Rashad and EL-Kabeir (2010) studied the coupled heat and mass transfer in transient flow by mixed convection past a vertical stretching sheet embedded in a fluid-saturated porous medium in the presence of a chemical reaction effect. Magyari and Chamkha (2010) considered the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface.

Finally, the effect of thermal radiation with chemical reactions on coupled heat and mass transfer has importance in such processes as the combustion of fossil fuels, atmospheric re-entry with suborbital velocities, plasma wind tunnels, electric spacecraft propulsion, hypersonic flight through planetary atmosphere photo-dissociation, photo ionisation, and geophysics. Analytical solutions for the overall heat and mass transfer on MHD flow of a uniformly stretched vertical permeable surface with the effects of heat generation/absorption and chemical reaction were presented by Chamkha (2003). The effects of radiation and chemical reaction on MHD free convective flow and mass transfer past a vertical isothermal cone surface were investigated by Afify (2004). Kandasamy et al. (2005) studied the non-linear MHD flow with heat and mass transfer characteristics on a vertical stretching surface with chemical reaction and thermal stratification effects. The effects of chemical reaction and the thermal radiation on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media in the presence of variable viscosity and magnetic field were considered by Seddeek et al. (2005). Mohamed and Abo-Dahab (2009) studied the effects of first-order chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar fluid flow over a vertical moving porous plate through a porous medium. Bakr (2011) explained the effect of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Chamkha et al. (2011) studied unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid with Joule heating, chemical reaction and radiation effects.

In the present work, the problem of unsteady heat and mass transfer mixed convection boundary-layer flow of a micropolar fluid near the region of the stagnation point on a double-infinite vertical flat plate in the presence of chemical reaction and thermal radiation effects is considered. The governing boundary-layer equations have been transformed into a non-similar form, and these have been solved numerically by an efficient implicit, iterative, finite-difference method. The effects of thermal radiation, chemical reaction and micropolar parameters on the velocity, microrotation, temperature and concentration profiles as well as the local skin-friction coefficient, local couple stress, local Nusselt number and the local Sherwood number have been shown graphically and discussed.

2 Governing equations

Consider unsteady, laminar, heat and mass transfer by mixed convection boundary layer flow on a double-infinite vertical flat plate which is placed in a micropolar fluid of uniform ambient temperature and concentration T_∞ and C_∞ with assisting external laminar flow in the presence of thermal radiation and chemical reaction effects. It is assumed that at time $t = 0$ the external flow starts in motion impulsively from rest towards the plate with a steady velocity $u_e(x)$. The flow configuration is shown schematically in Figure 1 together with the corresponding Cartesian coordinates in the vertical and horizontal directions. Either heating or cooling of the plate is assumed to begin simultaneously with the motion of the external stream. It is further assumed that the temperature and concentration of the plate $T_w(x)$ and $C_w(x)$ vary linearly with the distance x along the plate. Therefore, the flow and thermal field are no longer symmetric with respect to the stagnation line about the centreline plane, which contains the stagnation point. Thus, the wall temperature and concentration and the condition far from the plate surface are assumed to be given by $T_w(x) + T_\infty + T_0(x/L)$, $C_w(x) = C_\infty + C_0(x/L)$, and $u_e(x) = U_e(x/L)$ where u_e is a reference velocity, L is a characteristic length, $T_0 > 0$ is a reference temperature and $C_0 > 0$ is a reference concentration. The fluid is assumed to have constant properties except the density in the body force term. Under these assumptions along with the Boussinesq approximation, the unsteady laminar boundary layer equations governing the mixed convection flow are given by (see Lok et al., 2006):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \left(\frac{\mu + k}{\rho}\right)\frac{\partial^2 u}{\partial y^2} \\ &+ \frac{k}{\rho}\frac{\partial N}{\partial y} + g\left[\beta_T\left(T - T_\infty\right) + \beta_C\left(C - C_\infty\right)\right], \end{aligned} \tag{2}$$

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right), \tag{3}$$

$$\begin{aligned} &\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\frac{\partial^2 T}{\partial y^2} \\ &+ \frac{16\sigma}{3\left(a_r + \sigma_s\right)\rho C_p}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right), \end{aligned} \tag{4}$$

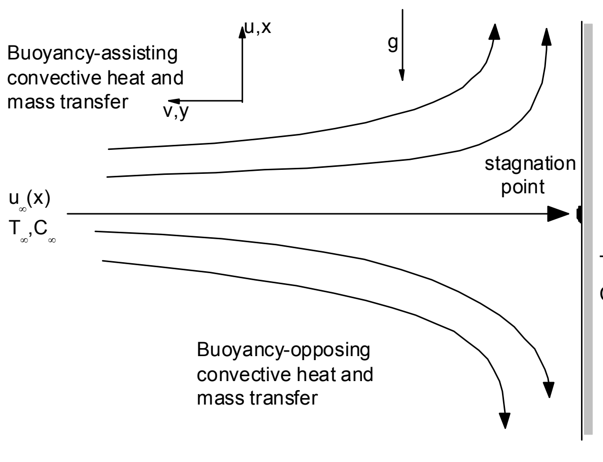
$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_C\left(C - C_\infty\right). \tag{5}$$

The corresponding initial and boundary conditions for this problem can be written as:

$$
\begin{aligned}
& t \prec 0: u(x, y) = v(x, y) = 0, N(x, y) = 0, T(x, y) = T_\infty, C(x, y) = C_\infty \quad \text{at any } x, y \\
& t \geq 0: u(x, 0) = v(x, 0) = 0, N(x, 0) = -n\frac{\partial u}{\partial y}(x, 0), \qquad x \geq 0 \\
& T(x, 0) = T_w(x) = T_\infty + T_0\frac{x}{L}, C(x, 0) = C_w(x) = C_\infty + C_0\frac{x}{L}, \qquad x \geq 0 \\
& u(x, \infty) = u_e(\text{x}) = U_e\frac{x}{L}, N(x, \infty) = 0, \qquad x \geq 0 \\
& T(x, \infty) = T_\infty, C(x, \infty) = C_\infty \qquad x \geq 0
\end{aligned} \tag{6}
$$

where u and v are the velocity components along x and y axes, N is the component of the microrotation vector normal to the x–y plane, T is the fluid temperature, C is the fluid concentration, g is the magnitude of the acceleration due to gravity, ρ is the density, μ is the absolute viscosity, κ is the vortex viscosity, γ^* is the spin-gradient viscosity, υ is the kinematic viscosity, j is the microinertia density, D is the mass diffusivity, σ_s is the scattering coefficient, a_r is Rosseland mean extinction coefficient, k_c is the chemical reaction, Pr is the Prandtl number and n is a constant $0 \leq n \leq 1$. It should be mentioned that the case $n = 0$, called weak concentration by Guram and Smith (1980), which indicates $N = 0$ near the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena and Mathur, 1981). The case $n = 1/2$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentrations (Ahmadi, 1976). The case $n = 1$, as suggested by Peddieson (1972), is used for the modelling of turbulent boundary layer flows.

Figure 1 Problem schematics and coordinate system



The governing equations and boundary conditions can be made dimensionless by introducing the stream function such that

$$
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{7}
$$

and using the following dimensionless variables

$$
\begin{aligned}
& \eta = \left(\frac{U_e}{L\nu}\right)^{\frac{1}{2}} \xi^{-\frac{1}{2}} y, \xi = 1 - e^{-t^*}, t^* = \frac{U_e}{L}t, \\
& u(x, y, t) = \frac{U_e x}{L} f'(\xi, \eta), v(x, y, t) = -\left(\frac{U_e \nu}{L}\right)^{\frac{1}{2}} \xi^{\frac{1}{2}} f(\xi, \eta) \\
& N(x, y, t) = \frac{U_e x}{L}\left(\frac{U_e}{L\nu}\right)^{\frac{1}{2}} \xi^{-\frac{1}{2}} h(\xi, \eta) \\
& T(x, y, t) = T_\infty + T_0\left(\frac{x}{L}\right)\theta(\xi, \eta) \\
& C(x, y, t) = C_\infty + C_0\left(\frac{x}{L}\right)\phi(\xi, \eta)
\end{aligned} \tag{8}
$$

Following the work of many recent authors (Rees and Bassom, 1996; Rees and Pop, 1998), γ^* is assumed to be given by

$$
\gamma^* = \left(\mu + \frac{k}{2}\right) j = \mu\left(1 + \frac{K}{2}\right) j. \tag{9}
$$

Substituting equation (9) into equations (2) to (5) yields:

$$
\begin{aligned}
& (1+K) f''' + Kh' + \frac{1}{2}\eta(1-\xi) f'' + \xi f f'' + \xi(1 - f'^2) \\
& + \lambda\xi(\theta + \Lambda\phi) = \xi(1-\xi)\frac{\partial f'}{\partial \xi},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
& \left(1 + \frac{K}{2}\right) h'' + \frac{1}{2}\eta(1-\xi) h' + \xi(f h' - f'\,h) + \frac{1}{2}(1-\xi) h \\
& - KB\xi(2h + f'') = \xi(1-\xi)\frac{\partial h}{\partial \xi},
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
& \frac{1}{\Pr}\left(1 + \frac{4}{3} R_d \left(c_T + \theta\right)^3\right)\theta'' + \frac{4R_d}{\Pr}\left(c_T + \theta\right)^2 \theta'^2 \\
& + \frac{1}{2}\eta\left(1-\xi\right)\theta' + \xi\left(f\theta' - f'\theta\right) = \xi(1-\xi)\frac{\partial \theta}{\partial \xi},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
& \frac{1}{Sc}\phi'' + \frac{1}{2}\eta\left(1-\xi\right)\phi' + \xi\left(f\phi' - f'\phi\right) \\
& - \gamma\xi\phi = \xi(1-\xi)\frac{\partial \phi}{\partial \xi},
\end{aligned} \tag{13}
$$

where equation (2) is identically satisfied. In equations (11) to (14), a prime indicates differentiation with respect to η, ξ is the dimensionless time ranges from 0 to 1 ($0 \le \xi \le 1$), $\Lambda = \frac{(C_w - C_\infty)\beta_C}{(T_w - T_\infty)\beta_T}$ is the concentration to thermal buoyancy ratio, $\lambda = \frac{Gr}{\mathrm{Re}^2}$ is the mixed convection parameter ($\lambda > 0$), $Gr = \frac{g\beta_T T_0 L^3}{v^2}$ is the Grashof number, $Re = \frac{U_e L}{v}$ is the Reynolds number, $Sc = \frac{v}{D}$ is Schmidt number, $\mathrm{Pr} = \frac{v}{\alpha}$ is Prandtl number, $B = \frac{Lv}{U_e j}$ is the spin gradient viscosity parameter, $K = \frac{k}{\mu}$ is the vortex viscosity parameter, $\gamma = \frac{k_C L}{U_e}$ is the dimensionless chemical reaction parameter, $R_d = \frac{4\sigma(T_w - T_\infty)^3}{[k_1(a_r + \sigma_s)]}$ is the thermal radiation parameter and $c_T = \frac{T_\infty}{T_w - T_\infty}$ is the surface temperature parameter.

The transformed initial and boundary conditions become:

$$
\begin{aligned}
&f(\xi,0) = f'(\xi,0) = 0, \theta(\xi,0) = 1, \phi(\xi,0) = 1,\\
&h(\xi,0) = -nf''(\xi,0)\\
&f'(\xi,\infty) = 1, \theta(\xi,\infty) = 0, \phi(\xi,\infty) = 0, h(\xi,\infty) = 0
\end{aligned} \tag{14}
$$

The local skin-friction coefficient C_f, local Nusselt number Nu, local wall couple stress M_w, and the local Sherwood number Sh are important physical properties. These can be defined in dimensionless form below:

$$
\begin{aligned}
C_f &= \frac{x/L}{\frac{1}{2}\rho u_e^2}\left[(\mu + k)\frac{\partial u}{\partial y} + kN\right]_{y=0}\\
&= 2\xi^{-\frac{1}{2}}\,\mathrm{Re}^{-\frac{1}{2}}\left[1 + (1-n)K\right]f''(\xi,0),
\end{aligned} \tag{15}
$$

$$
M_w = \frac{\gamma^* U_e^2 x}{\xi L^2 v}h'(\xi,0) = \frac{\gamma^* u_e(x)\mathrm{Re}}{\xi L^2}h'(\xi,0), \tag{16}
$$

$$
\begin{aligned}
Nu &= -\frac{L}{k_1(T_w - T_\infty)}\left[\left(k_1 + \frac{16\sigma T^3}{3(a_r + \sigma_s)}\right)\frac{\partial T}{\partial y}\right]_{y=0}\\
&= \xi^{-\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left[-\theta'(\xi,0)\left(1 + \frac{4R_d}{3}(1 + c_T)^3\right)\right],
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
Sh &= \frac{L}{C_w - C_\infty}\left[-\frac{\partial C}{\partial y}\right]_{y=0}\\
&= \xi^{-\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left[-\phi'(\xi,0)\right],
\end{aligned} \tag{18}
$$

3 Numerical method

The initial-value problem represented by equations (10) through (13) is non-linear and possesses no analytical solution. Therefore, a numerical solution is sought for this problem. The standard implicit, iterative, finite-difference method discussed by Blottner (1970) has proven to be adequate and accurate for this type of problems and therefore, it is chosen for the solution of equations (10) to (13) subject to equation (14). The computational domain is divided into 196 by 196 nodes in the ξ and η directions, respectively. Since the changes in the dependent variables are large in the immediate vicinity of the plate while these changes decrease greatly as the distance away from the plate increases, variable step sizes in the η direction are used. For the same reason, variable step sizes in the ξ direction are also employed. The initial step sizes employed were $\Delta\eta_1 = 0.001$ and $\Delta\xi_1 = 0.01$ and the growth factors were $K_\eta = 1.0375$ and $K_\xi = 1.0375$ such that $\Delta\eta_n = K_\eta\,\Delta\eta_{n-1}$ and $\Delta\xi_m = K_\xi\,\Delta\xi_{m-1}$. The convergence criterion used was based on the relative difference between the current and the previous iterations which was set to 10^{-5} in the present work. These values were found to give accurate grid-independent results as verified by the comparison mentioned below. For more details on the numerical procedure, the reader is advised to read the paper by Blottner (1970).

Table 1 Comparison of $f''(\xi,0)$ and $-\theta'(\xi,0)$ for different values of K and Pr with ($N = R_d = \gamma = 0$) at $\lambda = \xi = 1.0$ (steady-state flow)

Pr	*K*	*Lok et al. (2006)*		*Present results*	
		$f''(1,0)$	$-\theta'(1,0)$	$f''(1,0)$	$-\theta'(1,0)$
0.7	0	1.7064	0.7641	1.706493	0.764703
	1.0	1.1346	0.6957	1.134629	0.696394
	2.0	0.8798	0.6556	0.879789	0.657557
	3.0	0.7327	0.6300	0.732708	0.630615
7.0	0	1.5180	1.7226	1.518071	1.723200
	1.0	1.0106	1.5352	1.010596	1.536187
	2.0	0.7849	1.4315	0.784820	1.432323
	3.0	0.6549	1.3621	0.654854	1.363039
20	0	1.4486	2.4577	1.448609	2.458084
	1.0	0.9677	2.1807	0.967679	2.181632
	2.0	0.7528	2.0289	0.752855	2.027991
	3.0	0.6290	1.9287	0.629073	1.925720
40	0	1.4101	3.1023	1.410183	3.102175
	1.0	0.9443	2.7518	0.944433	2.746966
	2.0	0.7354	2.5624	0.735711	2.550047
	3.0	0.6149	2.4393	0.615327	2.418933

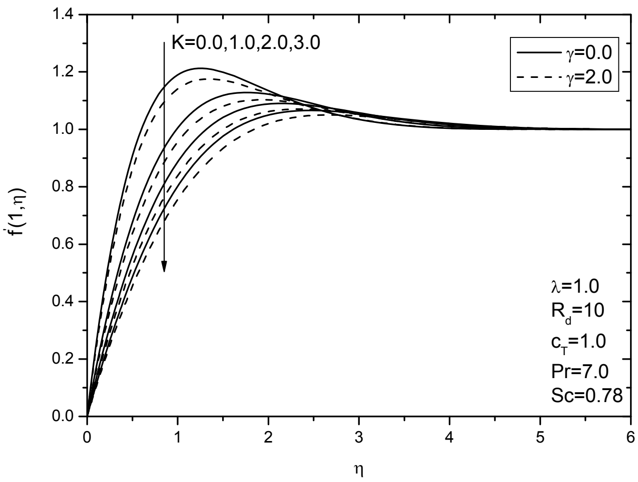
In order to access the accuracy of the numerical results, a comparison with previously published work reported by Lok et al. (2006) for the case of steady state flow ($\xi = 1$) in the absence of the thermal radiation, buoyancy force and chemical reaction effects ($R_d = N = \gamma = 0$) was performed.

This comparison is presented in Table 1. It is obvious from this table that excellent agreement between the results exists. This favourable comparison lends confidence in the graphical results to be reported in the next section.

4 Results and discussion

In this section, a detailed parametric study has been performed and the effects of the mixed convection parameter λ, micropolar vortex viscosity parameter K, thermal radiation parameter R_d, and dimensionless chemical reaction parameter γ, on the velocity, microrotation, temperature, and concentration profiles as well as the skin-friction coefficient, wall couple stress M_w, Nusselt number Nu and the Sherwood number Sh are presented graphically in Figures 2 through 17. All data are provided in the legends of these figures correspond to a micropolar fluid having a high vortex viscosity (K = 2), i.e., strongly non-Newtonian (unless otherwise indicated) with the case n = 0, i.e., weak concentration and B = 1.0. Since Λ = 2 in these figures, the thermal and species buoyancy forces are of the same order of magnitude and assist each other. Pr is set as 7.0 corresponding to water and Sc = 0.78 implies a dominance of momentum diffusivity over species diffusivity.

Figure 2 Effects of K and γ on velocity profiles



Figures 2 to 9 show the effects of the dimensionless chemical reaction parameter γ and the vortex viscosity parameter K on the velocity, microrotation, temperature, and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number. It is observed that as the viscosity parameter K increases, the velocity profile decreases while the opposite happens with the absolute value of microrotation, temperature, and concentration profiles. Also, the velocity and concentration profiles decrease while the temperature profiles increase with increasing values of the dimensionless chemical reaction parameter γ. In addition, the absolute value of microrotation profile decreases as γ increases for $\eta \prec 1$ and increases for $\eta \succ 1$. On the other hand, it can be seen that the skin-friction coefficient, Nusselt number and the Sherwood number increase as the dimensionless time ξ increases whereas the opposite behaviours happen as K increases. In addition, it is found that the wall couple stress increases as either ξ and K increases. Furthermore, the effect of the chemical reaction parameter is observed to cause reductions in the skin-friction coefficient, Nusselt number and the wall couple stress and increases in the Sherwood number.

Figure 3 Effects of K and γ on microrotation profiles

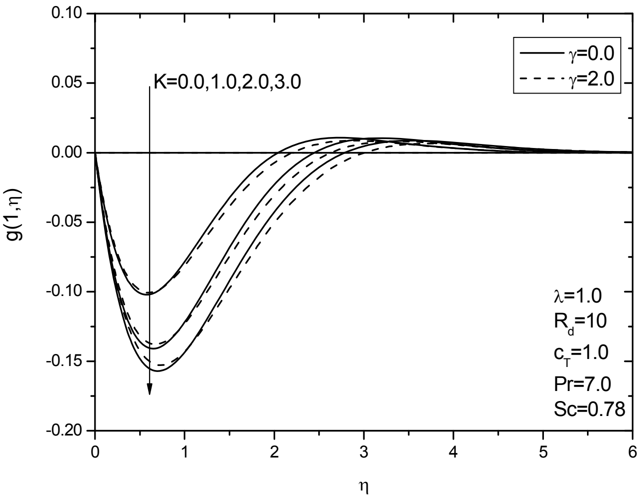
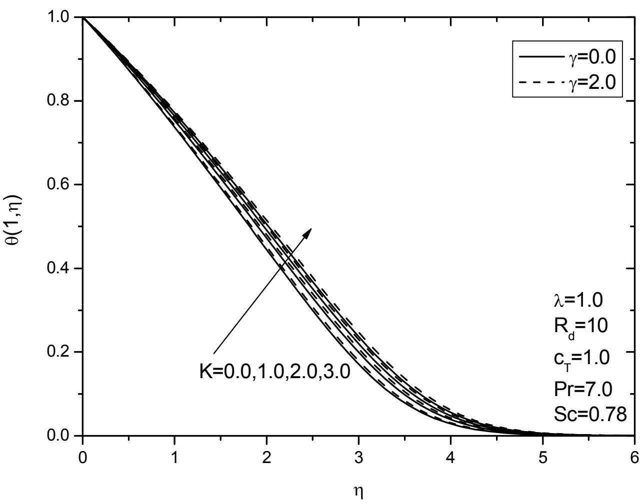


Figure 4 Effects of K and γ on temperature profiles



Figures 10 to 17 depict the influence of the mixed convection parameter λ and thermal radiation parameter R_d on the velocity, microrotation, temperature, and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number. We observe that increases in the values of λ and R_d cause strong increases in the velocity profile, absolute maximum values of microrotation, and the temperature profile while the reverse happens for the microrotation profile. Also, it is found that increasing λ and R_d leads to increases in all of the skin-friction coefficient, wall couple stress and the Sherwood number. Moreover, the Nusselt number increases as λ increases whereas the opposite happens as R_d decreases.

Figure 5 Effects of K and γ on concentration profiles

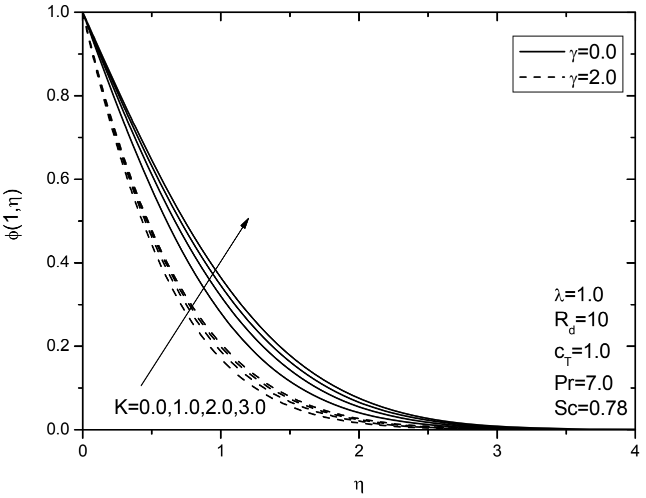


Figure 8 Effects of K and γ on the Nusselt number

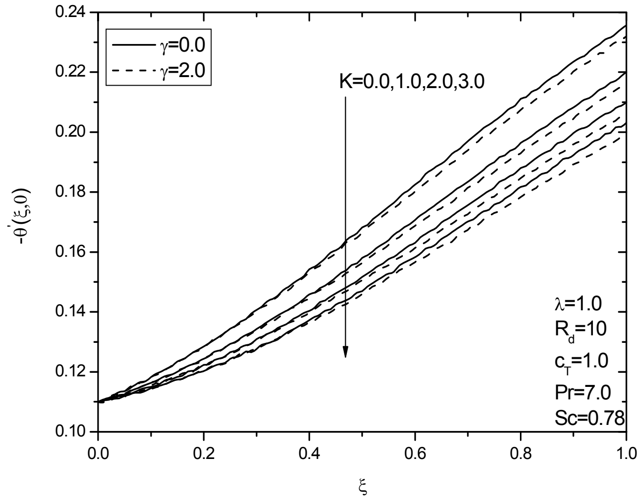


Figure 6 Effects of K and γ on the skin-friction coefficient

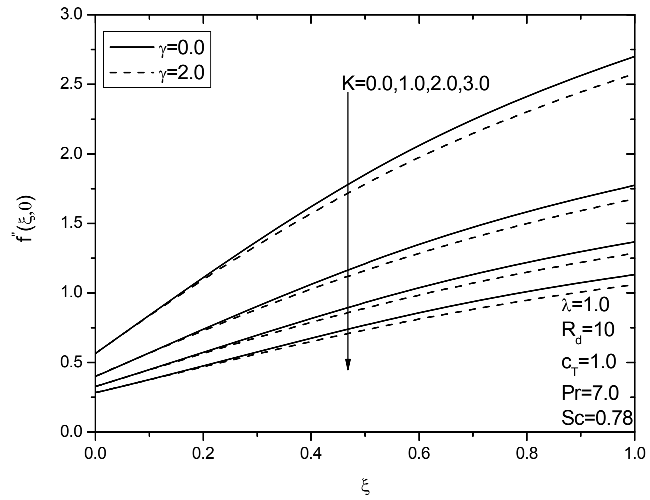


Figure 9 Effects of K and γ on the Sherwood number

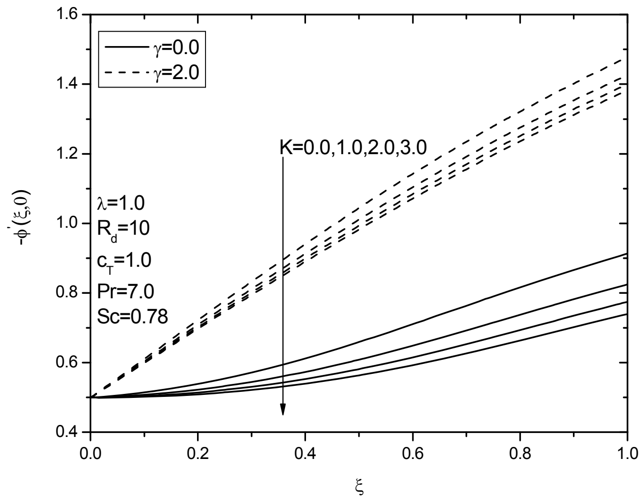


Figure 7 Effects of K and γ on the wall couple stress

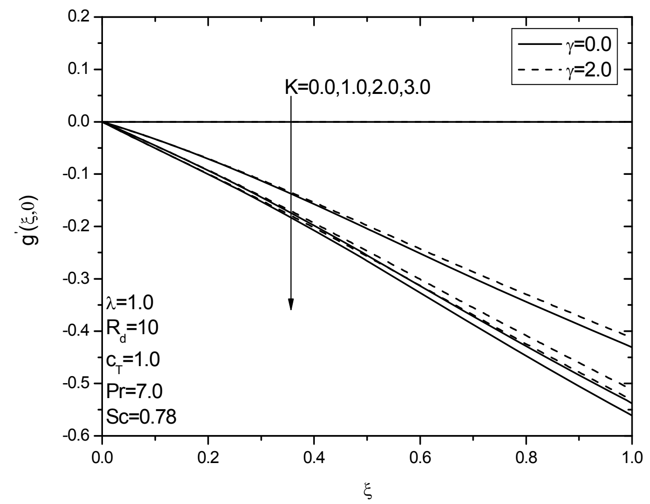


Figure 10 Effects of λ and R_d on velocity profiles

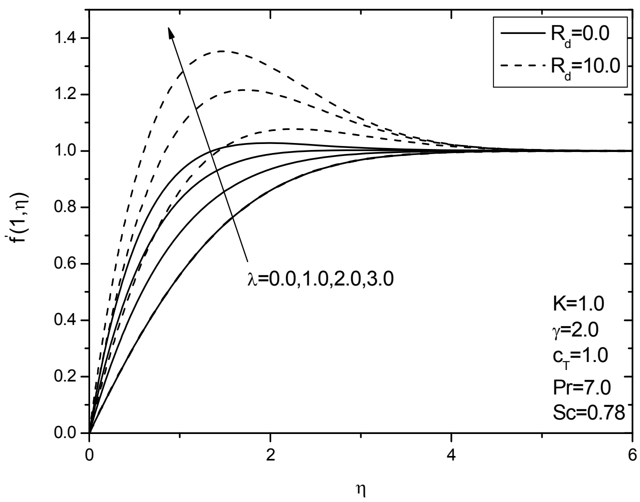


Figure 11 Effects of λ and R_d on microrotation profiles

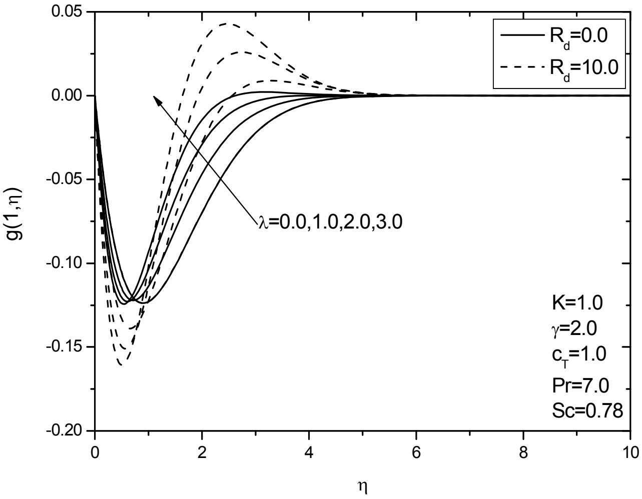


Figure 12 Effects of λ and R_d on temperature profiles

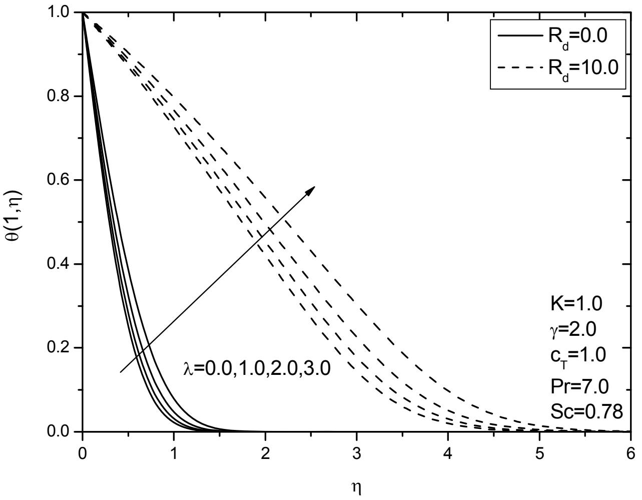


Figure 13 Effects of λ and R_d on concentration profiles

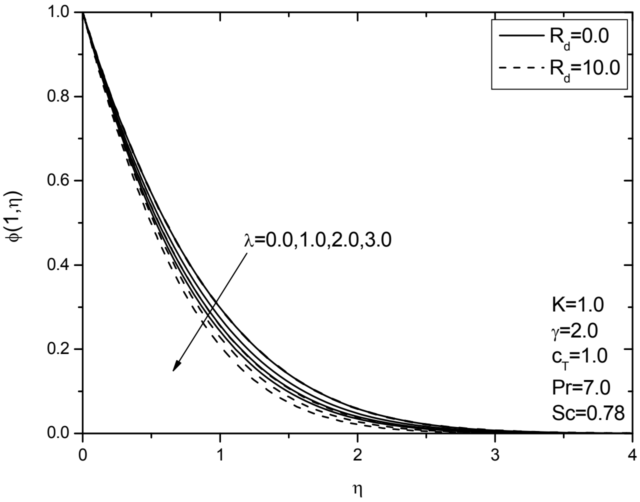


Figure 14 Effects of λ and R_d on the skin-friction coefficient

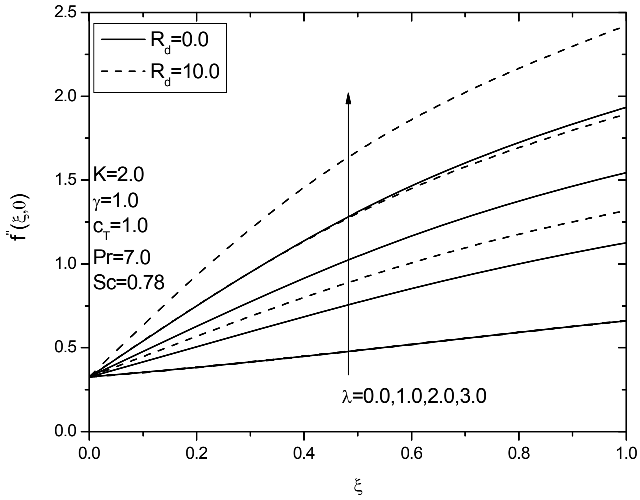


Figure 15 Effects of λ and R_d on the wall couple stress

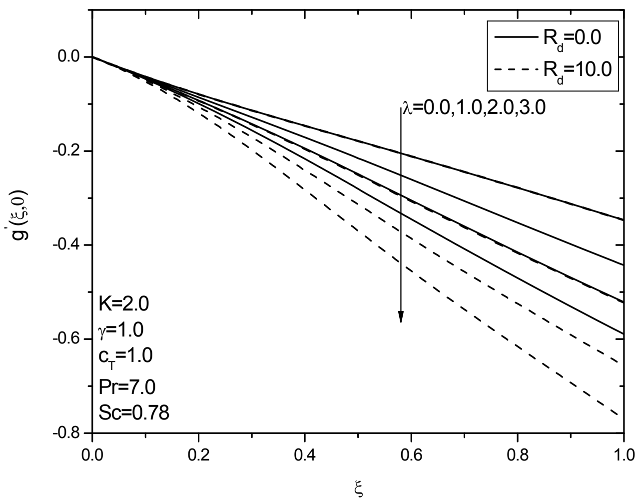


Figure 16 Effects of λ and R_d on the Nusselt number

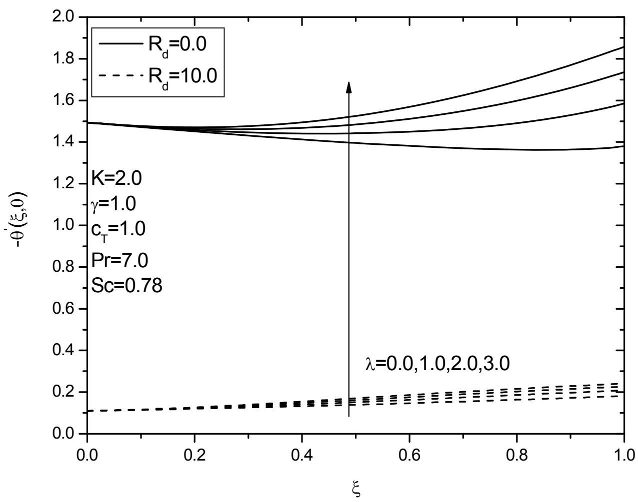
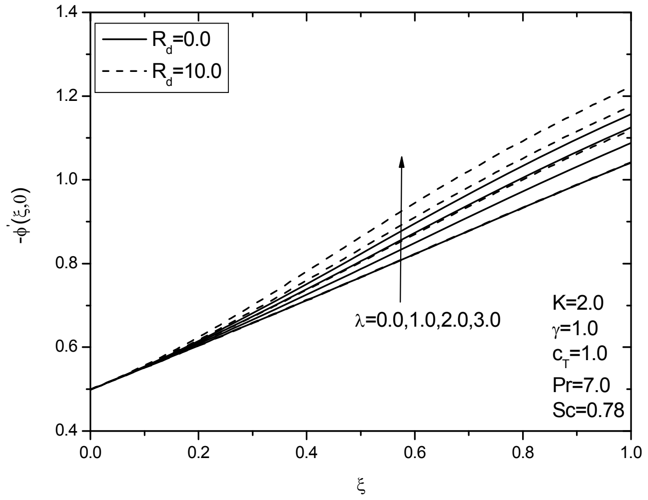


Figure 17 Effects of λ and R_d on the Sherwood number



5 Conclusions

In this paper, the effects of chemical reaction and thermal radiation on unsteady coupled heat and mass transfer by mixed convection in two-dimensional stagnation flows of a micropolar fluid on a vertical flat plate were studied. The governing boundary-layer equations were transformed into a non-similar form, and these equations were solved numerically. The effects of the mixed convection, thermal radiation, chemical reaction, and vortex viscosity parameter on the velocity, microrotation, temperature and concentration profiles as well as the skin-friction coefficient, wall couple stress, Nusselt number and the Sherwood number were shown graphically and discussed. It was found that increasing the micropolar fluid vortex viscosity parameter resulted in reductions in all of the skin-friction coefficient, Nusselt and Sherwood numbers, while the opposite happened for the wall couple stress. In addition, increasing the chemical reaction parameter led to decreases in the skin-friction coefficient, wall couple stress and the Nusselt number while the Sherwood number increased. Moreover, increasing the mixed convection parameter resulted in enhancements in all physical parameters such as the skin-friction coefficient, wall couple stress, and the Nusselt and Sherwood numbers. However, the presence of thermal radiation led to increases in the skin-friction coefficient, wall couple stress and the Sherwood number while the Nusselt number decreased.

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Nomenclature

a_r	Rosseland mean extinction coefficient
B	Spin gradient viscosity parameter
c_p	Specific heat at constant pressure
C_f	Local skin-friction coefficient
D	Mass diffusivity
f	Dimensionless stream function
g	Gravitational acceleration
h	Dimensionless microrotation
Gr	Grashof number, $\frac{g\beta_T T_0 L^3}{\nu^2}$
j	Microinertia density
K	Micropolar fluid vortex viscosity parameter
k_1	Thermal conductivity
k_c	Dimension of chemical reaction
L	Characteristic length
N	Component of the microrotation vector normal to x-y plane
Nu	Nusselt number
Pr	Prandtl number, ν/α
C_T	Surface temperature parameter
R_d	radiation-conduction parameter, $4\sigma T_\infty^3/\left[k\left(a_r+\sigma_s\right)\right]$
Re	Reynolds number $\frac{U_e L}{\nu}$
Sc	Schmidt number, ν/D
Sh	Sherwood number
T	Temperature
u	velocity component in the x-direction
U_e	Reference velocity
$u_e(x)$	Free stream velocity
v	Velocity component in the y-direction
x	Streamwise coordinate
y	Transverse coordinate
Greek symbols	
α	Thermal diffusivity
β_c	Coefficient of concentration expansion
β_T	Coefficient of thermal expansion
γ	Dimensionless of chemical reaction
γ^*	Spin-gradient viscosity
η	Pseudo-similarity variable
λ	(= constant) is the mixed convection parameter $\frac{Gr}{\mathrm{Re}^2}$
Λ	Concentration to thermal buoyancy ratio, $\beta_c(C_w - C_\infty)$ / $(\beta_T(T_w - T_\infty))$

Nomenclature (continued)

Greek symbols	
ϕ	Dimensionless concentration, $(C - C_\infty) / (C_w - C_\infty)$
θ	Dimensionless temperature, $(T - T_\infty) / (T_w - T_\infty)$
ξ	Dimensionless time
v	Kinematic viscosity
ρ	Density
Stefan-Boltzmann constant	
σ_s	Scattering coefficient
ψ	Stream function
Subscripts	
w	Condition at the wall
∞	Condition at infinity